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# IN MEMORIAM: BARRY COOPER
1943–2015

The logician and computability theorist Barry Cooper died on October 26, 2015, at the age of 72. Always a tireless campaigner and organiser, in the last few years of his life he had been heavily involved in setting up and acting as a coordinator for the Alan Turing year (and the ongoing Alan Turing years). He established the Alan Turing Centennial Advisory Committee, and played a large part in putting together many of the scores of events which formed the Turing year. The celebration had a global impact that few could have foreseen, eventually culminating in Turing's posthumous Royal pardon in 2013. During these last years of his life, Cooper travelled almost ceaselessly in his capacity as a spokesman for the Turing celebration, passionately espousing his views as to the significance of Turing's work and the fundamental role of computability theory in developing new scientific frameworks. Testament to his success in communicating these ideas, the volume *Alan Turing: His Work and Impact*, edited together with Jan van Leeuwen, won the Association of American publisher's prestigious R.R. Hawkins award.

Raised in Bognor Regis, England, Cooper studied mathematics at Oxford, graduating in 1966. He then went to Leicester to study for a Ph.D. under Reuben Goldstein. Since his principal interest was in the structural theory of the Turing degrees, however, he ended up working mainly with Mike Yates in Manchester, who was then the only established UK researcher in that field. He was appointed as a Lecturer at the University of Leeds in 1969, where he was to remain throughout his career. Becoming a Professor in 1996, he was also later awarded an Honorary Degree from Sofia University in 2011. During his career Cooper spent much time on research visits abroad. Later in life he often recalled very fondly his time at the University of California, Berkeley (1971–73), which was both an important formative period for his research in computability, but also for his left-wing politics which found kindred spirit with the civil rights movement and student activism which were such a part of Berkeley life at that time. Upon returning to the UK in 1973, he became very actively involved in the Chile Solidarity Campaign and is still well remembered amongst the Chilean community in Leeds and beyond for his role in coordinating the housing of refugees coming out of that crisis.

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Within the computability theory community, Cooper was known for his deep and technically complex research. A central focus of his work for many years revolved around one of the most significant open questions in the area, the bi-interpretability conjecture, and the question as to whether or not there exist nontrivial automorphisms of the Turing degrees (T-degrees). For philosophical reasons, he held the deep seated belief that nontrivial automorphisms should exist, and this was to be a question which would sustain his interest until the very end. A number of his theorems establishing basic structural properties of the T-degrees are now regarded as classics in the area. For a set of natural numbers $A$ of degree $\boldsymbol{a}$, the jump of $\boldsymbol{a}$ (denoted $\boldsymbol{a}'$), is defined to be the degree of the halting problem relative to $A$. A degree $\boldsymbol{a} > \mathbf{0}$ is minimal if there does not exist $\boldsymbol{b}$ with $\mathbf{0} < \boldsymbol{b} < \boldsymbol{a}$. In the global structure of the T-degrees, one of Cooper's best known results ([2]) asserts that every degree above $\mathbf{0}'$ is the jump of a minimal degree. Much of his work in the T-degrees was also centred around priority arguments establishing structural properties of the "local" degree structures, i.e., the c.e., or other sublcasses of the $\Delta^0_2$ degrees. He initiated the degree-theoretic analysis of the sets in the Ershov hierarchy, by showing in his thesis that there are properly d-c.e. degrees. Together with Harrington, Lachlan, Lempp, and Soare ([10]), he showed that there exist maximal incomplete d.c.e. degrees. He also made a very major contribution to the study of these local structures (together with Soare) by introducing the *jump classes*, which have become a central object of study, and one of the basic tools by which one may characterise the properties of degrees. A memorable result in this regard is that every high c.e. T-degree bounds a minimal pair of c.e. degrees ([3]).

A special interest of Cooper was in enumeration reducibility, and more generally in the so-called positive reducibilities, i.e., those reducibilities arising from models of relative computability in which only "positive" oracle information is available. These are models, in other words, in which questions of the form "is $x \in A$?" (for an oracle $A$), can in general only be answered if $x$ does belong to $A$. Another way of looking at positive reducibilities is by using models of relative computation in which partial functions are computed starting from oracle information stored, in turn, in a partial function. For this reason, the enumeration degrees (e-degrees) are also known as partial degrees. This interest dated back to when Cooper was in Berkeley, at a time when there was an ample philosophical debate about models of computation using imperfect, or partial, oracle information, and many important problems concerning positive reducibilities, such as the existence of minimal degrees, or density, were still open. Cooper was also very much intrigued by the fact that the T-degrees are a substructure of the e-degrees (they correspond exactly to the partial degrees which are "total"), and regarded as very important the task of exploring the context of the T-degrees within the larger degree structure.

In the early 80s, Cooper and his student McEvoy laid down the foundations for a revival of studies and interest in positive reducibilities, and in particular in enumeration reducibility. They introduced a jump operation on the e-degrees, and studied the resulting jump classes. This jump operation

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extends the Turing jump, if we regard the T-degrees as embedded in the e-degrees. Cooper put a considerable effort into studying the local structure, i.e., the e-degrees below the first jump, and he proved that the local structure is dense ([4]), although he later proved that the full structure is not dense (the proof of this result appeared as a sketch in Cooper's influential survey paper [6]; a complete and independent proof of nondensity was later given by Calhoun and Slaman). The interest of Cooper in density/nondensity problems for positive reducibilities in general is witnessed by the paper [5] which is still a valuable sorce of information and problems. Together with Copestake [9], he helped to clarify the distribution of the properly $\Sigma^0_2$ e-degrees within the local structure, proving in particular that below every high e-degree $\boldsymbol{h}$ there is an e-degree which is incomparable with all $\Delta^0_2$ e-degrees below $\boldsymbol{h}$, except $\mathbf{0}_e$, and $\boldsymbol{h}$. A great deal of what is known about the local structure is due to work done by Cooper himself, or coauthored with other collaborators: the existence of noncappable e-degrees ([13]); the existence of noncuppable e-degrees and the fact that they are downwards properly $\Sigma^0_2$ ([14]; the existence of nonbounding e-degrees and the fact that they are all downwards properly $\Sigma^0_2$ ([11]); the fact that every total degree in the local structure is the top of a diamond ([1]); there is a $\Pi^0_1$ e-degree $\mathbf{a} <_e \mathbf{0}'_e$ such that there are no $\mathbf{b}, \mathbf{c}$ with $\mathbf{b} \in \Pi^0_1$ and $\mathbf{c} \in \Sigma^0_2$ such that $\mathbf{a} \leq_e \mathbf{b}, \mathbf{c} <_e \mathbf{0}'_e$ and $\mathbf{b} \cup \mathbf{c} = \mathbf{0}'_e$ ([15]). (Note that the computably enumerable T-degrees embed onto the $\Pi^0_1$ e-degrees, and thus this result yields as a particular case the classical Harrington Non-Splitting Theorem. Results of this type, exploiting the context of the T-degrees within the e-degrees were particularly liked by Cooper.)

In 1994 Cooper was the main promoter of the European network *Complexity, Logic and Recursion Theory*, which grouped together several research centers in Europe, and in countries from the former Soviet Union. This very successful experience (which terminated in 1997) convinced him of the necessity of creating a larger European network, to form a reference point to all scholars (mathematicians and logicians, but also scientists from other disciplines, and philosophers) whose research is somehow related to computability and its interdisciplinary character. This was the origin of *Computability in Europe*, from 2008 a formal association, with members from all over the world, in which Cooper, as a dedicated founding father, has put so much enthusiasm and work, and of which he was president from 2012 until 2015. Among the achievements and the activities of the association, suffice it to list the annual conference series *Computability in Europe*, the book series *Theory and Applications of Computability*, the journal *Computability*.

In 2004, together with Jin-yi Cai and Ansheng Li, Cooper promoted and developed another series of conferences, *Theory and Applications of Models of Computation*, which are held annually, mostly in Asian universities and research centres. TAMC was the natural offspring of the International Joint Project *New Directions in the Theory and Applications of Models of Computation*, 2002–2006, based in China, in which Cooper was the main European scientist. The TAMC conferences nowadays bring together scholars and specialists in computability, complexity, algorithms, and models of

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computation, and, together with the CiE meetings, witness better than anything else Cooper's vision of computability as the unifying core of a vaster cultural and scientific interdisciplinary enterprise.

Later in his academic career, Cooper was particularly concerned with the applications of computability theory in forming new scientific frameworks [12], [7], [8]. He lamented the fact that most specialist recursion theorists have been concerned exclusively with mathematical questions, and argued that computability theory provides an appropriate setting and methodology for addressing gaps in our scientific worldview concerning interactions between the local and the global, and what appear to be breakdowns in the reductive structures commonly relied on in science and epistemology. In contexts where the deterministic structures we rely on are punctuated by what might be described as phase transitions between different levels of familiar relationships, Cooper argued that:

> "..the notion of mathematical definability is the key mathematical concept, with the potential to clarify a broad range of fundamental problems. Many questions in quantum theory, proof theory, and epistemology can be best understood as a breakdown of definability in an appropriate underlying mathematical model. At the same time, other mysteries, such as how the classical universe escapes the underlying quantum ambiguity, and how natural laws arise, can be traced back to the right notion of definability in the right mathematical structure."

According to Cooper, that "right mathematical structure" is to be provided by the T-degrees, or other degree structures arising in the context of computability. Accordingly, questions concerning the existence of nontrivial automorphisms for these degree structures, should be regarded as issues of significant scientific as well as mathematical interest.

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