Seventh Meeting, May 8, 1891.

R. E. ALLARDICE, Esq., M.A., F.R.S.E., President, in the Chair.

On the solitary wave.

By JOHN M'COWAN, M.A., B.Sc.

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On some applications of the pedal line of a triangle.

By Professor A. H. ANGLIN.

1. Taking the two following known properties of the pedal line of a triangle, viz.:

I. *The locus of a point, such that the feet of the perpendiculars from it on the sides of a triangle are collinear, is the circum-circle of the triangle;*

II. *The pedal line bisects the distance between the orthocentre and the corresponding point in the circumference of the circum-circle;*

we may apply them to establish the following known theorems:—

(1.) *The circum-circle of the triangle formed by three tangents to a parabola passes through the focus.*

For, the feet of the perpendiculars from the focus on the tangents lie on a straight line, viz., the tangent at the vertex; hence by (I.) the focus is on the circumference of the circum-circle.

(2.) *The orthocentre of the triangle is on the directrix.*

For, if O be the orthocentre, by (II.) SO is bisected by the pedal line corresponding to S, that is, by the tangent at the vertex; hence, if OX be perpendicular to the axis, SA = AX, and therefore OX is the directrix.

2. We may also, by reference to the pedal line, *find the equation to the circum-circle of a triangle formed by three lines the equations to which are given in Cartesian co-ordinates.*

We have to express analytically the fact that the area of the triangle formed by joining the feet of perpendiculars on the sides of the given triangle from the point (x, y) is zero, since it becomes a pedal line of the given triangle.

If, therefore, DEF be the pedal line of the triangle ABC corresponding to the point P, we have

$$\Delta PFE + \Delta PDE - \Delta PFD = 0,$$

that is,

$$PF \cdot PE \sin A + PD \cdot PE \sin C - PF \cdot PD \sin B = 0\,;$$

so that, if the equations to the sides of the triangle ABC be of the form $x\cos\alpha + y\sin\alpha - p = 0$, the equation to the circum-circle is

$$(x\cos\alpha_2 + y\sin\alpha_2 - p_2)(x\cos\alpha_3 + y\sin\alpha_3 - p_3)\sin(\alpha_2 - \alpha_3)$$
$$+ \text{two similar terms} = 0.$$

If the equations to the sides of the triangle be given in the form $ax + by + c = 0$, the equation to the circum-circle is

$$(a_2b_3 - a_3b_2)(a_1^2 + b_1^2)(a_2x + b_2y + c_2)(a_3x + b_3y + c_3)$$
$$+ \text{two similar terms} = 0.$$

[We may also show absolutely that this equation does represent the circum-circle. For the locus represented by it goes through A, B, C, since the co-ordinates of each obviously satisfy the equation; and, further, the equation represents a circle, since on examination it will be found that the coefficients of x^2 and y^2 are equal, while that of xy is zero.

The pedal line of a triangle and theorems connected with it.

By George A. Gibson, M.A

Most of the following theorems occur in a more or less explicit form in text-books on the geometry of the parabola; but it will not, I hope, be without interest and value to consider them independently, and to prove them by using only the propositions of Euclid.

1. If the perpendiculars AD, BE, CF of a triangle ABC are produced to meet the circumcircle of the triangle in X, Y, Z respec-

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