

# Influence of the astrometric accuracy of observation on the extrapolated ephemerides of natural satellites

J. Desmars<sup>1</sup>, J.-E. Arlot<sup>1</sup>, A. Vienne<sup>1,2</sup>

<sup>1</sup>Institut de Mécanique Céleste et de Calcul des Éphémérides - Observatoire de Paris, UMR 8028 CNRS, 77 avenue Denfert-Rochereau, 75014 Paris, France  
email: [desmars@imcce.fr](mailto:desmars@imcce.fr)

<sup>2</sup>Université de Lille, 59000 Lille, France

**Abstract.** The accuracy of planetary satellites ephemerides is determined not only by the accuracy of dynamical model (internal accuracy) but also by the accuracy of the observations (external accuracy) used to fit the initial parameters of a model. This external accuracy extrapolated in the future is unknown most of the time and tends to degrade the global accuracy of ephemerides. Even if we can estimate the quality of the ephemerides by comparison with observations, we do not know how to determinate the evolution of the accuracy outside the period of observations. We will present a statistical method, resampling of observations, which allows a better estimation of the extrapolated accuracy in the future.

**Keywords.** planets and satellites: individual (Saturn), astrometry, ephemerides

---

## 1. Introduction

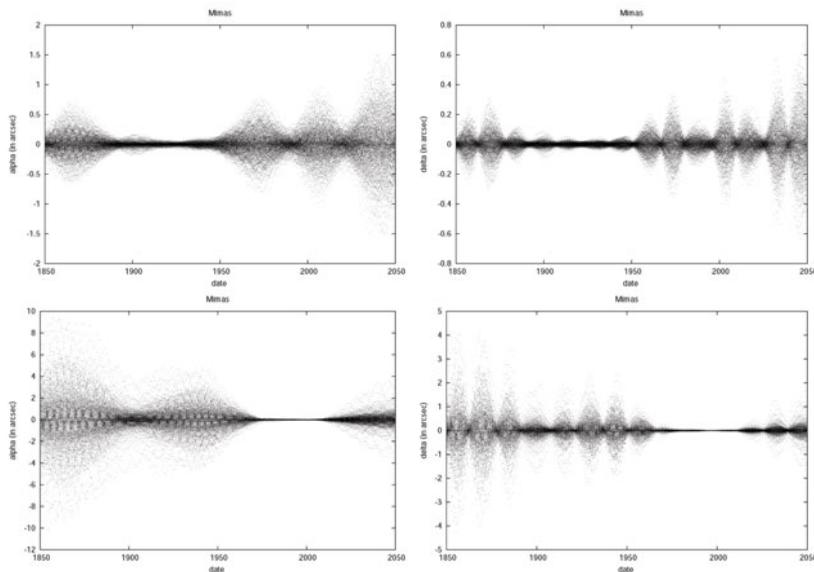
The accuracy of the ephemerides is determined by the accuracy of the model (internal accuracy) which is often well estimated and quite good, and by the accuracy of the observations (external accuracy) which depends on the quality and the distribution of the observations, and degrades the global quality of ephemerides. During the observational periods, the accuracy can be determined by the difference between observed positions and computed positions (O-C) and remains quite good. Outside the observational periods, the accuracy deteriorates and its estimation remains difficult. So the question is to know how to estimate the real accuracy of ephemerides outside the observational periods. The problem has already been studied for asteroids. Muinonen & Bowell (1994), Virtanen *et al.* (2001) use statistical methods to determine the orbital uncertainties of asteroids. They succeeded in rediscovering asteroids observed in past but lost. But our problem is quite different because asteroids have a slow motion and few observations exist, whereas the natural satellites have a fast motion and are frequently observed. Statistical methods, however, remain a good way of study.

## 2. Statistical methods

One of the statistical methods is the bootstrap resampling (Efron & Tibshirani, 1993). Bootstrap samples are generated from the original data set (with  $N$  elements). Each bootstrap sample has  $N$  elements generated by sampling with replacement  $N$  times from the original set. Thus, a great number of new samples is allowed.

Here we use TASS model of main Saturnian satellites (Vienne & Duriez, 1995). The parameters of the model have been fitted to each sample, positions have been computed during 1850-2050 period and compared with positions computed with initial parameters. A catalogue of real observations of Saturnian satellites from 1874 to 2007 has been

generated. Two observational periods can be separated. The first one, old observations, from 1874 to 1947, with a priori lower quality and the second one, recent observations from 1961 to 2007 with a priori better quality. The differences between positions in  $(\alpha, \delta)$  coordinates for Mimas are represented in Fig1. Fifty bootstrap samples were used. The differences are not very important during the observational period but becomes important outside the period. For the fit of old observations, the differences can reach 1.5" in  $\alpha$ . For the fit to recent epochs, the differences are even more important ( $\sim 10''$ ). It is amazing since recent observations are a priori better than the old ones. This difference can be explained because the old observational period stretches from 1874 to 1947 (73 years) whereas recent observations stretch from 1961 to 2007 (46 years). So, a long period of average observations seems to be better than a short period of accurate observations, for a better accuracy outside the observational period.



**Figure 1.** Difference for 50 bootstrap samples of old and recent observations in  $(\alpha, \delta)$  for Mimas

### 3. Conclusion and perspectives

Bootstrap resampling allows us to estimate the accuracy of ephemerides. The positions of satellites are not so accurate outside the observational period, especially if the model has been fitted on a short period of observations. Other statistical methods also enable to estimate the accuracy. For example, jackknife consists in resampling the original set by leaving out one observation at a time. Such a method leads us to define "one" observation and more precisely when two observations are independent. A next study on this problem of independence of observations will be undertaken.

### References

- Efron, B. & Tibshirani, R. J. 1993, in: Monographs on Statistics and Applied Probability *An Introduction to the Bootstrap*  
 Muinonen, K. & Bowell, E., 1993, *Icarus* 104, 255  
 Vienne, A. & Duriez, L., 1995, *A&A* 297, 588  
 Virtanen, J., Muinonen, K & Bowell, E., 2001, *Icarus* 154, 412