Hence
$$\frac{\frac{dX}{dt}}{X}=\frac{\frac{dY}{dt}}{Y}=\frac{\frac{dZ}{dt}}{Z}=\frac{\frac{dU}{dt}}{U};$$

$$\therefore \qquad \frac{X}{a}=\frac{Y}{b}=\frac{Z}{c}=\frac{U}{d}$$

where a, b, c, d are constants,

and since $$xX+yY+zZ+uU=0,$$

we have $$ax+by+cz+du=0,$$

which is the result required.

J. E. A. STEGGALL.

On the Solutions of $\dfrac{d^2u}{dt^2}+P\dfrac{du}{dt}+Qu=0.$

(Suggested in discussion at a meeting of the Society).

The following note gives a proof of the theorem that if x, y, z are any three values of u which satisfy the differential equation $\dfrac{d^2u}{dt^2}+P\dfrac{du}{dt}+Q=0$, then $lx+my+nz=0$ where l, m, n are constants. If the dots denote differentiation with respect to t, we have

$$\ddot{x}+P\dot{x}+Qx=0, \quad \ddot{y}+P\dot{y}+Qy=0 \quad \text{and} \quad \ddot{z}+P\dot{z}+Qz=0.$$

Hence
$$\begin{vmatrix} x & y & z \\ \dot{x} & \dot{y} & \dot{z} \\ \ddot{x} & \ddot{y} & \ddot{z} \end{vmatrix}=0. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Now x, y, z are functions of t, and therefore the locus of the point (x, y, z) is a curve in space. Equation (1) is the condition that the osculating plane at (x, y, z) should pass through the origin. But if the osculating plane at any point of a curve passes through a fixed point, the curve is a plane curve lying in a plane through the point. Therefore the point (x, y, z) lies in a plane through the origin or $lx+my+nz=0$.

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