Abstract: Variations in ambient density can largely account for: slope and extent of the observed $\Sigma$-D relations, the slope of about +1 of the cumulative N-D relation, and the increase of $E_0$ with D as derived from optical line data. $E_{\text{min}}$ increases during the evolution; the behaviour of $E_X$ remains uncertain.

Introduction

A sample of 84 SNRs at known distances in the Galaxy, the Magellanic Clouds, M31 and M33 observed at radio wavelengths formed the basis for a comparison of data at 1 GHz with X-ray data and [SII]-line data (Table 1 in Paper I = Berkhuijsen, 1986). A close correlation between the surface brightnesses in radio ($\Sigma_R$) and X-rays ($\Sigma_X$) was found, and a study was made of various SNR-properties as a function of apparent ambient density $n_0$ as derived from the X-ray data. A review of significant relationships is given in Table 1, some of which are discussed below. Details may be found in Papers I, II (Berkhuijsen, 1987) and III (Berkhuijsen, 1988).

$\Sigma$-D relations

Evolutionary models of X-ray SNRs (Fusco-Femiano and Preite-Martinez, 1984) indicate that during the adiabatic phase $\Sigma_X$ is fairly constant until it drops sharply at the beginning of the radiative phase. Similarly, since $\Sigma_R = \Sigma_X^{0.7}$, radio remnants may evolve at nearly constant $\Sigma_R$ through the observed strip in the $\Sigma_R$-$D_R$ diagram until they reach the maximum observable diameter at the beginning of the radiative phase. This conclusion is confirmed by the increase of the minimum energy in relativistic particles and magnetic fields with radius after correction to $n_0 = 1 \text{ cm}^{-3}$ (Fig. 1, Table 1), i.e. $E_{\text{min}}(n_0=1) = R_R(n_0=1)^{2.7 \pm 0.3}$, which implies $\Sigma_R(n_0=1) = R_R(n_0=1)^{0.9 \pm 0.5}$.

The slopes of the $\Sigma$-D relations observed in radio continuum and X-rays then may not reflect evolutionary tracks. Instead, the slopes of the distributions and part of the variations in $\Sigma$ and D can be explained by the dependence of $\Sigma$ and D on apparent ambient density $n_0$. It is remarkable that data of SNR-candidates in M82 and radio supernovae are consistent with this picture (Paper I).
Fig. 1: Dependence of $E_{\text{min}}(n_0=1)$ on $R_R(n_0=1)$ for shell-type SNRs. Two lines of constant $\Sigma_R$ are shown.

Table 1. Review of significant$^a)$ correlations for shell-type SNRs$^b)$
Orthogonal (o) and normal (n) regression lines

<table>
<thead>
<tr>
<th>Y</th>
<th>X</th>
<th>#pp. o/n</th>
<th>Slope (Y on X)</th>
<th>Zero point (Y on X)</th>
<th>Correl. coeff.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\log \Sigma_R$</td>
<td>$\log \Sigma_X$</td>
<td>37 o</td>
<td>0.58$\pm$0.07</td>
<td>-39.03$\pm$2.49</td>
<td>0.77$\pm$0.06</td>
</tr>
<tr>
<td>$\log \Sigma_R$</td>
<td>$\log n_0$</td>
<td>33 o</td>
<td>1.13$\pm$0.17</td>
<td>-19.34$\pm$0.11</td>
<td>0.75$\pm$0.08</td>
</tr>
<tr>
<td>$\log D_R$</td>
<td>$\log n_0$</td>
<td>33 o</td>
<td>-0.37$\pm$0.04</td>
<td>1.19$\pm$0.02</td>
<td>0.84$\pm$0.05</td>
</tr>
<tr>
<td>$\log E_{\text{min}}$</td>
<td>$\log n_0$</td>
<td>32 o</td>
<td>-0.54$\pm$0.12</td>
<td>49.69$\pm$0.03</td>
<td>0.62$\pm$0.11</td>
</tr>
<tr>
<td>$\log E_{\text{min}}(n_0=1)$</td>
<td>$\log R_R(n_0=1)$</td>
<td>33 n</td>
<td>2.72$\pm$0.31</td>
<td>47.22$\pm$0.28</td>
<td>0.84$\pm$0.05</td>
</tr>
<tr>
<td>$\log T_S$</td>
<td>$\log R_X(n_0=1)$</td>
<td>23 n</td>
<td>-1.57$\pm$0.33</td>
<td>1.62$\pm$0.28</td>
<td>0.71$\pm$0.11</td>
</tr>
<tr>
<td>$\log E_X$</td>
<td>$\log R_X(n_0=1)$</td>
<td>23 n</td>
<td>1.31$\pm$0.36</td>
<td>49.21$\pm$0.31</td>
<td>0.61$\pm$0.13</td>
</tr>
</tbody>
</table>

Units: $\Sigma_R$ in $\text{W Hz}^{-1} \text{m}^{-2} \text{sr}^{-1}$, $\Sigma_X$ in $\text{erg s}^{-1} \text{pc}^{-2}$, $n_0$ in $\text{cm}^{-3}$, $D_R$ and $R_R(n_0=1)$ in pc, $E_{\text{min}}(n_0=1)$ and $E_X$ in $\text{erg}$, $T_S$ in keV.

a) level of significance $P < 0.0027$; b) without Cas A; c) for 14 objects with known $T_S$. 

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N-D relation
The observed exponent of about +1 of the cumulative N-D relations could result from a random distribution of diameters, which may be largely ascribed to variations in $n_0$. After correction to $n_0 = 1 \, \text{cm}^{-3}$ the exponent is considerably larger than +1. For a total sample of 74 radio diameters the exponent of 2.5±0.7 suggests adiabatic expansion of the remnants in the sample. However, the statistical significance does not permit definite conclusions on their average expansion law (Paper II).

Energy content
The total energy in relativistic particles and magnetic fields, $E_{\text{tot}}(n_0=1)$, would be higher than $E_{\text{min}}(n_0=1)$ if the energy in particles were very different from that in magnetic fields. Indeed, in Kepler's SNR
(Matsui et al., 1984) the ratio of energy densities in particles and fields is much higher than in the ISM suggesting a decrease of this ratio with \( R_R(n_0=1)^{-1.5} \) during the evolution of a remnant. In this case the total energy in particles and fields possibly varies as \( E_{tot}(n_0=1) \propto R_R(n_0=1)^{2.0} \).

The thermal energy seen in X-rays was obtained from shock temperatures \( T_S \) derived from X-ray spectra by means of adiabatic models. The dependences of \( T_S \) and \( E_X \) on \( R_X(n_0=1) \) are shown in Figs. 2 and 3. Gronenschild and Mewe (1982) have pointed out that the low \( T_S \)-values in young remnants and the high \( T_S \)-values in old remnants found from CIE models may not apply. Disregarding such CIE points Figs. 2 and 3 yield \( T_S \propto R_R(n_0=1)^{-1.6^{\pm.3}} \) and \( E_X \propto R_R(n_0=1)^{1.3^{\pm.4}} \) (Table 1), whereas for adiabatic remnants exponents of -3 and 0 are expected, respectively.

The initial kinetic energy \( E_0 \) as derived from [SII]-line ratios increases with \( R^3 \), but is independent of \( R(n_0=1) \) in agreement with the assumption of adiabatic expansion. This result may be fortuitous, however, since for adiabatic remnants \( N_0(\text{SII}) \propto R(n_0=1)^{-3} \) should hold, whereas no dependence of \( N_0(\text{SII}) \) on \( R(n_0=1) \) was found. This disagreement may be caused by the inclusion of values of \( N_0(\text{SII}) \) for filaments interior to the shell not representative for shell conditions (Preite-Martinez, 1985).

The lack of direct proof for adiabatic expansion casts doubt on the reliability of estimates of \( E_X \) and \( E_0 \) obtained above. An estimate of the mean value of \( E_{tot}(n_0=1) \) indicates that a mean value of \( E_0 \approx 2 \times 10^{51} \text{ erg} \) might be consistent with the data (Paper III).

References