# THE

# **JOURNAL**

# OF

# SYMBOLIC LOGIC

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## **ERRATA**

## **VOLUME 34**

Page 310, line 12. For «Manningfaltigkeit», read «Mannigfaltigkeit».

Page 706, column 2, line 7. For «Manningfaltigkeit», read «Mannigfaltigkeit».

Page 742, column 1, line 9. For «XXXIII 120», read «XXXII 120».

## **VOLUME 35**

Page 148, line 20. For «[EPL] VII 170; [IFL] XXXV 115; Logic, combinatory», read «[IFL] VII 170; [EPL] Logic, combinatory».

#### **VOLUME 36**

Page 185, line 14. For «L. Johnathan Cohen», read «L. Jonathan Cohen».

Page 799, column 2, line 21. For «algebra», read «logic».

### **VOLUME 38**

Page 523. In the displayed formula, the subscript on the last occurrence of  $\psi$  should be i.

Page 536, lines 16, 13, 6, and 5 from below. Delete «Tłumaczyl».

Page 573. Between lines 5 and 6, for «K\*\*», read «\*K\*».

#### **VOLUME 39**

Page 303, lines 21-20 from below. For «segments up to the cofinality (in  $L_{\alpha}$ ) of  $\theta_{\nu}$ », read «segments  $\leq \theta_{\nu}$  ( $\varepsilon$  if  $\theta_{\nu}$  is singular)».

Page 303, lines 18 and 16 from below. For  $(e^{\eta}(\theta^{\eta}_{\nu}))$ , read  $(\theta^{\eta}(\epsilon))$ .

Page 304, line 4. For  $(\epsilon(\theta_{\nu}))$ , read  $(\theta_{\nu}(\epsilon))$ .

Page 304, lines 6 and 8. For  $(cf(\theta_v))$ , read  $(\theta_v)$ .

Page 335, line 4. For «structure», read «structures».

Page 366, line 30. For «XXXIX 355», read «XXXIX 355(2)».

Page 366, line 33. For «XXXIX 355», read «XXXIX 355(3)».

Page 621, line 13. Title should read \*Logic and ordinary language. Introduction ».

Page 622, line 14. This is a review of *The philosophy of mathematics*, edited by Hintikka, and not of *The philosophy of mathematics*, edited by Benacerraf and Putnam.

## **VOLUME 40**

Page 243, line 16 from below. For «separation», read «reduction»; and for «separates», read «reduces».

Page 261, lines 6 and 4 from below. For «XXXI 662(1)», read «XXXI 518(8)».

Page 262, line 1. For «XXXI 662(1)», read «XXXI 518(8)».

Page 267, line 10. For «WEINER», read «WIENER».

Page 268, line 13. For «XXXIX 609(2)», read «XXXIX 609(4)».

Page 271, line 4. For «XXXIV 552(10)», read «XXXIV 522(10)».

Page 272, lines 6-5 from below. For «Grossman», read «Grossmann».

Page 272, line 2 from below. For "Donald Lewis", read "David Lewis".

Page 294, line 1. Add period at end of line.

Page 297, line 15. For «H. G. SABADAŠ», read «N. G. SABADAŠ».

Attention is called to the fact that the JOURNAL invites submission of (1) original technical papers in the field of symbolic logic, (2) expository papers in this field, (3) philosophical papers which bear upon this field or make use of its methods, and (4) studies in the history of logic in which modern technical developments are taken into account.

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