Thus the philosophical theories at issue are construed as differing in what each takes logic to be. In a not dealt with in any detail, although the book is not lacking in interesting and provocative historical school would recognize any given reconstruction as a more or less faithful model of its own philosophy is conceptualistic, and realistic philosophical theories. The question of whether any given philosophical sense, therefore, this book replaces the idea that logic is philosophically neutral with the contrary idea truth-valuation function). Each philosophical theory reconstructed in this book is represented by a logic. (set of interpretations each with a (set of sequences of formulas), and a (semantic system)

deductive system

semantic system

logic in such a science? What are its explananda? What constraints are there on the explanans? Although many of the detailed discussions are helpful, the overall treatment is not very illuminating, being somewhat diffuse. Here, perhaps, the different sources have not been welded into a persuasive whole. Despite this complaint, the presentation overall is clear and vivid; the field is new enough to afford numerous intriguing prospects, and this volume is a valuable guide.


This book continues the logico-philosophical tradition which was founded by Frege and developed by Russell, Gödel, Carnap, Tarski, Quine, and others and which treats philosophical problems using devices and results of mathematical logic. The problems at issue here involve predicability, universality, and individuality. The book draws on ancient thought in taking a universal to be “predicable of things” and in taking an individual to be, as one might say, subjectable to predication. But it gives individuality a modern form by taking an individual to be a value of a bound individual variable. One of the issues that pervades the book is a traditional form of “the” problem of universals, viz. whether a universal is a linguistic expression, whether it is a “concept” (or mental idea), or whether it is an objective property or relation existing apart from language and thought. Nominalism, conceptualism, and realism are the traditional labels for these three views, each of which has many variants. In view of the fact that communication of predication involves not only an expression but also an intension to be conveyed and an extension to be referred to, intermixtures of the three views are to be expected. In Chapters III and V combined views are explicitly considered.

The book does not propose to solve the problem of universals or even to lend support to one or another philosophy. Rather, in a spirit of tolerance, objectivity, and neutrality sometimes associated with Rudolf Carnap, one of the author’s teachers, the book proposes logical reconstructions of various nominalistic, conceptualistic, and realistic philosophical theories. The question of whether any given philosophical school would recognize any given reconstruction as a more or less faithful model of its own philosophy is not dealt with in any detail, although the book is not lacking in interesting and provocative historical allusions.

In this review a logic is a three-part system composed of a language (set of sentences and formulas), a deductive system (set of sequences of formulas), and a semantic system (set of interpretations each with a truth-valuation function). Each philosophical theory reconstructed in this book is represented by a logic. Thus the philosophical theories at issue are construed as differing in what each takes logic to be. In a sense, therefore, this book replaces the idea that logic is philosophically neutral with the contrary idea that it is through logical differences that philosophical opposition is manifested. This idea, though
implicit in contemporary philosophy, emerges here in an especially clear, explicit, and perhaps even radical form.

This book consists of a substantial introduction and six chapters. The first three chapters are titled Nominalism, Conceptualism, and Realism. The fourth chapter deals with logics with predicate symbols occurring in subject position. The fifth chapter focuses on the predicate abstractor. The final chapter constructs and studies two logics thought to embody various Fregean views.

The introduction orients the reader to the philosophical problems at issue, the logico-mathematical framework of the book, and the methodology to be employed. There is a useful discussion of the important but often overlooked distinction between what the book calls internal semantics and external semantics. The point is not merely the familiar fact that two logics having the same language and the same system of deductions (and thus the same deducibility relation) can have semantic systems with fundamentally different mathematical characteristics, even when the two semantic systems give rise to one and the same (semantic) consequence relation. For example, consider two first-order logics, one with the standard semantics, the other admitting only countable interpretations. Familiar mathematical results imply that these two logics have the same consequence relation. The introduction goes beyond this mathematical point to the philosophical observation that one semantic system might represent (or even be) the way that a language is interpreted by a given philosophical school while the other may involve contrary philosophical intuitions or may even be an artificial mathematical construct not intended to represent any philosophy whatever. A semantic system representing philosophical intuitions is called internal, one intended as an auxiliary artifact is called external. The book points out that there is no philosophical justification for regarding the usual set-theoretic semantic systems as internal, not merely because in some cases this implies an ontology composed exclusively of pure sets or even because in most cases the universe of discourse of the intended interpretation of a set-theoretic language is excluded from being a set by the set theory in question. The book draws the conclusion that a philosophy of logic need not, and usually does not, rest on the epistemological and ontological presuppositions of set theory.

The internal-external distinction is closely related to distinctions that have already been made in the literature, for example, by Abraham Robinson, another of the author's teachers. This book, however, systematically employs the distinction in an interesting and original way. In particular, each logic that is to serve as a reconstruction of a philosophical theory has its own internal semantics but it is accompanied by a counterpart logic having the same language and the same deductive system but having an external set-theoretic semantics. This makes possible presentation of soundness and completeness (or incompleteness) results for the counterpart logic in cases where mathematically rigorous analogues using the internal semantic system are impossible.

Every philosophically significant logic constructed in this book, whether nominalistic, conceptualistic, or realistic, is second order (in the sense that bound predicate variables are involved) and classical (in the sense that all instances of all classically tautological schemes and all instances of all classically valid first-order schemes are provable). The reason that second-order languages are needed is that the book construes the three philosophies as differing in which predicates are held to exist. The fact that nominalism has been associated with first-order logic is adequately discussed, but the fact that conceptualism has been associated with intuitionistic logic is not explored. As can be expected given what has already been said, the deductive systems of the philosophically significant logics differ primarily in their respective “comprehension axioms,” the axioms that “assert” the existence of predicates. Roughly, nominalistic logics assert existence of expressible predicates, conceptualistic logics assert existence of predicates that are in various senses human constructs, and realistic logics make strong predicate-existence assertions.

The fourth chapter considers logics whose common language is obtained from an ordinary second-order language by allowing predicate symbols to occur in subject position thereby creating the potential for predicates (universals) to be (or to play the role of) individuals. This liberalization of well-formedness requirements entails reconsideration of deductive requirements and of semantic requirements. The new framework provides an opportunity to reassess various historically important discussions of Russell's paradox. Unfortunately, the neutral stance taken by the book seems to mute recognition of the participant-relative nature of paradoxes and the role that paradoxes play in the development of an actual philosophy. An argumentation that derives a contradiction from given premises is a paradox only to a person who believes the premises to be true (or at least consistent) and who believes the chain of
reasoning to be cogent. Tarski and others have emphasized the fact that it is through analysis of paradoxes that mistaken beliefs and logical fallacies are identified and corrected.

The fifth chapter deals with the ramifications vis-à-vis the three kinds of logics of including the predicate abstractor that produces a complex predicate symbol from a formula. Although this point is not mentioned in the book, the presence of the predicate abstractor is very closely connected to the presence of rules of definition that permit introduction of “simple predicates” definitionally coextensive with formulas. As mentioned already, the final chapter contains two logics associated with various aspects of Frege’s thought.

The above summary does not do justice to the richness and complexity of the book which to some extent is based on a series of research articles published by the author over the last two decades. The author has rendered a valuable service by collecting, interrelating, and putting into perspective a vast body of work. The author is to be admired for the serious, dignified, and respectful tone set early in the book and sustained throughout. The book has been very carefully planned and executed; it is a credit to the tradition which it continues.

It is unfortunate that the book lacks a concluding and summarizing chapter, a bibliography, an index of symbols, an index of persons, and an index of subjects. The latter three omissions only compound the heavy demands this book puts on a reader attempting to study it; the complexity of the philosophical and mathematical reasoning in the book makes casual reading impossible. The lack of a concluding and summarizing chapter, which would have provided an opportunity to list open problems and to suggest lines of further research, may tend to prevent the book from getting the recognition it deserves.

JOHN CORCORAN and WOOSUK PARK


CHRISTIAN THIEL. Einleitung des Herausgebers. Therein, pp. XXI–LXIII.


GOTTLOB FREGE. Erwiderung. Therein, p. 120. (Reprinted from Deutsche Litteraturzeitung, vol. 6 (1885), col. 1030.)

Anonymous. Review of the same. Therein, pp. 120–121. (Reprinted from Literarisches Centralblatt für Deutschland, vol. 36 (1885), cols. 1514–1515.)


The publication of a centenary edition of Frege’s Die Grundlagen der Arithmetik (1884) will be welcomed not only by Fregeans but by all who are interested in the history and philosophy of mathematics in general. The book has been edited with remarkable care, circumspection, and expertise. In particular, the editorial work displays comprehensive understanding of Frege’s philosophy and meticulous attention to questions of philology. Unfortunately, the original manuscript of this work has been lost. The lists that H. Scholz compiled of writings Frege left behind at his death (Frege’s Nachlaß) contain no reference to such a manuscript. Thus the text of Grundlagen is available only in the first edition and in various reprints (495, reprinted 1934; XXVII 368, reprinted 1977) containing no alterations. All corrections, supplementations, and comments provided by Christian Thiel refer to this text. In preparing the centenary edition the editor has taken into consideration J. L. Austin’s dual German/English edition of Grundlagen (XVI 67).

Thiel has set himself the important task of providing the first critical edition of Grundlagen. The aim has been (1) to eliminate all clearly identifiable misprints of the first edition; (2) to check the wording of