and, without loss of generality, we suppose $n \geqslant m$. By taking values of m where $1 \leqslant m \leqslant 5$, the solutions $m = 1$, $n = 2$ and $m = 4$, $n = 6$ are readily found. It remains to show that these are the only solutions for $n \geqslant m$ and we begin by noting that if

$$T_m = \frac{8(2m-1)!}{(m+1)!(m+2)!},$$

then

$$\frac{T_{m+1}}{T_m} = \frac{2m(2m+1)}{(m+3)(m+2)} = \frac{3(m-2)(m+1)}{(m+3)(m+2)} + 1.$$

So, for $m \geqslant 3$, T_m is increasing. But $T_6 = 11/7$, so $T_m > 1$ for $m \geqslant 6$. Hence, for $n \geqslant m \geqslant 6$,

$$\begin{aligned} nm(m+1)!(n+1)! &< m(m+1)!(n+1)!(n+2) \\ &< T_m\, m(m+1)!(n+1)!(n+2) \\ &= 4(2m)(2m-1)\ldots(m+3)(n+2)! \\ &\leqslant 4(m+n)! \end{aligned}$$

Thus $\binom{m+n}{m} > \binom{m+1}{2}\binom{n+1}{2}$ for $n \geqslant m \geqslant 6$.

Correct solutions were received from: S. Dolan, V. Everett, G. A. Garreau, D. M. Hallowes, N. Lord, R. Routledge, I. F. Smith, H. B. Talbot, R. Wakefield, M. Worboys, E. E. Wright.

G.T.Q.H.

Correspondence

Integral cuboids

DEAR EDITOR,

The problem about cuboids with integral edges and integral facial diagonals, raised by Peter Mason, in the December 1984 *Gazette*, has a parametric solution.

Let x, y, z be the edges and p, q, r the facial diagonals. Let integers a, b, c be the sides of a right-angled triangle such that $a^2 + b^2 = c^2$. Take

$$x = a(4b^2 - c^2),\ y = b(4a^2 - c^2),\ z = 4abc,$$

$$p = b(4a^2 + c^2),\ q = a(4b^2 + c^2),\ r = c^3.$$

Then

$$p^2 = x^2 + z^2, \quad q^2 = y^2 + z^2, \quad r^2 = x^2 + y^2.$$

These formulae do not give the complete solution. Here are some that cannot be obtained from them.

x	85	187	195	231	275	855	1105	1155	1155
y	32	1020	748	792	252	2640	9360	1100	6300
z	720	1584	6336	160	240	832	35904	1008	6685

From any solution we can derive a new one by taking $u = yz$, $v = zx$, $w = xy$.

All the above may be found in Kraitchik's *Mathematical recreations* but it still does not find a cuboid of this sort having its special diagonal also integral.

Yours sincerely,

D. M. HALLOWES

17 St Albans Road, Halifax

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