DEVELOPING HIGH ACCURACY NUMERICAL THEORIES OF MOTION OF OUTER SATELLITES OF JUPITER AND SATURN

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ABSTRACT. The effect of different perturbing factors on the rectangular coordinates of outer satellites of Jupiter (VI-XII) and Saturn (VI-IX) is investigated by the numerical integration of the equations of motion. The effect of relativistic terms in the equations of motion is considered among other perturbations. The effect of weak perturbations is compared with the accuracy of modern observations of satellites.

The present paper deals with the problem of developing high accuracy theories of motion of outer satellites of Jupiter (VI-XII) and Saturn (VI-IX).

The existing theories of motion of Jupiter VI-XII (P.Herget, 1968, T.V.Bordovitsyna and L.E.Bykova, 1978, T.V.Bordovitsyna, T.S.Boronenko, L.E.Bykova and A.M.Chernitsov, 1980) represent the modern observations with the error of 1.5-2.0" (in the geocentric coordinate system) that corresponds to the error of 5000-7500 km in the planetocentric Cartesian coordinates of the satellite. The theories of motion of Saturn VI-VIII, i.e. Titan, Hyperion, Iapetus (A.T.Sinclair, 1977, 1974, J.Woltjer, 1928) represent the observations with higher accuracy of 0.1-0.7", i.e. 700-4800km in the Cartesian coordinates. The numerical theory of motion of Phoebe, Saturn IX, which has recently been developed by us (L.E.Bykova and V.V.Shikhalev, 1982, 1984) and at the same time by A.Bec-Borsenberger and P.Rocher (1982) gives the error in the presentation of observations of order 1.5" that corresponds to 10400 km.

The main source of so large errors is insufficient accuracy of satellites observations and initial parameters of their orbits (constants of theories) determined from the observations.

It may be expected that the use of modern methods of

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observing with the help of spacecrafts and ground radar equipment will increase 10 to 100 times the accuracy of measuring outer satellites positions up to 50-100 km (D.Pascu, 1980).

With the purpose of developing satellites motion models corresponding to the given accuracy of 50-100 km we carried out comparative analysis of the effect of different disturbing factors on the satellites motion. At the same time we estimated the relativistic perturbations. Weak perturbations were compared with the accuracy of modern observations and time intervals on which these perturbations begin to act are given. Since the existing theories of Saturn VI-VIII are sufficiently complete we dealt only with the relativistic effects for this series of satellites.

Our investigation of perturbations due to the Sun, major planets and satellites was based on the numerical integration of the differential equations of motion by Everhart method (E.Everhart, 1974). The equations of motion were taken in the form given by T.V.Bordovitsyna and L.E.Bykova (1978).

The perturbations due to the central planet oblateness were computed on the basis of the solution of the generalized problem of two fixed centres (E.P.Aksenov, 1977).

The relativistic effects were estimated on the secular change of the satellite orbital elements (V.A.Brumberg, 1972). This is the Schwarzschild displacement of the pericentre π caused by the central planet, the displacement due to the proper planet rotation and the geodesic precession, i.e. the relativistic perturbations due to the planet revolution around the Sun.

The results of the perturbations analysis are presented in tables 1-4. Here the Jupiter satellites are designated by JN, those of Saturn by SN, where N is the number of a satellite, Δr is the perturbations in the position vector of a satellite:

$$\Delta r = (\Delta x^2 + \Delta y^2 + \Delta z^2)^{1/2} ,$$

where Δx, Δy, Δz are the differences of the integration results in the Cartesian planetocentric coordinates calculated with the considered disturbing factor being taken into consideration and without it, $\Delta \varphi$ is a corresponding angular displacement in the geocentric system of coordinates: $\Delta \varphi = \Delta r / \rho$, where ρ is a geocentric satellite distance.

The estimations of the maximum values of perturbations in the 100 years interval are given in table 1. In estimation of the perturbations due to the Galilean satellites, Titan and the ring of Saturn the masses of these ob-

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Table 1. The effect of different disturbing factors in 100 years time interval

Disturbing factor	JVI,VII,X		JVIII,IX,XI,XII		SIX	
	Δr, km	Δφ,"	Δr, km	Δφ,"	Δr, km	Δφ,"
Sun	25000000	6700	45000000	11900	10000000	1500
Galilean satellites	3300000	860	2100000	560	–	–
Titan	–	–	–	–	3400000	490
Saturn's ring	–	–	–	–	82000	12
Oblateness	16000	4.3	3200	0.9	4900	0.7
Jupiter	–	–	–	–	94000	13
Saturn	14000	3.8	65000	17	–	–
Uranus	24	0.006	400	0.13	460	0.08
Neptune	9	0.002	122	0.04	110	0.03
Earth	140	0.04	470	0.14	30	0.008
Venus	110	0.03	370	0.11	25	0.007
Mars	15	0.004	50	0.01	3	0.0008
Relativistic effects	8	0.002	5	0.001	2	0.0003

Table 2. Perturbations due to the oblateness (J_2) of the central planet

Satellite	Time interval in years					
	20		60		100	
	Δr, km	Δφ,"	Δr, km	Δφ,"	Δr, km	Δφ,"
SVI	360000	55	1000000	145	1600000	235
JVI	2800	0.8	8200	2.2	16000	4.3
SIX	1000	0.2	3000	0.5	4900	0.7
JIX	360	0.05	1100	0.2	2000	0.3

Table 3. SIX: perturbations due to Titan

Estimated value	Time interval in years					
	20		60		100	
	Δr, km	Δφ,"	Δr, km	Δφ,"	Δr, km	Δφ,"
Total perturb.	630000	91	2300000	330	3400000	490
σ_T	2300	0.3	7900	1.1	12000	1.7

Table 4. Relativistic perturbations

Satellite	Schwarzschild displacement due to the central planet	Displacement due to the proper rotation of the central planet		Geodesic precession
	$\Delta\pi$,"/cent.	$\Delta\pi$,"/cent.	$\Delta\Omega$,"/cent.	$\Delta\pi=\Delta\Omega$,"/cent.
SVI	3.1	-0.14	0.07	0.007
SVII	1.9	-0.08	0.04	0.007
SVIII	0.2	-0.005	0.003	0.007
SIX	0.009	0.0002	0.00006	0.007
JVI	0.07	0.0012	0.0007	0.03
JIX	0.012	0.0002	0.00006	0.03

jects were taken correspondingly as $M_G = 1.7\cdot10^{-4}M_J$, $M_T = 2.41\cdot10^{-4}M_S$, $M_{RS} = 6\cdot10^{-6}M_S$ (M_J is the mass of Jupiter, M_S is the mass of Saturn). In computing the motion of outer satellites perturbations due to other satellites of the system are usually determined by including the masses of disturbing satellites into the central planet's mass. On the example of Phoebe (SIX) table 3 shows that part of perturbations due to Titan (σ_T) which is usually neglected in the approach of this kind. The value of this part is σ_T=1.7" per century. For Jupiter VI and IX the neglected part of perturbations due to Ganimed amounts to 2.3" and 0.1" per century respectively.

The given estimates show that for Jupiter VI,VII,X and Saturn IX only relativistic effects and the perturbations due to the major planets with the exception of Jupiter and Saturn are beyond the scope of the accuracy of modern photographic observations. For Jupiter VIII,IX,XI and XII perturbations due to the major planets excluding Mars and Pluto amount to accuracy of modern photographic observations. To obtain the accuracy of 50-100 km it is necessary to take into account practically all the perturbations considered except the relativistic ones. For Saturn VI,VII and VIII the relativistic effects are given in table 4 and for Saturn VI they amount to 20 km per century in the planetocentric satellite position vector.

Moreover tables 5, 6, 7 show how much the satellite position is affected by the errors in the value of the following constants: the mass and the gravitational field coefficient J_2 of the central planet as well as the error in the planetocentric coordinates of the Sun. Here the variation of values of the enlisted parameters was made within the limits of the mean-root-square error σ of the parameter under consideration: $\sigma(M_J) = 0.005$, $\sigma(M_S) = 0.2$, $\sigma(J_2)_J = 4\cdot10^{-5}$, $\sigma(J_2)_S = 3\cdot10^{-5}$.

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Table 5. The estimate of the effect of the error in the mass value of the central planet

Satellite	Time interval in years					
	20		60		100	
	Δr, km	$\Delta\varphi$,"	Δr, km	$\Delta\varphi$,"	Δr, km	$\Delta\varphi$,"
JVI	20000	5	58000	15	90000	24
JIX	12000	3	30000	8	47000	13
SIX	140000	20	530000	75	780000	110

Table 6. The estimate of the effect of the error in the value of coefficient J_2

Satellite	Time interval in years					
	20		60		100	
	Δr, km	$\Delta\varphi$,"	Δr, km	$\Delta\varphi$,"	Δr, km	$\Delta\varphi$,"
SVI	650	0.09	2000	0.2	3300	0.4
JVI	8	0.002	23	0.006	40	0.01
SIX	2	0.0003	6	0.0009	9	0.001
JIX	1	0.0002	3	0.0007	6	0.001

Table 7. The estimate of the effect of the error in the planetocentric coordinates of the Sun

Satellite	Time interval in years					
	20		60		100	
	Δr, km	$\Delta\varphi$,"	Δr, km	$\Delta\varphi$,"	Δr, km	$\Delta\varphi$,"
JVI	10	0.003	30	0.006	50	0.01
JIX	46	0.01	160	0.04	200	0.06
S IX	3	0.0005	10	0.002	16	0.005

For the Jovicentric Solar coordinates we took $\sigma = 6\cdot10^{-7}$ a.u. and for the Saturnocentric Solar coordinates $\sigma = 1\cdot10^{-6}$ a.u.

The data given in these tables show that the errors in modern determination of the planetary masses exceed the effect due to all the disturbing factors except the solar ones. Inaccuracy of the coefficient J_2 and of the coordinates of the major planets, i.e. Jupiter and Saturn, gives the errors of 50-100 km in determining position of outer satellites. The value of this error is equal to or even exceeds the relativistic effects.

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