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Stanley Corrsin

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7.1 Early years

On 3 April 1920, a few years after G.I. Taylor's far-reaching observations of turbulent diffusion aboard the SS Scotia (Taylor, 1921), and at the time Lewis Fry Richardson was imagining vast weather simulations of atmospheric flow by human 'computers' (Richardson, 1922), across the Atlantic in the city of Philadelphia, Stanley Corrsin was born. His parents, Anna Corrsin (née Schorr) and Herman Corrsin had both emigrated to the United States only 13 years before. They came from Romania, where many Russian Jews had settled after leaving Russia in the late 19th and early 20th century. Following further hostilities in Romania, many emigrated again, this time to America. Anna and Herman Corrsin arrived separately at Ellis Island in 1907, Anna in July, and Herman in October. After brief stays in the New York and New Jersey area, where they met and married in 1912, they settled in the city of Philadelphia, in a mixed middle-class neighborhood, not far from the University of Philadelphia. They went into business in the clothing industry and raised their children. Their first son Eugene died young and their second, Lester, was born in 1918. Stan was their third and youngest son.

As a child, Stan Corrsin attended school in Philadelphia and, showing early signs of a highly gifted analytical mind, went on to skip two grades. He enjoyed following the ups and downs of his favorite baseball team, the Philadelphia Athletics. An appreciation for the game would accompany him throughout his life, including a keen interest in the subtle aerodynamic effects that can determine how balls fly through the air. Young Stanley enjoyed frequent visits to Philadelphia's Leary, a large used books store, and to Wanamaker's, Philadelphia's main department store. On these outings to downtown, he would have witnessed rapid developments thanks to mechanization and engineering. Like many American cities, Philadelphia in the 1920s saw ambitious projects of modernization with erections of steel and concrete skyscrapers, electrification of old buildings, widening of streets and construction of bridges, such as the Benjamin Franklin bridge over the Delaware river. The 1930s brought the Great Depression, and Philadelphia only began to recover with the massive further industrialization triggered by World War II and the expansion of giant shipyards that would – in time – supply the war effort.

After graduating from West Philadelphia High School in 1936, a bright and ambitious 16-year-old Stan Corrsin decided to study mechanical engineering. He had always been interested in how things work, and study in a technical field would allow an ambitious son of immigrants to make his mark. A 'mayor's scholarship' from the city of Philadelphia enabled his enrollment at the prestigious University of Pennsylvania located nearby in downtown Philadelphia. As a student, he distinguished himself with outstanding marks. And, presaging an unusual facility with the pen, it is said that he was the first engineering student at the University of Pennsylvania to receive the prize for a freshman essay in English. At some point he even considered becoming a professional writer. In his last year at U. Penn, he participated in a technical lecture competition organized by the American Society of Mechanical Engineers. His presentation entitled An Optical Method for Visualizing Low Velocity Air Flow earned him first prize. A proud headline from the 2 May 1940 university newspaper The Daily Newsletter proclaims "Corrsin dethrones Princeton as Pennsylvania takes first prize in engineer's Tourney".

In 1940, at the age of 20, Stan Corrsin graduated from U. Penn with a bachelors degree in mechanical engineering. Being drawn to the fundamentals underlying the engineering practice, he chose to continue his education and apply to graduate school in engineering. At the time, however, many top institutions still placed limits and quotas on students of Jewish heritage. It is said that Clark B. Millikan at Caltech heard of young Corrsin's promise and decided on the spot to offer him admission to Caltech. Thus Corrsin was accepted into the Caltech graduate program in aeronautics and he traveled to Southern California to begin graduate studies, just as the US war effort was ramping up.

7.2 First contributions at Caltech

Corrsin arrived at Caltech during what has come to be known as the golden age of aeronautics. Caltech was the US epicenter of aerodynamics at the time. The United States had found itself challenged to manufacture, on very short order, a massive fleet of war planes and needed to develop a basic understanding of the fluid dynamics of flight to aid in the design of modern aircraft. A vigorous



Figure 7.1 A 22-year-old Stan Corrsin in 1942 at the Graduate Aeronautical Laboratories, Caltech (GALCIT). The verso of the photograph states, in Corrsin's handwriting "Man doing research". The note is addressed to his mother, and adds for reassurance: "I'm really not so thin, it's just the light". Photograph courtesy of Dr. Stephen D. Corrsin.

program in fundamental and applied fluid dynamics research had been developing at Caltech's Guggenheim Aeronautical Laboratory (GALCIT), under the direction of Theodore von Kármán and Clark B. Millikan. Hans Liepmann, who in 1939 had just been hired by von Kármán after finishing his PhD in Zürich, became Corrsin's main academic adviser. At the time, Liepmann had begun experimental studies on boundary layers, transition to turbulence, and various turbulent shear flows. Corrsin began working in Liepmann's laboratory and distinguished himself for his dexterity in experimental science.

His first project at Caltech, which became his thesis in partial fulfillment of the requirements of Aeronautical Engineer, dealt with measurements of the decay of turbulence behind various grids. The subject of isotropic turbulence was in the air: on a visit to Caltech in 1936 and 1937, Leslie Howarth had collaborated with von Kármán and developed the equation for two-point correlation functions in decaying isotropic turbulence (von Kármán and Howarth, 1938). Corrsin's initial experiments provided data on the decay of standard deviations of two of the three turbulent velocity components behind three types of grids. More will be said later about ingenious measurement techniques of the time. In a photograph taken in the laboratory (Figure 7.1), he is seen reading a manometer, pencil tucked behind his ear. The results of the experiments, it turns out, were rather inconclusive. It was unclear whether turbulence was, or was not, observed to be sufficiently isotropic, or what the decay rate was. At the end of the thesis, which was never published in journal format, Corrsin writes: "The [...] conclusions are rather tentative; it is hoped that more certain results, and in particular the reasons for them, will come out of further investigation". Thus were laid the early seeds for Corrsin's work elucidating the fundamentals of isotropic turbulence. He completed the Masters thesis in 1942 (Corrsin, 1942) but, as further described in §7.6, his definitive experiments on decaying isotropic turbulence would have to await over two decades to become reality.

Corrsin then began to work in earnest towards his doctoral research and this work led to important, and no longer tentative or uncertain, results. The 1930s had seen initial developments in documenting basic properties of what are now known as the 'canonical' turbulent shear flows. By applying Prandtl's boundary layer concept to turbulent shear layers, thus assuming that they become asymptotically thin (although many never do), simplified parabolic equations had been developed describing the mean velocity in plane and round wakes and jets, in mixing layers, and in turbulent boundary layers along walls. The use of similarity variables and the eddy-viscosity assumption with local velocity and length-scales led to further simplifications. A series of experiments, most notably the measurements of mean velocity profiles in wakes by Townsend, had already begun to establish the validity and limitations of this approach. The popular textbooks by Townsend (1956), Hinze (1959) and Tennekes and Lumley (1972) provide excellent accounts of the accomplishments of that era.

By the early 1940s, after several of the canonical shear flows had been measured and documented in terms of mean velocity and Reynolds stresses, attention began to turn to the distribution of scalar fields. Examples of scalar fields include the temperature or the concentration associated with species being transported by turbulence. They are termed 'passive scalars' if they do not affect the motion, which therefore excludes cases with buoyancy effects that often occur in geophysical flows, or with strong volumetric expansion that accompany combustion. In the early 1940s, not much was known about distributions of passive scalars in turbulent shear flows. Of natural importance to propulsion and mixing, the turbulent jet was of great interest to von Kármán, Millikan, and the National Advisory Committee for Aeronautics (NACA). Thus, Corrsin's doctoral research project was on detailed measurements of the velocity and temperature fields in round jets. The work was closely followed by von Kármán, Millikan and supervised by Liepmann. Financial support was provided by NACA.

Corrsin went to work and designed and, with helpful laboratory technicians, built the experiment out of an existing, open-return 6 1/2 feet diameter wind tunnel. It was retrofitted with a contracting nozzle unit near its exit, thus creating a jet. Electrical units upstream would provide heating for the air, and warm air was also ducted outside the nozzle to improve uniformity of the temperature profile exiting the jet. In characteristic style, he writes:

That this scheme was not completely successful can be seen from the temperature distribution measured at the mouth. It did represent, however, a distinct improvement over the wooden nozzle first tried.

The mean velocity and temperature readings were photographically recorded on automatically traversing photo-sensitive paper illuminated by a light beam. The latter was continually being deflected by a mirror mechanically connected to a Pitot pressure line for mean velocity measurements, and by a galvanometer connected to a thermocouple for mean temperature. Fluctuating velocity was measured using a platinum hot wire. In the second part of the report, an oscilloscope was used and the screen photographically recorded.

The results were reported in a NACA Wartime Report (Corrsin, 1943). The report contains 43 figures with profiles of mean velocity and temperature at various downstream distances, profiles of standard deviation of velocity fluctuations, log-log plots for downstream scaling, calibration curves, etc. Rather than simply showing experimental results, much of the effort was spent in detailed comparisons with profiles predicted using several variants of eddyviscosity models. Based on the measurements, Corrsin reached conclusions about limits of validity of the similarity assumption and commented on differences between the scalar and momentum diffusivities (he confirmed in his measurements that the turbulent Prandtl number is less than unity). Notably, his first conclusion was "In a fully developed turbulent jet with axial symmetry, a completely turbulent flow exists only in the core region ...". The conclusion was based on his observations of hot-wire signals on the 'oscillograms', with fully turbulent signals when the probe is located in the centerline of the jet, but showing spotty turbulent regions interspersed with smooth, quasi-laminar portions of the signal when the probe was located off-center of the jet axis. This observation and conclusion already point to his keen interest in the detailed fundamental structure of turbulence. Corrsin would maintain interest in the phenomenon of what became known as 'outer intermittency' for several decades to come; some of this work addressing the geometry and intermittency of turbulence will be described in §7.7. A second experiment, on an array of plane jets, and a second NACA Wartime Report (Corrsin, 1944) soon followed. All the while, he contributed to important developments of the hot-wire anemometer.

According to Liepmann (1989), by 1943 Corrsin had completed the bulk of his PhD research. With the Second World War in full swing, however, he was

charged with the instruction of Navy pilots and other military personnel on the basics of aerodynamics, and thus he remained actively involved in teaching at Caltech even after the end of the war, all the while writing his doctoral dissertation. His doctoral degree was awarded in May 1947, for his two-part dissertation entitled *I. Extended Applications of the Hot-Wire Anemometer; II. Investigations of the Flow in Round Turbulent Jets* (Corrsin, 1947).

At Caltech Corrsin had met a young woman, Barbara Daggett, who would become his wife. She was originally from the Los Angeles area, and worked as part of the Caltech administrative staff. They were soon married. Then came the call from Johns Hopkins University to join its faculty as Assistant Professor.

7.3 Arrival in Baltimore

The end of the Second World War and the transformative GI bill that provided college support for returning servicemen brought a renewed sense of direction that was felt on many campuses across the United States. At the Johns Hopkins University, located in the east coast city of Baltimore, there was talk of creating a department that would focus on the new science of aeronautics. This would be a new and forward-looking department, part of the university's School of Engineering. Johns Hopkins had been founded in 1876, at first occupying temporary spaces downtown, and only between 1914 and 1916 did classes move to the university's definitive seat on the Homewood campus in what not too long before had been farmland but was fast becoming a leafy suburb of the city. The School of Engineering had existed as part of the university almost since its inception, and counted the traditional departments of Mechanical, Electrical, Civil, Chemical, and Sanitary Engineering. The addition of Aeronautical Engineering would develop synergies with laboratories in the area such as the Applied Physics Laboratory, Aberdeen Proving Ground, and the Naval Ordnance Laboratory that all pursued research and development in the rapidly developing field of aerodynamics.

The department began in 1946, under the direction of Francis H. Clauser, who was brought in as the department chair. Clauser, an earlier Caltech graduate who had worked under von Kármán, became well known as the developer of the 'Clauser plot' method to determine skin friction coefficients from measurements of mean velocity in turbulent boundary layers (Clauser, 1954). He began to hire faculty and among the first two was recently graduated Stan Corrsin who, together with his wife Barbara, thus moved across the country back to the East Coast. He began his work at the Johns Hopkins University in 1948, and would remain at the same institution for the rest of his life. The Corrsins lived several miles north of the Johns Hopkins campus in the suburban, almost rural, Towson area, in a house they bought in 1953. They had two children, Nancy E. Corrsin and Stephen D. Corrsin. Meanwhile, his parents Anna and Herman would retire to the state of Florida.

The Johns Hopkins Aeronautics Department continued to grow in the following years. In 1950 Leslie G. Kovasznay was appointed to the faculty, followed by Mark Morkovin and Robert Betchov who were appointed as research scientists (Hamburger Archives, JHU, 2009). In one of the most visible early contributions of the department, the faculty participated in a number of episodes of the critically acclaimed television series *The Hopkins Science Review*. Episodes included *Flight at Supersonic Speeds*, which aired on 2 February 1949, and a series *Man Will Conquer Space*, that aired in October 1952 and featured Wernher von Braun as the guest.

With him from Caltech, Corrsin brought Mahinder Uberoi, who had received his Masters degree there in 1946. Uberoi moved to Baltimore to become Corrsin's first doctoral student. They also brought along a hot-air jet unit that they had built at Caltech (Corrsin and Uberoi, 1950). It was more compact than the original wind tunnel add-on facility Corrsin had used earlier (Corrsin, 1947). The unit consisted of a centrifugal blower pushing air through a horizontal chamber with heating coils. A 90 degree elbow then turned the flow upward and, after passing through further screens and a smooth contraction, the 1 inch diameter heated jet emanated up into the laboratory. They also brought hot-wire anemometry from California. The experiment was set up in a laboratory in the Aeronautics Building (later known as Merryman Hall), an unassuming, grey concrete block building next to a wooded hillside at the edge of campus.

The velocity and temperature measurements from this experiment are described in some detail in a new NACA report (Corrsin and Uberoi, 1951). While this report was in press, Corrsin had performed initial analysis of the velocity data and published a rather remarkable brief communication in the *Journal of the Aeronautical Sciences* (Corrsin, 1949). This would be his third publication in 1949 and since joining Johns Hopkins. [He had written two other short notes published earlier in 1949, with Kovasznay on a hot-wire length correction (Corrsin and Kovasznay, 1949) and on transformation formulae between one and three-dimensional scalar spectra (Kovasznay et al., 1949).] He begins the short *Journal of Aeronautical Sciences* note with the statement that "The most significant idea contributed to the problem of turbulent shear flow in many years is the hypothesis of local isotropy due to Kolmogorov" (a statement of remarkable longevity still valid, some would say, to this day). He goes on to present a plot of the correlation coefficient between band-pass filtered signals of stream and cross-stream components (the normalized cross-spectrum). The data were taken at the maximum shear region in the jet using X-hot wires and the voltage readings from both wires were band-pass filtered using analog filter banks. The difference of their mean-square voltages, evaluated using vacuum thermocouple units, are proportional to the co-spectrum. The correlation coefficient as function of frequency decays rapidly to zero at the high frequencies characteristic of small-scale motions. Statistical isotropy demands that the two fluctuating components be uncorrelated at high frequencies. Corrsin's observation, therefore, gave significant and direct support to the notion that small scales in turbulence are isotropic, in a flow where the large scales clearly are not isotropic. This would be his first of many direct experimental examinations of theories pertaining to the small-scale structure of turbulence.

7.4 Structure of scalar fields in isotropic turbulence

Having begun to ponder the fine-scale structure of turbulence and having now temperature and velocity data available from the experiments with Uberoi, Corrsin turned his attention to the expected forms of the temperature two-point correlations and spectra in isotropic turbulence. In Corrsin (1951a), he applied the methodology of von Kármán and Howarth (1938) to derive the equation for scalar correlations $\langle \theta(\mathbf{x} + \mathbf{r})\theta(\mathbf{x}) \rangle = \langle \theta^2 \rangle m(r)$. He also used the von Kármán and Howarth (1938) argument about the vanishing pressure–velocity correlations in isotropic turbulence to reason that the temperature–velocity correlation vanishes, and established the dimensionless third-order scalar-variance velocity correlation and its cubic behavior with distance at small displacements. He went on to define integral and Taylor micro-scales appropriate for the scalar field, namely

$$L_{\theta} = \int_{0}^{\infty} m(r) dr$$
, and $\lambda_{\theta}^{2} = -2/m''(0, t)$, (7.1)

respectively. He also examined various possible consequences of assuming self-preserving solutions and discussed the role of the invariant,

$$N = \langle \theta^2 \rangle \int_0^\infty r^2 m(r) \, dr, \tag{7.2}$$

during the decay of scalar fluctuations.

In parallel, he examined the spectral structure of the scalar fluctuations by repeating the arguments presented by Kolmogorov (1941) using Fourier space.

Taking the Fourier transform of the equation for scalar correlations, Corrsin (1951b) derived the spectral equation for the temperature spectrum, G(k). He derived the solution in the case of small Péclet number, where the nonlinear transfer terms are negligible. In this case the problem reduces to the heat equation with its characteristic exponential decay as $\sim \exp(-2\gamma tk^2)$, where γ is the scalar diffusion coefficient and k is the magnitude of the wavenumber. At low wavenumbers, Corrsin showed that the spectrum grows as k^2 , a result intimately related to the existence of the invariant N mentioned above. For the intermediate range of wavenumbers, he went on to generalize the Kolmogorov (1941) approach using dimensional arguments.

Of crucial relevance are the rates of dissipation of kinetic energy

$$\epsilon = 2\nu \int_0^\infty k^2 E(k) dk$$

and of scalar variance

$$\epsilon_{\theta} = 2\gamma \int_0^\infty k^2 G(k) dk.$$

Quoting directly from Corrsin (1951b):

The dimensions of the pertinent quantities are:

$$k = L^{-1}$$
$$G = L\mathcal{T}^{2}$$
$$\epsilon_{\theta} = \mathcal{T}^{2}T^{-1}$$
$$\epsilon = L^{2}T^{-3}$$

where *L*=Length, \mathcal{T} =temperature, and *T*=time. Hence, the only possible arrangement is

$$G(k) = A\epsilon_{\theta}\epsilon^{-1/3}k^{-5/3},$$
(7.3)

where A is a dimensionless constant.

Thus Corrsin arrived at the -5/3 spectral scaling in the inertial-convective range of wavenumbers. Unbeknownst to him, on the other side of the iron curtain Obukhov (1949) had undertaken very similar steps and arrived at the same form for the spectrum of scalar fluctuations. Consequently, the dimensionless constant *A* in (7.3) is now called the 'Obukhov–Corrsin constant' C_{θ} .

Having produced this well-known prediction for the power-law decay of the scalar spectrum, the experimental evidence for it in Corrsin's papers from that time is, however, not overwhelming. The NACA report (Corrsin and Uberoi,



Figure 7.2 One-dimensional power spectra of velocity (circles) and temperature (squares) measured in a jet near the peak shear off-center position, adapted from Corrsin and Uberoi (1951).

1951) is only one of the very few publications where measured scalar spectra are reported. Figure 7.2 shows a reproduction of the measured spectra in the maximum shear region in the heated round jet. The velocity and scalar spectra display similar decay, not inconsistent with -5/3; but with the scatter of the data, as well as slightly different results obtained on the jet centerline where the scalar spectra were a bit flatter, Corrsin never argued that the data really supported his predictions of -5/3 scaling. The subject of the scaling of power spectrum continued to elicit many further studies over the subsequent decades, including several by his students and junior collaborators (Kistler et al., 1954; Mills et al., 1958; Sreenivasan et al., 1980; Sreenivasan, 1996).

7.5 Scalar transport and diffusion

Have you ever stopped to watch smoke billowing out of a chimney? On a calm day it moves generally upward; but there is much irregular meandering in its small-scale motions. On a windy day the wind overshadows this slow buoyant rising and washes the smoke along. Again, the smoke-cloud motion, now chiefly due to the wind, shows not only a gross pattern but also a thoroughly chaotic motion of various parts relative to each other.

Thus opens Corrsin's general interest article 'Patterns of Chaos' published in the *Johns Hopkins Magazine* (Corrsin, 1952). He wrote the article to introduce his research to the university community just a few years after arriving on campus. Ever since his PhD thesis in his study of the heated turbulent jet, a central theme of his research was the turbulent transport of scalars, e.g. smoke, temperature, and chemical species. He continued work on this topic throughout his career, making a number of major contributions.

In his thesis research (Corrsin, 1947), in addition to his important observations of the turbulent velocity field, he was – for the first time – able to measure the turbulent heat flux and, therefore, to present data to test some of the existing theories for turbulent heat transfer. This was followed, in particular, by a detailed study of diffusion of heat from a line source in isotropic turbulence (Uberoi and Corrsin, 1953), another piece of work he completed with Uberoi after they had moved to Johns Hopkins. The line source experiment was meant to address Taylor's theory of 'diffusion by continuous movements' (Taylor, 1921). In the paper were listed the following important statistical measures of the diffusive powers of turbulence:

- (1) average rate of dispersion of particles from a fixed point;
- (2) average rate of increase of the spacing between different fluid particles;
- (3) the average rate of transport of particle concentration under a given mean concentration gradient;
- (4) the average rate of increase of the length of a fluid line;
- (5) the average rate of increase of the area of a fluid surface.

These and closely related topics were to consume the attention of much of Corrsin's future research.

In addressing the first of these topics, Uberoi and Corrsin realized the approximate correspondence between the average temperature, $\overline{\Theta}$, downstream from a heated wire, stretched perpendicular to the flow direction, and the probability density of *Y*, the fluid particle displacement in the direction normal to both the wire and the flow direction. Using this correspondence the mean-square displacement $\overline{Y'^2}$ versus downstream distance was measured, and compared with Taylor's theory. The data offered one of the first consistency checks

of Taylor's theory, and an estimate for the turbulent heat transfer coefficient (turbulent diffusivity) from the Lagrangian analysis. In addition Eulerian and Lagrangian micro-scales were measured, and a correction and generalization of a theoretical expression of Heisenberg (1948) relating them was made.

After these ground-breaking results obtained in the late 1940s and early 1950s, institutional challenges called on Corrsin to lead the Mechanical Engineering Department. Thus in 1954 he moved from the Aeronautics Department in Merryman Hall on the campus periphery, to the more centrally located Maryland Hall where Mechanical Engineering was housed. As chairman of ME, he would be expected to devote some part of his time to administrative duties such as dealing with faculty hirings, teaching assignments, and managing the departmental infrastructure. There were teaching laboratories and halls containing large machinery, steam engines and, as remarked by John Lumley, other "examples of man's ingenuity" (Lumley and Davis, 2003). Lumley had arrived at Johns Hopkins in 1952, and would become Corrsin's third PhD student after Uberoi and Kistler. He recalls that one of Corrsin's major efforts was to modernize the department by removing the old machines and replacing them with wind tunnels. This did not occur without some resistance by more tradition-bound alumni and administrators in the Dean's office at the time.

A move away from engineering towards engineering science was to become one of the hallmarks of mechanics at Johns Hopkins. This move culminated with the closure of the engineering school altogether and the creation of the Mechanics Department in 1960. As recalled by Phillips (1986), Corrsin happily relinquished the chairmanship to George Benton and had a rubber stamp made that said "let George do it". He would use it with gusto on the incessant paperwork that could now proceed to be dealt with somewhere else.

With his family, he would continue to live in their house in Towson, the calm suburban area north of the city. Every morning he would drive the children to their school along tree-lined Charles Street, on his way to the Homewood campus. He never left Baltimore for extended periods of time. He did not absent himself for sabbatical leaves, preferring to host extended visitors rather than being a visitor himself. More will be said in §7.9 about the extraordinary environment at Johns Hopkins at that time, and about Corrsin's role in shaping it.

He and his students continued to work on the diffusive properties of turbulence through the next three decades. Taylor's theory, which gives a prediction of the Eulerian heat transfer coefficient, is expressed only in terms of Lagrangian quantities, namely, the Lagrangian mean-square displacement $\overline{Y'^2}$ and the Lagrangian velocity time autocorrelation $R_L(\tau) = \overline{V(t)V(t+\tau)}$, where V is a Lagrangian velocity and τ the time separation. Almost all data are taken, however, in an Eulerian frame. This led Corrsin to carefully define and then address the so-called Euler–Lagrange problem. That is, given the statistical properties of the Eulerian velocity field, say $\mathbf{u}(\mathbf{x}, t)$, what are the statistical properties of interest of the Lagrangian field, in particular the mean-square displacement and velocity time autocorrelation. Corrsin (1959; and later in more detail, 1962b) pointed out the exact relationship between the Lagrangian velocity time correlation, $R_{Lij}(\tau)$, the joint Lagrangian displacement/Eulerian velocity probability density, and the Eulerian space-time velocity autocorrelation, $R_{Eij}(\zeta, \tau) = \overline{u_i(\mathbf{x}, t)u_j(\mathbf{x} + \zeta, t + \tau)}$. Assuming that, for large time separations, the Eulerian velocity field becomes independent of the Lagrangian displacements ζ field, he then obtained

$$R_{Lij}(\tau) = \iiint p_Y(\zeta, \tau) R_{Eij}(\zeta, \tau) \, d\zeta \,, \tag{7.4}$$

where $p_Y(\zeta, \tau)$ is the Lagrangian displacement probability density. This result has been utilized by many turbulent dispersion model developers, and is widely known as the 'Corrsin independence hypothesis'.

Corrsin (1963) made some of the first estimates of the relationship between various Eulerian and Lagrangian length and time scales. Assuming high Reynolds numbers and the existence of inertial ranges for both the Eulerian velocity spatial spectrum and the Lagrangian velocity frequency spectrum, he concluded that, for homogeneous turbulence, $v'_1T_{11}/L_{11} \sim 1$ where v'_1 is the Lagrangian root-mean square velocity in a flow direction of interest (which equals the Eulerian rms velocity u'_1 for a homogeneous flow; Lumley, 1962), T_{11} is the Lagrangian integral time scale for the velocity v_1 , and L_{11} is the Eulerian integral spatial scale for u_1 . He found this result surprising, given the complex relationship between the Eulerian and Lagrangian autocorrelations; see, for example, equation (7.4) above. Continuing with this same reasoning, he concluded that $T_{11}/\Theta_{11} \sim 1$, where Θ_{11} is the integral time scale of u_1 . Finally, he also concluded, using similar arguments, that $\theta_{11}/\alpha_{11} \sim 1$, where θ_{11} and α_{11} are temporal Taylor microscales corresponding to u_1 and v_1 , respectively. Corrsin expressed doubts about this last relationship, reasoning that this estimate disagrees

with a plausible intuitive expectation: since the Eulerian time autocorrelation involves new fluid continuously wandering past the observation point, while the Lagrangian one follows along a material point, we might expect the latter to be more persistent.

The testing of these predictions awaited accurate measurements of both the Eulerian and the corresponding Lagrangian quantities. The appropriate measurements of the Eulerian quantities were completed by Comte-Bellot and Corrsin (1966, 1971) in their landmark measurements of homogeneous, isotropic turbulence in a frame of reference moving with the mean flow. These experiments will be described in more detail in the next section. Shlien and Corrsin (1974) repeated the heated wire experiments of Uberoi and Corrsin (1953), but with greater precision. They found, in particular, that $1.25 T_{11} \approx \Theta_{11}$, verifying the first estimate. On the other hand, they found that $\alpha_{11} \approx 12 \theta_{11}$, contradicting the second estimate, but consistent with his speculation.

Turbulent particle dispersion had often been modeled in a way similar to random walks in Brownian motion (see, for example, Einstein, 1905 and Goldstein, 1951). An innovative extension of this idea, addressing specifically the Euler–Lagrange problem, was Corrsin's work with John Lumley (Lumley and Corrsin, 1959) on random walks with both Eulerian and Lagrangian statistics. Limiting the problem to one dimension, they defined an Eulerian grid in space and time with specific rules of motion at each point, which defined the Eulerian space/time statistics. Then particles were allowed to 'walk' on this grid, determining the Lagrangian statistics. Analytical expressions were obtained for the relationships between various Eulerian and Lagrangian quantities, although the results could not be immediately applied to turbulence. This work was followed up by that of Patterson and Corrsin (1966), where more complex Eulerian fields and rules were defined, and computer simulations were used to obtain the various statistics. Although

...it was hoped that some empirical connection might be discovered between these two kinds of functions, the results show that no single Eulerian two-point correlation function is a good approximation to the Lagrangian function, ...

a result that is probably true also for the turbulence case.

As mentioned earlier, in dealing with turbulent dispersion, Corrsin realized the importance of understanding the bending and folding of iso-surfaces of transported scalars, and hence the growth of lines embedded in these surfaces, and the growth of the surfaces themselves. In first addressing this problem (Corrsin, 1955), he realized that the length of a scalar iso-line in two dimensions could be related to the number of crossings of that surface with a straight line through the fluid. Then using a theorem from Rice (1944, 1945) and assuming the scalar, say $\phi(\mathbf{x})$, is normally distributed, the line length \mathcal{L}_{ϕ} can be related to the autocorrelation of the scalar as

$$\mathcal{L}_{\phi} = \frac{1}{\pi} \left\{ -\frac{\Psi''(0)}{\Psi(0)} \right\}^{1/2} \exp(-\phi_c^2/2\Psi(0)), \qquad (7.5)$$

where $\Psi(\sigma)$ is the auto-correlation $\overline{\phi(s)\phi(s+\sigma)}$, and ϕ_c is the constant value of interest of ϕ . This result can easily be extended to an iso-surface in three



Figure 7.3 Evolution of lines formed with tiny hydrogen bubbles in a turbulent water channel flow (reprinted from Corrsin and Karweit, 1969).

dimensions. The work was further extended by Corrsin and Phillips (1961) to include contour lengths and surface areas of multiple-valued random variables. These results have proven very useful, in particular, in theories of turbulent combustion, where the area of the flame surface is often directly modeled (see, for example, Poinsot and Veynante, 2001).

Corrsin and Karweit (1969) were the first to measure the fluid line growth in turbulence. Michael Karweit was a graduate student pursuing his Masters degree and would remain at Johns Hopkins as a long-time junior collaborator of Corrsin. Their experiment utilized a water tunnel with a test section of dimension 8 in. square by 48 in., and approximately homogeneous turbulence generated by a bi-plane grid of mesh size $\frac{1}{2}$ in. The grid Reynolds number was 1360. They used the 'hydrogen bubble' electrolysis method, with a platinum wire stretched normal to the flow to generate the hydrogen bubble lines. These lines were photographed at various distances downstream from the wire, and their lengths were determined using an analysis relating length to the number of cuts and the angle of the line with respect to a straight reference line (Corrsin and Phillips, 1961). Photographs and movies from this experiment are now used in many classes in turbulence throughout the world, and a photograph is shown in Figure 7.3. Unfortunately, because of the limitation of the length of the water tunnel, only short-time growth of the lines could be observed. This growth was consistent with short-time growth estimates, but the measurements could not confirm the conjectured long-time growth of the lines.

Batchelor (1952) had conjectured that, for long times in stationary, homogeneous turbulence, the number of eddies of each size acting to stretch the line is proportional to the line length. This leads immediately to the conclusion that the line will grow exponentially. This result was proven more rigorously by Cocke (1969) and Orszag (1970). Corrsin (1972) offered a simpler geometric proof of this result. His conclusions were weaker than the previous ones, but without restrictions to isotropy or constant density being required.

While pointing out the problems of a fundamental nature in the use of a turbulent scalar diffusivity (see below), Corrsin made theoretical and experimental estimates of this quantity. He extended Taylor (1921)'s theory to include a homogeneous, isotropic, stationary shear flow (Corrsin, 1953) and found, for example, for long times, the cubic dependence on time of the streamwise dispersion, i.e.

$$\overline{X_1'^2} \sim \frac{2}{3} \left(\frac{d\overline{u}_1}{dx_2} \right)^2 \overline{v_2'^2} T_{22} t^3 , \qquad (7.6)$$

where $d\overline{u}_1/dx_2$ is the uniform mean shearing of the u_1 component of the velocity in the x_2 direction. This result was extended by Riley and Corrsin (1974) for the non-isotropic case. In particular, they computed the turbulent diffusivity tensor \mathcal{K}_{ij} and found it to be non-diagonal, and to depend on the mean shear and the correlations of v'_1 and v'_2 . For example, the \mathcal{K}_{11} component was found to be

$$\mathcal{K}_{11}(t) = \int_0^t R_{L11}(\tau) \, d\tau + \frac{d\bar{u}_1}{dx_2} \int_0^t \tau R_{L12}(\tau) \, d\tau \,. \tag{7.7}$$

Riley and Corrsin (1971) also performed computer simulations of fluid particle dispersion of homogeneous shear flows using an artificially constructed Eulerian flow field consisting of spatially and temporally varying Fourier modes, with amplitudes defined so that the statistics of the flow were similar to the laboratory measurements of Champagne et al. (1970). Their computed results were consistent with the analysis.

Almost all turbulence models employ, at some point, a linear gradient model, where the turbulent flux of a quantity (e.g. mass, heat, species concentration, momentum, kinetic energy) is assumed proportional to the linear gradient of that quantity. Corrsin would often express skepticism about closure models, in particular the eddy-diffusivity, gradient-based models and their motivating analogies to kinetic theory of gases. His arguments about the fundamental limitations of these models have served as motivating force to many researchers who in subsequent decades have attempted to develop more general and intricate closure models of turbulence.

In an influential paper entitled *Limitations of gradient transport models in* random walks and in turbulence, Corrsin (1974) presented a systematic analysis of closure models. Following ideas from continuum mechanics in deriving a relationship between the molecular flux of the quantity, the properties of the fluid, and the space and time gradients of the quantity, he assumed a general functional relationship between the turbulent flux of a quantity in the, say, z direction, $\overline{F}(z)$, and a functional of the average quantity $\overline{\Gamma}$ and the statistical properties of the velocity field. He then determined the assumptions required for the turbulent flux to be linearly related to the gradient of the average quantity. With ℓ , a length scale of the turbulence, τ , the time scale, and $V = \ell/\tau$ the corresponding turbulent velocity scale, the necessary conditions for the linear gradient model are found to be the following, where a subscript denotes a derivative with respect to that quantity:

- (i) $|\bar{F}_{zzz}/\bar{F}_z|\ell^2 \ll 1$, i.e. the turbulent length scale should be much smaller than the distance over which the curvature of $\bar{\Gamma}$ changes appreciably;
- (ii) $\tau |\bar{\Gamma}_{tz}/\bar{\Gamma}_z| \ll 1$, i.e. the turbulence time scale must be much smaller than the time over which $\bar{\Gamma}$ changes appreciably;
- (iii) $|\ell_z/\ell + V_z/V| \ll |\overline{\Gamma}_z/\overline{\Gamma}|$, i.e. the changes in the turbulence properties must be very small over a distance for which $\overline{\Gamma}$ changes appreciably;
- (iv) $|V_z/V| \ll |\ell_z/\ell|$, i.e. the turbulent velocity must be appreciably more uniform than ℓ ; and
- (v) the relative change in $\overline{\Gamma}$ must be very small over the turbulent time scale τ .

Corrsin then went on to compute these inequalities for several flows, pointing out that

the archival literature is replete with data showing, either directly or indirectly, for both scalar and momentum transport, that the mean gradients vary considerably over distances comparable to the length scales characteristic of the 'eddies'.

He also argued that, for the turbulent flux of a scalar $\overline{\gamma u_i}$ (where $\gamma = \Gamma - \overline{\Gamma}$ is the scalar fluctuation and u_i is the fluctuating part of the velocity vector), the turbulent diffusivity must be considered as a second-order tensor, i.e.

$$\overline{\gamma u_i} = -\mathcal{K}_{ij} \frac{\partial \bar{\Gamma}}{\partial x_i}.$$
(7.8)

He pointed out that there is no reason to assume that the diffusivity matrix is diagonal, as assumed in many models. In fact, using estimates from various

sets of data, he argued that the off-diagonal terms are often comparable to the diagonal terms.

Several years later, he and co-workers (Sreenivasan et al., 1981) revisited these conditions and analyzed experimentally obtained turbulent heat flux and temperature gradients across various types of homogeneous and inhomogeneous shear flows. They identified additional conditions and concluded that there was a need for models based on more than just the mean field properties of the flow.

7.6 Homogeneous turbulence: decay and shear

Ever since his early experiments as part of his Masters thesis at Caltech, Corrsin had been interested in quantifying the precise decay rate of kinetic energy in isotropic turbulence unconstrained by boundaries. Several statistical theories and models predicted different decay rate exponents n of kinetic energy with time, i.e. $\overline{u'^2} \sim t^n$ for a particular component of turbulence kinetic energy. Depending on what quantity (invariant) was assumed to be constant during the decay, different values of n were obtained. As discussed in Davidson (2004), most well-known are n = -10/7 (Kolmogorov, 1941), n = -6/5 (Saffman, 1967), or n = -1 for complete self-preservation which at the time had been discarded (Batchelor, 1948). Careful new experiments were needed to provide accurate data. Such data could be produced in a wind tunnel with a test section of large enough cross-section to prevent wall effects and long enough to enable turbulence to decay significantly. Also, the turbulence should be truly isotropic at the entrance of the test section. Many earlier attempts at generating isotropic turbulence, including Corrsin's own trials at Caltech, typically provided for larger velocity fluctuations in the streamwise direction than in the cross-stream directions. In order to take advantage of the unexpected availability of wood and of a team of carpenters who, it is said, had finished working on a Johns Hopkins building project earlier than planned, Corrsin designed a large closed-loop wind tunnel made almost entirely out of wood. The design called for a very large primary contraction with an area ratio of 25 to 1, in order to create a smooth, constant velocity air flow at the core of the test section.

Construction of the two-story facility occupying large areas of the basement and first floor of Maryland Hall proceeded quickly. Procurement and installation of a two-stage axial fan with adjustable pitch connected to a 150 horsepower electric motor on the first floor completed what would become a major facility for turbulence research. Cooling was needed to prevent excessive thermal contamination of the hot-wire probe readings. It was provided by a



Figure 7.4 Professor Stan Corrsin in later years explaining decaying isotropic turbulence behind a grid in a wind tunnel, including a secondary contraction.

cross-flow heat exchanger installed before the fan. Through the heat exchanger circulated cooling water siphoned off from a pond which, at the time, graced the east side of campus. For years, this arrangement would cause friction with the university's ground maintenance personnel.

The design also included a secondary contraction which would be located downstream of the turbulence producing grid. By forcing the initially somewhat anisotropic turbulence to go through the secondary contraction, vorticity aligned in the streamwise direction would get amplified due to vortex stretching, and the cross-stream turbulence variance would be increased relative to the streamwise turbulence component. Corrsin settled for a 1.27:1 secondary contraction which would greatly reduce the initial anisotropy of the turbulence. Years later, he would often explain the principle of the secondary contraction on a blackboard (Figure 7.4).

Geneviève Comte-Bellot arrived to Baltimore in 1963 as a Fulbright and postdoctoral fellow, having recently obtained her doctorate from the University of Grenoble working with Antoine Craya. She went to work with Corrsin and implemented various improvements in hot-wire instrumentation and analog data acquisition. In two seminal papers on the decay of isotropic turbulence that arose from their collaboration, they presented what has become one of the most celebrated datasets of fluid mechanics. In the first paper (Comte-Bellot and Corrsin, 1966), they documented the performance of the secondary contraction in promoting isotropy of the turbulence behind the grid. They also showed that the decay of turbulence variances proceeded according to a power law,

$$\frac{\overline{u'^2}}{U_0^2} \approx \frac{\overline{v'^2}}{U_0^2} \approx C \left(\frac{x - x_0}{M}\right)^n,$$
(7.9)

where $\overline{u'^2}$ and $\overline{v'^2}$ are the variances of streamwise and cross-stream velocity, respectively, U_0 is the mean velocity in the tunnel (that ranged between 10 and 20 m/s), M is the mesh-size of the turbulence-producing grid of bars (M ranged from 1 to 4 inches), and $x - x_0$ is the downstream distance to a virtual origin. In the presence of the secondary contraction, the isotropy requirement $\overline{u'^2} = \overline{v'^2}$ was met to a remarkable degree. Moreover, the data yielded decay exponents that fell mostly in the range between n = -1.2 and n = -1.3, over more than one decade of scaling. It was the most convincing experimental result showing that predictions from theories leading to either a $t^{-10/7}$ or a t^{-1} decay were not reproduced.

The second work was published sometime later (Comte-Bellot and Corrsin, 1971) and provided a detailed analysis of measured two-point correlation functions and spectra at various downstream distances from the grid. Hot-wire probes recorded velocity signals over a wide range of frequencies. Spectra for high frequencies were obtained using an HP wave analyzer. Since it was sensitive only down to 20 Hz, lower frequencies were captured by recording the signals to tape and replaying the tapes at higher speeds later on. Measuring correlation functions also involved playing back the tapes with varying time-delays. Additional analog signal processing included band-pass filters, multipliers and an electro-chemical integrator whose output finally corresponded to the timeconverged correlation coefficients among narrow band-pass filtered signals.

The results show that correlation functions for band-pass filtered velocities decay at time-scales commensurate with the eddy-scale highlighted by the band-pass filtering. Also, all curves could be collapsed by an appropriate time scale, combining effects at various scales.

Comte-Bellot and Corrsin (1971) also report, in great detail, the precise energy spectra at various times (distances) during the decay. They used the measured one-dimensional energy spectrum $E_{11}(k, t)$ to deduce the radial threedimensional energy spectrum using the assumption of isotropy. The resulting radial spectra E(k, t), carefully tabulated, have been used by many researchers since to test and validate spectral closures such as eddy-damped quasi-normal theories and, in recent decades, subgrid-scale models for large eddy simulations (Moin et al., 1991). It has taken three decades for this ground-breaking experiment to be replicated using direct numerical simulations (de Bruyn Kops and Riley, 1998) as well as for a similar experiment to be remade at higher Reynolds number in the same wind tunnel, this time using an active grid (Kang et al., 2003).

The question of the dynamics of narrow-band effects in turbulence continued to interest Corrsin for many years. He had been following the theoretical efforts of R.H. Kraichnan, who at the time lived in relative isolation, north in the New Hampshire woodlands. Once a year, Corrsin would travel to New Hampshire to visit with Kraichnan and discuss turbulence. One of the central quantities of the Kraichnan direct interaction approximation is the response function of turbulence to a spectrally local disturbance. Partly motivated by the discussions with Kraichnan, Kellogg and Corrsin (1980) performed an experiment in which the wake of a fine wire stretched across otherwise isotropic grid turbulence introduced a narrow-band disturbance. They recorded its decay and compared it to the linear perturbation response predicted by Kraichnan, noting 'fair agreement'. Interest in the dynamics of Fourier modes also led Corrsin to consider early uses of computer simulations. With J. Brasseur, a postdoctoral fellow at Hopkins in the early 1980s, they performed numerical experiments and followed the time-evolution of individual Fourier modes and observed their interactions within wave-number triads (Brasseur and Corrsin, 1987).

Towards the late 1970s and early 1980s Corrsin directed a concerted effort to study the most elemental non-isotropic turbulent flow, namely homogeneous shear flow in which the mean flow has a linear profile. Champagne et al. (1970) and Harris et al. (1977) produced such a mean velocity profile by forcing air flow through a set of parallel plates, each channel being associated with a screen of different solidity. The side with larger solidity corresponds to lower speeds due to the increased head losses suffered by the flow there. The evolution of turbulence, the growth of length-scales, and the resulting anisotropy were measured and to this day form a dataset used to calibrate turbulence models and compare to simulations.

Returning to the question of scalar transport, Tavoularis and Corrsin (1981a) made direct measurements of the turbulent diffusivity in a homogeneous shear flow. They used an experimental setup similar to that used in the homogeneous shear flow experiments of Harris et al. (1977), but with the exit turbulence-generating rods replaced with heating rods. This produced a uniform temperature gradient in the cross-stream (x_2) direction, to go along with their uniform velocity gradient across the same direction. Detailed measurements were made of the velocity field and temperature field statistics, including joint temperature/velocity statistics, spectra, autocorrelations, microscales and integral scales. In particular, with $d\bar{T}/dx_2 = \text{constant}$, and $d\bar{T}/dx_1 = d\bar{T}/dx_3 = 0$, from measuring $\bar{u_1\theta}$ and $\bar{u_2\theta}$ they were able to determine $\mathcal{K}_{12} = -\bar{u_1\theta}/\frac{d\bar{T}}{dx_2}$ and $\mathcal{K}_{22} = -\bar{u_2\theta}/\frac{d\bar{T}}{dx_2}$. The result was that $\mathcal{K}_{12}/\mathcal{K}_{22} \simeq -2.2$. In re-examining

existing data for heated turbulent boundary layers and heated pipe flows, they found approximate values of -2.4 and -2.1, respectively.

This work was extended by Tavoularis and Corrsin (1981b), using the same flow field, to the case with the mean temperature gradient transverse to the direction of the mean flow and the mean shear, i.e. $d\bar{T}/dx_3 = \text{constant}$, and $d\bar{T}/dx_1 = d\bar{T}/dx_2 = 0$. The only significant heat flux component was $\overline{u_3\theta}$ (the other two components were approximately zero by symmetry), and gave the results that $\mathcal{K}_{33} = -\overline{u_3\theta}/\frac{d\bar{T}}{dx_2} \approx 1.6\mathcal{K}_{22}$.

7.7 The geometry and intermittency of turbulence

In his PhD thesis on the circular turbulent jet, Corrsin (1947) computed many of the statistical properties of the turbulent velocity and temperature field. But in observing oscillograms of the axial velocity signal he noticed that

the 'turbulent' jet is completely turbulent only from the axis out to approximately $r = r_0$. For $r > r_0$, there exists first an annular transition region, in which the flow at a point alternates between the turbulent and laminar regimes.

(Here r_0 is the radial location where the mean axial velocity \overline{U} drops to half of its peak value.) He went on to note that

the general location of the transition region in the jet is about the same as the location of the u'/\bar{U} maximum. This may mean that a part of the 'turbulence' is not due to the usual turbulent velocity fluctuations, but to actual differences in local mean velocity at a point, as the flow oscillates between the laminar and turbulent states.

Corrsin had discovered the intermittent layer between a laminar and a turbulent flow which is now known to be characteristic of any turbulent flow with a freestream boundary (i.e. not a solid boundary) such as turbulent boundary layers, jets, wakes, shear layers, and other related flows.

The first definitive study of the intermittent regions between a laminar and a turbulent flow was by Corrsin and Kistler (1955), who addressed such regions for a turbulent boundary layer, a plane wake, and a circular jet. Although interesting experimental data were obtained in this study, one of its principal contributions was conceptual, in defining and clarifying the overall processes involved. The first issue is how to distinguish the turbulent and the non-turbulent regimes. Corrsin and Kistler realized that it was not the random motion that distinguished the turbulent region, since the flow in the laminar region was also quite random. They concluded that the characterizing feature of the turbulent region was its high vorticity, compared to the essentially irrotational flow of

the non-turbulent region. Thus they concluded to apply "the word 'turbulent' to random rotational fields only".

They surmised that the rotational turbulent region must propagate into the non-turbulent region, much as "a flame front propagates through a combustible mixture". From the vorticity equation they reasoned that

the random vorticity field ... can propagate only by direct contact, as opposed to action at a distance, because rotation can be transmitted to irrotational flow only through direct viscous shearing action. This assures that ... the turbulent front will always be a continuous surface; there will be no islands of turbulence out in the free stream disconnected from the main body of turbulent fluid.

Corrsin and Kistler reasoned that a very thin layer, which they called the laminar superlayer, separated the turbulent and non-turbulent regions. The turbulent side was characterized by strong vorticity amplification by vortex stretching, while the superlayer itself was characterized by viscous diffusion of vorticity across this layer. From simple physical/mathematical arguments, they concluded that the superlayer was very thin, with a width on the order of the Kolmogorov scale.

In order to address the intermittency of turbulence in the flow, Corrsin and Kistler followed Townsend (1948) and defined the intermittency γ as "the fractional time spent by the (fixed) probe in the turbulent fluid". Experimentally the intermittency γ was determined by electronically differentiating the hotwire signal for the axial component of the velocity, then rectifying, smoothing and clipping the resulting signal. A signal discriminator was used to determine whether the resulting signal was strong enough such that the region was turbulent; this signal discriminator was set by comparing the results of the signal output to a visual oscillogram output. In addition to the usual measurements of the velocity statistics, they were able to measure the intermittency γ and the position of the front Y as functions of time and downstream coordinate x. They were thus able to determine the intermittency γ , which is, in terms of $Y, \gamma(y) = \text{prob}\{y \le Y(t) \le \infty\}$. In addition, they could determine the average position of the turbulent front, \bar{Y} , and its standard deviation $\sigma = \{\overline{(Y - \bar{Y})^2}\}^{1/2}$, which is a measure of the width of the intermittent zone, which they termed the wrinkle amplitude of the turbulent front.

From their data and using theoretical arguments, they found that the rate of increase of the wrinkle amplitude of the turbulent front was roughly predicted by Lagrangian analysis as $\sigma(x) \simeq \sqrt{2(v'/\bar{U})v'T_L}$, where v' and T_L are the local Lagrangian velocity fluctuation and velocity integral time scale, respectively. They also found that the downstream growth of the turbulent front, as measured by *Y*, was proportional to the growth of the shear-layer thickness.

Corrsin and Kistler drew two important additional conclusions from their study. First, they concluded "that the presence of the turbulent front with its attendant detailed statistical properties will have to be included in basic research on turbulent shear flows with free-stream boundaries". Secondly, they speculated that, in considering a scalar (e.g. heat, mass) in the flow for Prandtl and Schmidt numbers not much smaller than unity, "the front should apply equally well to heat or chemical composition. Oscillographic observations ... in a hot jet show a temperature fluctuation intermittency, presumably coincident with the vorticity intermittency".

Corrsin saw indications of outer intermittency in many other fluid dynamical systems. In a noteworthy interview in *Sports Illustrated* (Terrell, 1959), he was asked to explain the mechanism underlying the so-called 'knuckle ball'. It was the hallmark of Hoyt Wilhelm, a then famous pitcher for the Baltimore baseball team, the Orioles. Hoyt could throw a ball that would then move in unpredictable trajectories, thus confusing the opposing team's batter. A photograph in the article shows Corrsin in front of the blackboard with a sketch of the flow-field at the rear side of a baseball during flight. A jagged boundary line encloses the separated turbulent region. It is used to show that the unpredictable trajectories of the knuckle ball can be due to slight changes in lift and drag forces associated with the complicated geometry of the separated region. Quoting from the article:

If the separation line was perfectly straight, the ball would go straight, for the pressure forces would be even. But since the separation line is highly irregular, so is the course of the ball. And since the separation line is constantly shifting and changing... the course of the knuckle ball can change direction several times in flight.

Following Kolmogorov's (1941) important theory of local isotropy and similarity hypotheses regarding turbulent velocity fine-structure, and in fact his own along with Obukhov's (1948) theory for fine-scale scalar fields (Corrsin, 1951b), Corrsin became interested in the intermittent behavior and the geometric properties of fine-scale turbulence. Measurements by Batchelor and Townsend (1949) indicated that the fine-structures were strongly intermittent, localized in relatively small regions which were distributed somewhat randomly in space. This led many to question Kolmogorov's original theory (see, for example, Landau and Lifshitz, 1959), which led to a number of attempts to address the structure of the fine-scales as well as modifications of Kolmogorov's theory.

Corrsin's interest in understanding the spatial structure associated with the turbulent cascade of kinetic energy comes to light in a passage of his general-interest article published in *American Scientist* in 1961 (Corrsin, 1961b). Quoting from the article:

From a geometrical viewpoint, the spectral transfer process in turbulence can be seen in the (empirical) fact that any blob of fluid momentarily having a fairly uniform local velocity, is stretched and twisted by its own motion (and that of neighboring fluid) into even longer, thinner and more convoluted 'strings' and 'sheets'. Since it is difficult to sketch such a locally coherent velocity field, we can illustrate this aspect via a similar phenomenon: turbulent mixing of a passive contaminant, like dye spots in a turbulent liquid.

He earned the 1961 American Scientist Prize for this article.

Corrsin (1962c) used a simple phenomenological model and some existing data to suggest that the fine-structure was distributed into thin sheets, with thickness on the order of the Kolmogorov scale, and separation distance of the order of the integral scale. This suggested that the velocity derivative flatness factor should scale linearly with the Taylor scale Reynolds number, $R_{\lambda} = u'\lambda/\nu$. On the other hand, Tennekes (1968) suggested the fine-structure was distributed as vortex tubes, with diameters of the order of the Taylor scale λ . This led to the prediction of the flatness factor scaling as $R_{\lambda}^{3/2}$. In addition, Obukhov (1962) and Kolmogorov (1962), attempting to take the fine-scale intermittency into account, assumed that the logarithm of the average energy dissipation rate over a very small volume had a normal distribution, and from this they were able to obtain modified expressions for the energy spectrum and structure functions. These hypotheses remained to be tested.

Working with A. Kuo (Kuo and Corrsin, 1971) in both grid-generated, nearly isotropic turbulence and on the axis of a round jet, Corrsin first addressed the size of the fine-scale regions, the dependence on Reynolds number, and the probability density of the locally averaged dissipation rate. Hot-wire anemometers were employed to make the velocity measurements, and three kinds of circuits were used to extract fine-scale signals from the outputs of the anemometers: differentiation circuits, band-pass filters, and high-pass filters. They found that there was a decrease in the relative fluid volume occupied by fine-structure of a given size as the turbulence Reynolds number R_{λ} increased. They also found that, for a fixed Reynolds number, the relative volume is smaller for smaller fine-structures. In addition, the average linear dimension of a volume of fine-structure (L_r) was found to be much larger than the size of the fine-structure r itself. For example, at $R_{\lambda} = 110$, they found that L_r/r varied from 15 to 30, decreasing with r. Finally, they found that $(\partial u/\partial t)^2$ was approximately log-normally distributed, at least when probabilities fall between about 0.3 and 0.95, in partial agreement with the assumptions of Obukhov and Kolmogorov.

Realizing that information about the shape of the fine-scale structures might eventually help understand the physical processes related to energy transfer to these scales, Kuo and Corrsin (1972) then attempted to determine the geometric character of the structures. Using the measurement technique of twoposition coincidence functions for the presence of velocity fine-structure, they tried to distinguish the structures as being 'blobs', 'rods', or 'slabs'. Again hotwire anemometer measurements were made in nearly isotropic turbulence. In order to determine the geometry of the structures, Kuo and Corrsin developed mathematical, geometric models for each structure; these models predicted, for each assumed structure, the simultaneous detection event rate as well as the simultaneous intermittency factor. Comparisons of the experimental results for these quantities to the predictions of the models then allowed the determination of the type of fine-scale structures.

Their tentative conclusion was that the fine-scale regions are more rod-like than blob-like or sheet-like. This implies a tendency for slightly 'stringy' structures, which may overlap with each other. Two other classes of structures were not eliminated by the measurements, ribbon-like structures, and a mixture of blobs and rods. Kuo and Corrsin suggested coincidence measurements using three or more probes to help determine among these alternatives. They also suggested using similar models for fine-scaled scalar fields to help distinguish the structures. These detailed results motivated many subsequent publications by other researchers on the intermittency statistics of turbulence, as well as an influential paper by Kraichnan (1974). He dealt with an analysis of the energy cascade along wavenumber bands arranged in octaves in an effort to provide a possible dynamical explanation for the spatial concentration of energy fluxes in smaller and smaller subregions of the flow during the cascade. A number of subsequent developments are recounted in some detail in the book by Frisch (1995).

7.8 Turbulence and chemical reactions

Corrsin was the first to apply statistical theory to turbulent, reacting flows. In a series of papers spanning the 1950s and 1960s he applied statistical analysis and results from turbulence and turbulence mixing developed over the past 20 years to determine the statistical properties of simple chemical reactions in turbulent flows. His research set the stage for much of the work on such flows that followed.

As was usually the case for him, Corrsin idealized the problem and considered homogeneous, isotropic turbulence, and assumed negligible effects of heat release on the fluid properties so that, in particular, the fluid density, the reaction-rate coefficients, and diffusion coefficients remain constant. Furthermore, he assumed that all of the reactant species concentrations but one were in great excess, so that the concentration of only the latter, say Γ , changes significantly in time. Finally, he assumed that the chemical reaction rates were of simple power-law form, i.e. proportional to Γ^n , and considered the cases where n = 1 or n = 2. Therefore, Γ satisfies the following convection–diffusion–reaction equation:

$$\frac{\partial\Gamma}{\partial t} + u_i \frac{\partial\Gamma}{\partial x_i} = \mathcal{D}\nabla^2\Gamma - \Phi(\Gamma), \qquad (7.10)$$

where $\Phi(\Gamma) = k_n \Gamma^n$, k_n is the reaction-rate constant, \mathcal{D} is the molecular diffusivity of Γ , and *n* is either 1 or 2.

First-order reactions (n = 1): With the assumption of homogeneity, Corrsin (1958) averaged equation (7.10) and obtained the solution for the average as $\overline{\Gamma}(t) = \overline{\Gamma}_0 \exp(-k_1 t)$. He then obtained the equation for the mean-square fluctuation $\overline{\gamma^2}(t)$:

$$\frac{d\overline{\gamma^2}}{dt} = -2\mathcal{D}\frac{\overline{\partial\gamma}}{\partial x_i}\left(\frac{\partial\gamma}{\partial x_i}\right) - 2k_1\overline{\gamma^2}.$$
(7.11)

He introduced a microscale for γ , say λ_{γ} , and, based upon previous experiments, assumed $\lambda_{\gamma}/\lambda \simeq 2\mathcal{D}/\nu$ and obtained solutions for $\overline{\gamma^2}$ for different assumptions regarding λ . Here λ is the usual Taylor microscale. His solutions were of the form

$$\overline{\gamma^2}(t) = \overline{\gamma_m^2}(t) \exp(-2k_1 t), \qquad (7.12)$$

where $\overline{\gamma_m^2}(t)$ is the solution for the nonreacting case $(k_1 = 0)$. Therefore, he found that the effect of the first-order chemical reaction is to cause exponential decay in both $\overline{\Gamma}$ and $\overline{\gamma^2}$.

In a subsequent paper (Corrsin, 1961a), he focused on the spectral behavior of γ for first-order reactions. Using extensions of the spectral cascade arguments of Onsager (1949), and the mixing theories of Batchelor (1959) and Batchelor et al. (1959), he derived the following expressions for the energy spectrum of γ , say G(k), for three different spectral subranges:

(i) inertial-convective subrange, $k \ll (\epsilon/\nu D^2)^{1/4}$ if $\nu/D \gg 1$, or $k \ll (\epsilon/D^3)^{1/4}$ if $\nu/D \ll 1$

$$G(k) \simeq Bk^{-5/3} \exp(3k_1 \epsilon^{-1/3} k^{-2/3}).$$
 (7.13)

(ii) viscous–convective subrange, $k \ll (\epsilon/\nu^3)^{1/4}$

$$G(k) \simeq Nk^{-(1+4k_1\nu^{1/2}\epsilon^{-1/2})} \exp\{-2(k/k_B)^2\}.$$
 (7.14)

(iii) inertial–diffusive subrange, $(\epsilon/\mathcal{D}^3)^{1/4} \ll k \ll (\epsilon/\nu^3)^{1/4}$

$$G(k) \simeq \frac{1}{3} \frac{\epsilon_{\theta}^* \epsilon^{2/3}}{\mathcal{D}} \frac{1}{k^{5/3} [\mathcal{D}k^2 + k_1]^2}.$$
 (7.15)

Here *B* and *N* are constants determined from the analysis, and $k_B = (\epsilon/\nu D^2)^{1/4}$ is the Batchelor wave number. It is easy to see the effects of the reaction rate on the spectra. For example, in the inertial–convective subrange, where the spectrum is proportional to $\epsilon_{\gamma} \epsilon^{-1/3} k^{-5/3}$, the effect of chemical reaction is given by the factor $\exp(3k_1 \epsilon^{-1/3} k^{-2/3})$. Corrsin points out that, for wave numbers above $k_c = k_1^{3/2} \epsilon^{-1/2}$, the effect of the chemical reaction on the spectral shape is negligible.

Having obtained results for the concentration of the reactant, Γ , Corrsin (1962a) then addressed the concentration of the product of the reaction, say *P*, for first-order reactions. The product concentration for this case satisfies the following equation:

$$\frac{\partial P}{\partial t} + u_i \frac{\partial P}{\partial x_i} = \mathcal{D}_{\mathcal{P}} \nabla^2 P - k_1 \Gamma.$$
(7.16)

Assuming equal diffusivities for the reactant and the product, i.e. $\mathcal{D} = \mathcal{D}_{\mathcal{P}}$, the equation for \overline{P} is closed and the solution is easily found to be $\overline{P} = \overline{P}_0 + \overline{\Gamma}_0 \{1 - \exp(-k_1t)\}$. The equation for the mean-square fluctuations about \overline{P} , say $\overline{p^2}$, is similar to equation (7.11), except that the last term is now $+k_1 \overline{p\gamma}$, introducing a new unknown. Arguing that the *p* and $\underline{\gamma}$ fields are perfectly correlated, Corrsin was then able to obtain a solution for $\overline{p^2}$ analogous to equation (7.12):

$$\overline{p^2}(t) = \overline{\gamma_m^2}(t) \{1 - \exp(-k_1 t)\}^2.$$
(7.17)

He also obtained equations for the energy spectra of the product concentration for the inertial–convective, viscous–convective, and inertial–diffusive subranges, but these results will not be repeated here. These various predictions for both mean values and energy spectra are available to the community to guide experiments and modeling, and have been extensively utilized.

Second-order reactions (n = 2): Due to the nonlinearity of the reaction term, $k_2\Gamma^2$, the equation for mean reactant $\overline{\Gamma}$ is no longer closed, but contains the unknown $\overline{\gamma^2}$. In addition, the chemical reaction now causes a spectral flux in $\overline{\gamma^2}$. Corrsin (1958) again formed the equation for $\overline{\gamma^2}$, which now contained, in addition to the dissipation-rate term, the additional unknown $\overline{\gamma^3}$, for which he

introduced an additional equation. Introducing microscales for the dissipationrate terms in the equations for $\overline{\gamma^2}$ and $\overline{\gamma^3}$, he was left with equations for $\overline{\Gamma}$, $\overline{\gamma^2}$ and $\overline{\gamma^3}$ with a number of additional unknowns requiring additional assumptions. To simplify the problem further he then addressed three separate limiting problems:

- (i) extremely low fluctuation levels, $\gamma'/\bar{\Gamma} \ll 1$;
- (ii) very slow reactions;
- (iii) very fast reactions;

and obtained solutions for each case. Corrsin (1958) also briefly addressed the equation for the spatial autocorrelation function $\overline{\gamma(\mathbf{x})\gamma(\mathbf{x}+\mathbf{r})}$.

Corrsin (1964) went on to consider the energy spectra for second-order reactions. Assuming small fluctuation levels ($\gamma' \ll \overline{\Gamma}$), extending the cascade method of Onsager (1949), and again following the approach of Batchelor (1959) and Batchelor et al. (1959), he developed expressions for the reactant energy spectra's inertial–convective, inertial–diffusive, and viscous–convective subranges.

7.9 The Johns Hopkins environment

During Corrsin's tenure at Johns Hopkins, the Mechanics Department was an exciting environment in which to work, and was considered one of the top centers for turbulence research in the world. Some of this is described in the *Annual Reviews of Fluid Mechanics* article by John Lumley and Steve Davis (Lumley and Davis, 2003). One of the authors of this chapter (J.J. Riley) was a graduate student there in the late 1960s and early 1970s, so this section relates mostly to that time period.

Owen Phillips has referred to the period in the late 1960s and early 1970s in the Mechanics Department at Johns Hopkins as the 'golden years', and much of the credit for creating and sustaining such an environment goes to Corrsin. The fluid mechanics faculty, which he had helped to build, was very active. Owen Phillips had completed his now classical work on surface wave and internal wave resonances, among many other things. Robert Long had finished his pioneering studies on stratified flow over complex terrain, and was now delving into stratified, turbulent flows. Leslie Kovasznay was continually improving experimental methods and studying structures in turbulent boundary layers. Francis Bretherton had just joined from Cambridge University and was embarking on several studies of geophysical flows which have now become famous; and Stephen Davis arrived from Imperial College London and immediately developed a very active program on various aspects of nonlinear instabilities. Clifford Truesdell and Jerald Eriksen were world-renowned for their work in theoretical continuum mechanics, and James Bell, Robert Pond and Robert Green had well-established programs in solid mechanics.

A cornerstone of the Hopkins environment was an unapologetic promotion of fundamental aspects of research in mechanics. According to Phillips (1986), Corrsin would say that science begins by asking simple questions about complex phenomena, but advances by asking more penetrating questions about simpler systems, whose solution could be obtained with rigor and explained with clarity. Again, quoting Phillips (1986), if an unwary colleague commented that some question was academic, Corrsin's inevitable response was, "This is an academic institution, where we consider academic questions". According to Michael Karweit the atmosphere was one of genuine pleasure in doing research on fundamentally important problems. Perhaps it was the realization of working on transcendental knowledge which resulted in the uniquely joyful atmosphere at that time.

Along similar lines, Corrsin had definite views about the importance of fundamental science in the context of the education of engineers. He did not favor those who would constantly clamor for an education mainly characterized by 'applied relevance'. His strong views come into focus in a 1968 letter he wrote to the editors of a publication that at the time reigned above all other illustrated magazines in the United States: *Life Magazine*. The letter was in response to the publication of an article advocating more "engineering relevance" in education.

Sirs: I observe with wonder the demands of some college students and some faculty members (Professor Jerome in "The System Really Isn't Working," *Life*, 1 November) for an education characterized by 'relevance'. The primary weakness of the US engineering education from its inception until World War II was its wholehearted devotion to relevance: spending more time in shop and lab than in classroom, the students were well trained to cope with the design, operation and repair of the world's machinery, possibly to improve it a bit. Then along came new machines based on scientific principles no one guessed engineers would ever need to know. The result: many science-trained people had to be hastily recruited into doing engineering work. Their weakness in engineering principles led to mistakes – but at least they hadn't been given tunnel vision by a totally 'relevant' education. Very truly yours, Stanley Corrsin, Professor.

The department seminar series was particularly interesting and lively. Both novice and established researchers would visit and present the results of their work, usually with much lively discussion from the faculty. Douglas Lilly from the National Center for Atmospheric Research discussed some new numerical simulations of turbulence by him and his colleague, James Deardorff; these happened to be the first large-eddy simulations of turbulence. Chris Garrett discussed ideas about ocean internal waves which would later lead to the now-famous Garrett-Munk energy spectrum for ocean internal waves. There were always a number of visitors for extended visits. For example, Geneviève Comte-Bellot had two long visits while engaged in her now-famous collaboration with Corrsin on decaying grid turbulence (§7.6). Tim Pedley was visiting from Cambridge and lectured on hydrodynamic stability. Keith Moffatt, who was working on magnetohydrodynamic turbulence, was another visitor from Cambridge. The Swedish oceanographer Pierre Welander was a visiting Professor who lectured on ocean currents. James Serrin from the University of Minnesota was a visiting Professor lecturing on mathematical fundamentals of fluid mechanics. Frank Champagne arrived from Boeing to study turbulent shear flows, while John Foss from Michigan State University worked with Corrsin on turbulent diffusion experiments. Many postdoctoral fellows working on various topics in fluid mechanics subsequently have had outstanding careers. They included, among others, John Allen, James Brasseur, John Dugan, Fazle Hussain, Wolfgang Kollmann, Martin Maxey, and Katepalli R. Sreenivasan. Of the students who received their PhD degrees from the Mechanics Department during the late 1960s and early 1970s, many are today well known for their research, and occupy top positions in academia and research laboratories in industry and government.

There was considerable interaction with Hopkins researchers 'off-campus' as well. For example, Akira Okubo, from the Chesapeake Bay Institute, at the time located on the Homewood campus, often discussed his latest ideas on turbulent dispersion. Vivian O'Brien of the JHU Applied Physics Laboratory, located between Baltimore and Washington, DC, would give seminars on her latest research, often on bio-fluid mechanics. Besides turbulence, fluid mechanics in and around living things was another emerging area to which Corrsin made major contributions.

The morning coffee period was legendary, always drawing a large number of faculty, students, and visitors. The late 1960s and early 1970s were the time period of the Vietnam war, and feelings were very strong on all sides. Many discussions were political. At the same time, Baltimore was rife with racial and social problems. The downtown area of the city showed signs of considerable neglect. The harbor area in particular consisted of not much more than a collection of abandoned and derelict warehouses. Racial tensions exploded following the assassination of Martin Luther King Jr. and led to the Baltimore riot of 1968, which lasted over a week. National Guard and federal troops had to be called in to restore order. It was not until the early 1980s that, in a highly successful example of American urban renewal, the entire inner harbor area was redeveloped.

Besides political discussions at the coffee period, the latest technical ideas were argued at length; if the ideas could survive a discussion at coffee somewhat intact, there was some hope for them. And, the coffee period was also a time of camaraderie and joking, especially if Corrsin were around. Many stories from this coffee period, perhaps often somewhat embellished, would be recalled at conference meetings for many years. And if it was an especially good, or bad, week, some students would go off and buy a supply of wine and cheese on Friday, and the coffee period would become an even livelier party.

By the late 1970s, however, internal disagreements in the department (by then called the Department of Mechanics and Materials Science) that had been developing for some time finally boiled to the surface. Moreover, the recognition that engineering required a separate administrative structure led to the closing of the department and the reestablishment of a distinct engineering school at Hopkins. It consisted of several traditional engineering departments which continued to nurse a distinctly strong science flavor.

7.10 Final years

In 1984, Corrsin became ill with cancer. He underwent an apparently successful operation followed by a none-too-aggressive therapy. Preparations for a conference in his honor, the 'Corrsin Birthday Symposium', were in full swing. It took place in Evanston, Illinois, early in 1985. The happy event celebrating his 65th birthday reunited many of his former students and postdocs. The contributions from the symposium are recorded as a collection of papers in a well-known book, *Frontiers in Fluid Mechanics*, edited by Davis and Lumley (1985).

Soon however the illness returned, this time much worse. There were months of treatments, extended hospital stays, and distress. Stanley Corrsin died on 2 June 1986. He was sixty-six. Another symposium that had been planned in his honor on occasion of the award of the American Society of Civil Engineers' Theodore von Kármán Medal took place in Minneapolis, Minnesota, on the day after his death (George and Arndt, 1988). The medal was awarded posthumously.

A memorial service was held on the Johns Hopkins campus in September at the beginning of the Fall semester. It was attended by many of Corrsin's students, postdocs and collaborators from around the world, his family, and university colleagues and staff. Corrsin's impact on the field has been felt beyond his own scientific contributions, through those he instructed directly and through others he inspired, directly and indirectly. He had been honored by many professional awards, such as fellowship in the American Academy of Arts and Sciences, the American Physical Society and American Society of Mechanical Engineers, membership of the US National Academy of Engineering, and being named the Theophilus Halley Smoot Professor of Fluid Mechanics.

During his lifetime, Corrsin saw turbulence research progress from rudimentary single-probe hot-wire measurements in small bench-top shear flow experiments, all the way to large-scale turbulence measurement campaigns in large wind tunnels, and in the atmosphere and oceans. He saw turbulence theory develop from simple one-point and two-point closures to path-diagrammatic methods, and to the first several successful direct numerical and large-eddy simulations on supercomputers. His own contributions form the backbone of our present understanding of turbulent scalar transport, of fine-scale structure of passive scalars in turbulence, and of the phenomenon of outer intermittency. His contributions to homogeneous turbulence, decaying and sheared, as well as chemically reacting turbulence, are considered pivotal. Yet, what he called "the theoretical turbulence problem" (Corrsin, 1961b) remains to this day unsolved. The lack of systematic methodologies to make analytical predictions for even the simplest statistical objects continues to pose a serious challenge to the many fields where turbulence plays a crucial role. In the absence of a definitive theoretical framework to attack the problem, Corrsin's approach of joyful empiricism and fundamental analysis of canonical and carefully chosen example problems remains to this day the best approach to turbulence research.

Towards the end of his life, on occasion of Liepmann's 70th birthday in 1984, Corrsin penned the 'Sonnet to Turbulence'. It was read at the event by Anatol Roshko and loosely follows the form of William Shakespeare's Sonnet #18 and Elizabeth Browning's poem "How do I love thee? Let me count the ways ...". In the form received from SC by William K. George (1990), it is reproduced below as closing words about Corrsin's life. The sonnet evokes relevant turbulence phenomena and provides insights about his views on several new approaches that were being proposed at the time. For instance, in juxtaposing low-dimensional strange attractors versus supercomputing, he correctly predicted that the latter would be needed due to the very large number of degrees of freedom of turbulence (it is useful to recall that at the time the most powerful supercomputer was the Cray 2).

Sonnet to Turbulence (by S. Corrsin):

Shall we compare you to a laminar flow? You are more lovely and more sinuous. Rough winter winds shake branches free of snow, And summer's plumes churn up in cumulus. How do we perceive you? Let me count the ways. A random vortex field with strain entwined. Fractal? Big and small swirls in the maze May give us paradigms of flows to find. Orthonormal forms non-linearly renew Intricate flows with many free degrees Or, in the latest fashion, merely few – As strange attractor. In fact, we need Cray 3's. Experiment and theory, unforgiving; For serious searcher, fun ... and it's a living!

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