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FINITE PRESENTABILITY OF ARITHMETIC GROUPS OVER GLOBAL FUNCTION FIELDS

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1. Introduction and survey

Arithmetic subgroups of reductive algebraic groups over number fields are finitely presentable, but over global function fields this is not always true. All known exceptions are "small" groups, which means that either the rank of the algebraic group or the set S of the underlying S-arithmetic ring has to be small. There exists now a complete list of all such groups which are not finitely generated, whereas we only have a conjecture which groups are finitely generated but not finitely presented. The present situation is as follows:

Let F denote a global function field, $S=\{v_1, v_2, \ldots, v_s\}$ a finite non-empty set of primes of F; furthermore G is a linear algebraic group defined over F, which we can assume to be absolutely almost-simple (the reductive case can be reduced to this one—on the other hand the results become much easier) and which has rank r over F and r_i over the completion F_{v_i} of F $(v_i \in S)$; finally let Γ be a S-arithmetic subgroup of $G(F)$.

Theorem. *Γ is not finitely generated if and only if*

$$s=r=r_1=1.$$

The positive part (that means finite generation) is old (cf. [**6**]); the counter-examples for classical groups are due to Keller (cf. [**14**]), whose proof is modelled after Serre's for SL_2 ([**18**]), so only one group of exceptional Lie-type was left, which was settled in ([**14a**]). According to Tits' classification in [**21**] there exist S-arithmetic subgroups Γ, which are not finitely generated if G is equivalent up to central isogeny to exactly one of the following groups, where all forms are non-degenerate and of Witt-index (=rank) 1 over F and $F_v := F_{v_1}$:

(a) Special linear group $SL_2(D)$, where D is a central division algebra over F and $D \otimes F_v$ is also a division algebra.

(b) Special unitary group $SU_n(F', h)$ for $n=3$ or 4, where F' is a quadratic extension of F and h a hermitian form with respect to F'/F.

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(c) Special unitary group $SU_2(D,h)$, where D is a central division algebra over a quadratic extension F' of F with an involution of the second kind, such that F' splits over F_v (which implies $SU_2 \otimes F_v \simeq SL_2$).
(d) Special unitary group $SU_n(D,h)$ for $n=2,3$, where D is a quaternion skew-field and h a hermitian form with respect to the standard-involution of D; provided char $F \neq 2$.
(e) Special unitary group $SU_n(D,s)$ for $n=4,5$, where D is a quaternion skew-field and s an antihermitian form with respect to the standard-involution of D, provided char $F \neq 2$.
(f) G is of type ${}^6D_{4,1}^9$ and $G \otimes F_v$ of type ${}^2D_4$.

Remarks.

(1) In case (d) and (e) there is another description for char $F=2$ (cf. [**21**]).
(2) Special orthogonal groups in 3, 5 or 6 variables are isogenous to groups in (a), (d) or (e) respectively, for 4 variables they are not absolutely almost simple.

Conjecture. Γ *is not finitely presentable if and only if*

$$\sum_{i=1}^{s} r_i \leqq 2 \quad \textit{and} \quad r>0.$$

This conjecture has been proved in the following cases:

(a) $G=SL_2$ (Stuhler [**20**]);
(b) G split with constant root-length, $s \geqq 1$, $r=r_1 \geqq 3$ (Rehmann–Soulé [**17**], Splitthoff [**19**]);
(c) G split, not of type G_2, $\Gamma = G(\mathbb{F}_q[t,t^{-1}])$, $r=r_1=r_2 \geqq 2$ (Hurrelbrink [**13**]) or $G=SL_n$, $n \geqq 3$, $s \geqq 2$ (Splitthoff, loc. cit.);
(d) G arbitrary, $s=1$, $r=r_1=2$ (Behr [**7**], [**16a**]; McHardy [**16**]);
(e) G arbitrary, $r=1$, $\sum_{i=1}^{s} r_i \geqq 3$ (see main theorem below).

Remarks.

(1) These are "positive results" (i.e. proving finite presentability) in case (a) for $s \geqq 3$, case (b), (c) and (e) and "negative results" (i.e. giving counter-examples) in case (a) for $s=2$ and case (d).
(2) A positive result remains positive (for the same group) if one enlarges the set S: This can be shown in the same way as Kneser did (in [**15**]) in the number field-case.
(3) Γ is finitely presented if $r=0$: For an anisotropic group G the quotient $G(F_v)/\Gamma$ is compact by Godement's criterion (cf. Section 2) and since $G(F_v)$ is compactly presented (look at the Bruhat–Tits-building), the assertion for $s=1$ follows by Reidemeister–Schreier.
(4) The result (d) can be translated into an explicit list of groups: As above—in the case of nonfinitely generated groups—one obtains classical groups in low dimensions and some exceptional types.

(5) If the conjecture is true there should be further counterexamples with

(i) $s=1$, $r=1$, $r_1=2$: this is known only for some very special examples with the same proof as in (d),

(ii) $s=2$, $r=r_1=r_2=1$: Stuhler's proof for SL_2 can be generalized for these groups.

These are the only remaining cases for the negative part of the conjecture. For the positive part it would be enough—according to (b), (c) and Remark 2—to settle the cases

(iii) $s=1$, $r\geqq 2$, $r_1\geqq 3$;

(v) $s=2$, $r\geqq 2$

for non-split groups—or split groups excluded above.

(6) The proofs use very different methods: For (b) and (c) one applies pure group-theory and algebraic K-theory, for (a) and (d) also topological methods come in via the operation on Bruhat–Tits-buildings.

All examples dealt with in (a)–(d) are either split (or Chevalley) groups or at least the global and local ranks of the group G coincide. For better support of the conjecture it is therefore important to look at a situation, in which these ranks are different, this may happen in case (e).

Main theorem. *If the absolutely almost simple algebraic group G has rank* 1 *over F, a S-arithmetic subgroup Γ of G is finitely presentable in each of the following three cases:*

(a) $s=3$, $r_1=r_2=r_3=1$;

(b) $s=2$, $r_1\geqq 2$ *or* $r_2\geqq 2$;

(c) $s=1$, $r_1\geqq 3$.

The proof uses old and new tools. In the next two sections we have to describe reduction theory and the operation on products of Bruhat–Tits-buildings: In this way we can pass from Γ to $\Gamma\cap P(F)$ for a parabolic subgroup P (defined over F). If and only if the global rank of G equals 1 we can translate the question of finite presentability of $\Gamma\cap P(F)$ to the problem of compact presentability of $P^0(F_{v_i})$ (for an appropriate subgroup P^0 of P) and for the latter one we can dispose of Abels' new techniques like contracting automorphisms and amalgamation of subgroups (cf. [**4**]), but unfortunately we cannot apply his explicit criterion, because its proof is only valid for characteristic 0. Then it is easy to show part (a) (which includes the positive part of Stuhler's result for SL_2), a little bit harder for (b), but part (c) can be settled only by a tedious and lengthy case-by-case proof. Therefore we shall merely give the list of groups and carry out the details for some example. But all these examples suggest that Abels' criterion remains true in arbitrary characteristics.

We freely use the theory of reductive groups, especially over local fields and of their corresponding buildings as it is given in [**8**], [**10**] and [**22**], only special results will be cited precisely. To make things simpler we will assume that G is simply-connected: we can do this because the image of an arithmetic subgroup $\tilde{\Gamma}$ of the simply-connected covering $\tilde{G}$ of G has finite index in Γ (cf. [**5**], Satz 1).

2. Reduction theory of arithmetic groups

We describe the main results of reduction theory over global function fields in the formulation of Harder in [12] together with some supplements due to [6]. For this purpose we need the following list of notation:

Let be

F a global function field;
V the set of all primes of F;
A the ring of adeles over F;
G a connected reductive algebraic group, defined over F;
T a maximal F-split torus of G;
Δ $=\{\alpha_1,\ldots,\alpha_r\}$ a system of simple roots of G with respect to T;
P a minimal parabolic subgroup of G, defined over F, which contains T and corresponds to Δ;
P_Θ a F-parabolic subgroup of G of type $\Theta\subseteq\Delta$, which contains P, i.e. Θ is a system of simple roots of the semi-simple part of P_Θ, in particular:
Q_i the maximal parabolic subgroup of type $\Delta-\{\alpha_i\}$ above P;
Z the centre of G;
K the product $\prod_{v\in V}K_v$ with open and compact subgroups K_v of $G(F_v)$ for all completions F_v of F;
$H^\circ(A)=\{h\in H(A)\,\|\,\chi(h)|=1$ for all $\chi\in\hat{H}(F)\}$ for a subgroup H of G and its group $\hat{H}$ of characters, where $|\ |$ denotes the idele-norm;
H^g $=g^{-1}Hg$ for $H\subseteq G(A)$, $g\in G(A)$;
S a finite non-empty set of primes of F;
G_S $=\prod_{v\in S}G(F_v)$ (for $H\subseteq G(A)$ denote by H_S the projection of H on G_S);
Γ a S-arithmetic subgroup of G.

Remark. The roots α_i are in general not contained in P, but it is possible to extend the idele-norm of α_i to a function on $P(A)$ (cf. [12], page 47).

The numerical invariants $v_i(P,K)$ $(i=1,\ldots,r)$, defined by Harder, have the following properties:

(1) $v_i(P^\gamma,K^\gamma)=v_i(P,K)$ for $\gamma\in\Gamma$;

(2) $v_i(P,K^p)=v_i(P,K)|\alpha_i(p)|^{-1}$ for $p\in P(A)$;

(3) For two compact-open subgroups $K=\prod_v K_v$ and $K'=\prod_v K'_v$ there exist real constants d and d' with $dv_i(P,K)\leqq v_i(P,K')\leqq d'v_i(P,K)$.

Theorem A. *There exists a constant $C_1>0$, depending on K, such that for each $g\in G(A)$ there is a minimal parabolic F-subgroup of G with*

$$v_i(P,K^g)\geqq C_1 \quad \textit{for all} \quad i=1,\ldots,r.$$

We can reformulate Theorem A with respect to the action of Γ and sharpen it by choosing a particular K: For each $v\in V$ we take K_v to be the stabilizer of a "special point" in the Bruhat–Tits-building, then we have the Iwasawa-decomposition $G(F_v)=K_vP'_v(F_v)$ with a minimal parabolic F_v-subgroup P'_v (cf. [22], 3.2 and 3.3.2), which implies that there is only one conjugacy class of $P \supseteq P'_v$ relative to $K = \prod_v K_v$; thus we have the

Corollary A′. *There exists a maximal compact subgroup K of $G(A)$, such that for a fixed parabolic F-subgroup P_0 of G and each $g \in G(A)$ there is a $\gamma \in G(F)$ with $v_i(P_0, K^{g\gamma}) \geqq C_1$ for all $i = 1, \ldots, r$ (where C_1 is the constant of Theorem A).*

Theorem B. *Let $C_1 > 0$ be a constant for which Theorem A holds: There is a second constant $C_2 > 0$ (depending on C_1) with the following property:*

If $v_i(P, K^g) \geqq C_1$ for all i and some minimal parabolic F-subgroup P and even $v_i(P, K^g) \geqq C_2$ for all i with $\alpha_i \in \Theta \subseteq \Delta$, then each minimal parabolic F-subgroup P' is contained in $P_{\Delta-\Theta}$ if $v_i(P', K^g) \geqq C_1$ for all i.

Corollary B′. *If $v_i(P, K^g) \geqq C_1$ and $v_i(P, K^{g\gamma}) \geqq C_1$ for all i and $\gamma \in G(F)$ and even $v_i(P, K^g) \geqq C_2$ for all i with $\alpha_i \in \Theta$, then γ is an element of $P_{\Delta-\Theta}$.*

Proof. According to Theorem B and property (1) we have $P' = \underline{P^{\gamma^{-1}}} \subseteq P_{\Delta-\Theta}$ which implies $\gamma \in P_{\Delta-\Theta}$ (cf. [9], 2.6).

Theorem C.

(a) *$M \subseteq G(A)$ is relatively compact* modulo $Z(A)G(F)$ *if and only if for each $g \in M$ there exists a minimal parabolic F-subgroup P with $C_1 \leqq v_i(P, K^g) \leqq C'$ for all i, constant C_1 from Theorem A and some constant C'.*

(b) *"Godement's compactness criterion": $G(A)^0/G(F)$ is compact if and only if G is anisotropic, i.e. there exists no proper parabolic subgroup defined over F.*

(c) *For a unipotent subgroup U of G the quotient $U(A)/U(F)$ is compact.*

For assertion (c) compare [6], Satz 3; it is valid for all groups, which can be trigonalized over the separable closure of F.

We have to transfer these results on adelized groups to finite products $G_S = \prod_{v \in S} G(F_v)$, which we consider as subgroups of $G(A)$ (taking all components outside S to be 1) and which contain S-arithmetic groups as discrete subgroups. We assume that G is concretely given as a matrix group defined over O_S, the ring of S-integers in F. Thus we have $\Gamma = \{g \in G(F) \mid g \in G(O_v) \text{ for all } v \notin S\}$, where O_v denotes the ring of integers in F_v. Moreover we fix K_v for $v \notin S$ to be $G(O_v)$ and suppose that it is the stabilizer of a special point—changing to a commensurable group Γ' if necessary.

The Theorems A, B, C and Corollary B′ remain true for the pair (G_S, Γ) instead of $(G(A), G(F))$; the constants C_1 and C_2 depend on the choice of the groups K_v, but only for $v \in S$. The Corollary A′ has to be weakened; we use

Theorem D. *$P(F) \backslash G(F)/\Gamma$ is a finite set and therefore the number of Γ-conjugacy classes of P is also finite.*

Corollary D′. *Let $P_1, \ldots, P_h$ be a complete set of representatives for the Γ-conjugacy classes of minimal parabolic F-subgroups. For suitably chosen K_S there exist for each $g \in G_S$ an element $\gamma \in \Gamma$ and an index $j \in \{1, \ldots, h\}$ such that $v_i(P_j, K^{g\gamma}) \geqq C_1'$ for all $i = 1, \ldots, r$ and an appropriate constant $C_1' \leqq C_1$.*

For the proof compare [**11**], no. 12 and [**6**] nos. 8 and 9; for $v \in S$ one has to choose again K_v as stabilizer of a special point, because one needs Iwasawa's decomposition.

3. Action on products of Bruhat–Tits-buildings

For each $v \in S$ denote by X_v the Bruhat–Tits-building of the group $G(F_v)$ and define $X := \prod_{v \in S} X_v$. Henceforth we make the following

Assumption. *G is semi-simple and simply-connected.*

This implies that all X_v and also X are polysimplicial complexes with the following properties (cf. [**22**], 3.1–3.2):

$G(F_v)$ is a group with BN-pair (or Tits-system), B being the stabilizer of an open chamber C_v of X_v and N the normalizer of a maximal F_v-split torus. The maximal compact subgroups of $G(F_v)$ are precisely the stabilizers of the vertices of X_v, which are uniquely determined by their stabilizers. There are finitely many conjugacy-classes with respect to $G(F_v)$ and as a set of representatives we can choose the stabilizers of the vertices of a fixed chamber C, we call it $\mathfrak{R}_{v,C}$, such that $\mathfrak{R} := \{\prod_{v \in S} K_v \mid K_v \in \mathfrak{R}_{v,C}\}$ is a finite set of representatives for the conjugacy-classes of maximal compact groups in G_S. In this way we can identify all vertices of X with the groups K^g for $K \in \mathfrak{R}$ and $g \in G_S$. Using property (3) of no. 2 we obtain a version of Theorem A which yields for all $K \in \mathfrak{R}$ the same parabolic subgroup P—of course for a smaller constant c_1. We denote a polysimplex by $\{K^g\}$, K running through all K in $\mathfrak{R}$.

(A) *For each polysimplex $\{K^g\}$ there exists a minimal parabolic F-subgroup P of G with $v_i(P, K^g) \geqq c_1$ for $i = 1, \ldots, r$ and all $K \in \mathfrak{R}$.*

The Corollary A′ is only valid for a special K (in general there exist finitely many K-conjugacy-classes of minimal parabolic subgroups), but if we use once more property (3) of no. 2, we may assume this corollary for all $K \in \mathfrak{R}$, provided we take a smaller constant. Again we pass from $G(A)$ to G_S and obtain the following generalization of the Corollary D′, observing that the representative P_j does not depend on K:

(D) *Let $P_1, \ldots, P_h$ denote a complete system of minimal parabolic F-subgroups of G. There exists a constant $c_1' > 0$ such that for each polysimplex $\{K^g\}$ there is a Γ-equivalent polysimplex $\{K^{g\gamma}\}$ and an index $j \in \{1, \ldots, h\}$ with the following property: $v_i(P_j, K^{g\gamma}) \geqq c_1'$ for all $i = 1, \ldots, r$ and all $K \in \mathfrak{R}$.*

Theorem B and its Corollary B′ are true simultaneously for all $K \in \mathfrak{R}$ with a constant c_2 or c_2' corresponding to c_1 or c_1' respectively.

We are going to construct a covering of X by subcomplexes.

Definition. (a) Let Q be a parabolic F-subgroup of type Θ (i.e. $Q=P_\Theta$) and $X_Q(c_2)$ a subcomplex of X, defined by the following condition:

A polysimplex $\{K^g\}$ of X belongs to $X_Q(c_2)$ if and only if there is a minimal parabolic F-subgroup P of Q such that $v_i(P,K^g)\geqq c_1$ for all i and even $v_i(P,K^g)\geqq c_2$ for all i with $\alpha_i\in\Theta$ and for all $K\in\mathfrak{R}$ in both cases.

(b) The subcomplex X_0 of X is given as follows:

A polysimplex $\{K^g\}$ of X belongs to X_0, if and only if for each minimal parabolic F-subgroup P with $v_i(P,K^g)\geqq c_1$ for all $K\in\mathfrak{R}$ and all $i\in\{1,\dots,r\}$ there is no index i such that $v_i(P,K^g)\geqq c_2$ for all $K\in\mathfrak{R}$.

From the results of reduction theory and the definition above we deduce immediately the following

Proposition 1. (a) *$X=X_0\cup X'$ with $X'=\bigcup_Q X_Q(c_2)$, Q running over all proper parabolic F-subgroups of G.*

(b) *X_0 and X' are Γ-invariant, $X_0 \bmod \Gamma$ is finite.*

In statement (b) we consider the action of Γ on X, the invariance of X_0 and X' follows from property (1) in no. 2. For the second assertion we make use of Theorem C, part (a), observing that the centre Z of a semi-simple group G is finite: If $\{K^g\}$ belongs to X_0, we have $v_i(P,K^g)<c_2$ for each index i and some $K\in\mathfrak{R}$; by property (3) of no. 2 this implies $v_i(P,K^g)\leqq\bar{c}_2$ for some constant $\bar{c}_2\geqq c_2$ and all $K\in\mathfrak{R}$. On the other hand $v_i(P,K^g)\geqq c_1$ and Theorem C(a) shows that the set of all $g\in G_S$ with $\{K^g\}\in X_0$ is relatively compact modulo Γ, which means that $X_0 \bmod \Gamma$ is finite, because all K are also open subgroups.

For the special case of a group G with F-rank 1 we may specialize these results: all proper parabolic F-subgroups are minimal and there is only one invariant v. We construct a subcomplex Y of X, which contains representatives modulo Γ. We have to use statement (D) with constant c_1' and choose a constant $c_2'>c_1'$ such that Theorem B and its Corollary B′ are valid simultaneously for all $K\in\mathfrak{R}$. With the system $\{P_1,\dots,P_h\}$ from (D) we define for $j=1,\dots,h$:

$$Y_j:=Y_j(c_2'):=\{\{K^g\}\,|\,v(P_j,K^g)\geqq c_2' \quad \text{for} \quad K\in\mathfrak{R}\} \qquad \text{and}$$

$$Y_0:=\left\{\{K^g\}\,\middle|\,\begin{matrix} c_1'\leqq v(P_j,K^g) & \text{for some } j \text{ and all } K\in\mathfrak{R} \text{ but} \\ v(P_j,K^g)\leqq c_2' & \text{for at least one } K\in\mathfrak{R}\end{matrix}\right\}.$$

Proposition 2. (a) *For each polysimplex in X there exists a Γ-equivalent polysimplex in*

$$Y:=Y_0\cup\bigcup_{j=1}^{h} Y_j.$$

(b) *Y_0 is a finite complex and the complexes Y_j are mutually disjoint.*

(c) *If there exists for given $y_j \in Y_j$ and $y_k \in Y_k$ with $j>0$, $k \geqq 0$ an element $\gamma \in \Gamma$ with $y_j^\gamma = y_k$, then we have $j=k$ and $\gamma \in P_j(F) \cap \Gamma$.*

In addition we can assume that Y is connected by blowing up the finite complex Y_0 in such a way that it becomes connected and meets all Y_j $(j=1,\dots,h)$, which are themselves connected.

4. Reduction to parabolic subgroups

From the action of Γ on the complex X we deduce now a presentation of Γ using the subcomplex Y of representatives $\operatorname{mod}\Gamma$ given in Proposition 2. This presentation contains—besides some finite set of generators and relations—the free product of the groups $\Gamma_j := P_j(F) \cap \Gamma$ with amalgamation of their mutual intersections. We use the following principle:

Theorem E. *The group Γ acts (on the right) on the poly-simplicial complex X with simplicial operation on each factor of X. We suppose X to be connected and simply-connected. Let Y be a subcomplex of X such that for each polysimplex $x \in X$ there exists a polysimplex $y \in Y$ and $\gamma \in \Gamma$ with $y\gamma = x$. Then $E := \{\gamma \in \Gamma \mid Y\gamma \cap Y \neq \emptyset\}$ is a set of generators for the group Γ and $R := \{\gamma_1 \cdot \gamma_2^{-1} \cdot (\gamma_2\gamma_1^{-1}) \mid Y\gamma_1 \cap Y\gamma_2 \cap Y \neq \emptyset\}$ is a system of defining relations in E.*

Theorem E is proved in [2], Example 4.6 for simplicial complexes.

We apply this theorem to Γ, X and Y from Section 3 (denoted in the same way):

A Bruhat–Tits-building and therefore X is contractible and we can assume that Y is connected. The operation of Γ on X is simplicial on each factor of X: if S contains more than one prime, the operation is defined on each factor X_v $(v \in S)$ separately; if G is semi-simple the simple factors of G are acting only on one simplicial factor of the building.

Corresponding to the decomposition of Y we divide E and R into the following parts:

$$E_0 := \{\gamma \in \Gamma \mid Y_0\gamma \cap Y_0 \neq \emptyset\}$$

is a finite set, because Y_0 is a finite complex and all stabilizers of vertices are finite as intersections of a compact and a discrete group;

$$E_j := \{\gamma \in \Gamma \mid Y_j\gamma \cap Y_j \neq \emptyset\}$$

is contained in $\Gamma_j = P_j(F) \cap \Gamma$ (cf. Proposition 2(c));

$$R_0 := \{\gamma_1 \cdot \gamma_2^{-1} \cdot (\gamma_2\gamma_1^{-1}) \mid Y_0\gamma_1 \cap Y_0\gamma_2 \cap Y_0 \neq \emptyset\}$$

is finite;

$$R_j := \{\gamma_1 \cdot \gamma_2^{-1} \cdot (\gamma_2 \gamma_1^{-1}) \mid y_1 \gamma_1 = y_2 \gamma_2 = y_3 \text{ with } y_\lambda \in Y_j \text{ for some } \lambda \in 1, 2, 3\}.$$

R_j consists only of relations in the group Γ_j, because we have the following implications from Proposition 2(c):

$$y_1 \in Y_j \Rightarrow \gamma_1, \gamma_1 \gamma_2^{-1} \in \Gamma_j;$$

$$y_2 \in Y_j \Rightarrow \gamma_2, \gamma_2 \gamma_1^{-1} \in \Gamma_j;$$

$$y_3 \in Y_j \Rightarrow \gamma_1^{-1}, \gamma_2^{-1} \in \Gamma_j.$$

Moreover we have to take into consideration that the same generator may belong to different sets E_0 or E_j whereas our relations are products in a fixed set; therefore we must add identifying relations:

$$R_{0j} := \{\gamma_1 \cdot \gamma_2^{-1} \mid \gamma_1 \in E_0, \gamma_2 \in E_j\}$$

is finite for all j;

$$R_{jk} := \{\gamma_1 \gamma_2^{-1} \mid \gamma_1 \in E_j, \gamma_2 \in E_k\}$$

identifies the intersection $\Gamma_j \cap \Gamma_k$ with subgroups of Γ_j and Γ_k respectively.

We sum up these considerations in

Proposition 3. *Γ is finitely presentable if all subgroups $\Gamma_j = \Gamma \cap P_j(F)$ are finitely presented and all intersections $\Gamma_j \cap \Gamma_k$ are finitely generated.*

5. Compact presentability

We start with the observation that Γ_j is not contained in $(P_j)_S = \prod_{v \in S} P_j(F_v)$ but even in $(P_j)_S^0$, since $|\chi(\gamma)|_v = 1$ for $v \notin S$ and we have the product formula $\prod_{v \in V} |\chi(\gamma)|_v = 1$.

By application of the Reidemeister–Schreier-principle we are now able to translate our problem of finite presentability into a question of compact presentability—several definitions of this notion are given in [**1**]—using the following

Theorem F. *Let Γ be a S-arithmetic subgroup of the algebraic group H and suppose that H_S^0/Γ is compact; then we have:*

Γ is finitely generated or finitely presented if and only if H_S^0 is compactly generated or respectively finitely presented.

Theorem F is proved in [**15**] in the more difficult situation of number fields where one has to reduce the problem to the finite generation (presentation) of an ordinary

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arithmetic group (defined for $S=S_\infty$, the set of archimedean primes); we can replace this group by the finite group $H(\bar{F})$, where $\bar{F}$ denotes the field of constants in F. Furthermore we have to use H_S^0 instead of the product $\prod_{v\in S\setminus S_\infty} H_v$, according to the fact that the quotient $\prod_{v\in S\setminus S_\infty} H_v(k_v)/\Gamma$ is always compact for a number field k and a S-arithmetic group Γ.

In our case we have to show that P_S^0/Γ_P for $\Gamma_P=\Gamma\cap P(F)$ is compact, if P denotes a minimal parabolic F-subgroup. But P is a semi-direct product of a reductive group Z_P and its unipotent radical U_P and we have the decomposition $P_S^0=(Z_P)_S^0(U_P)_S$. For the second factor we have part (c) of Theorem C and for the first one we can apply part (b) of the same theorem since a minimal parabolic group has "semi-simple rank 0", which means that P^0 is anisotropic. Thus we obtain the following consequence of Proposition 3:

Proposition 4. *Γ is finitely presentable if for minimal parabolic F-subgroups P and P' of G the group P_S^0 (and also $(P')_S^0$) is compactly presentable and the intersection $P_S^0\cap(P')_S^0$ is compactly generated.*

For the proof of compact presentability we dispose of the following results due to Abels:

Theorem G. (a) *Let $1\to A\to B\to C\to 1$ be an exact sequence of locally compact topological groups.*

(i) *If B has a compact presentation and A contains a compact subset K, such that A is the smallest closed normal subgroup of B containing K, then C has a compact presentation too.*

(ii) *If A and C are compactly presentable, the same is true for B.*

(b) *Let H be the semi-direct product of a torus T and a unipotent subgroup U, both defined over the local field F_v. If $T(F_v)$ contains an element, which acts on $U(F_v)$ as a contracting automorphism, then $H(F_v)$ has a compact presentation.*

Remark. An automorphism α of a locally compact topological group N is called contracting if the sequence α^n, $n\in\mathbb{N}$ converges to the map $N\to\{e\}$ uniformly on compact subsets.

Statement (a) is part of the "diagram-lemma" of [1]. Assertion (b) is proved in [4], for the case of a semi-direct product $\langle t\rangle \ltimes N$ with $\langle t\rangle\simeq\mathbb{Z}$ and N locally compact, but this is enough in view of (a) and the fact that $T(F_v)$ is compactly presentable. We shall use part (a) of this theorem in the following way: We assume for simplicity that G is absolutely almost-simple and simply connected of F-rank 1 and we consider P_S^0 for a minimal parabolic F-subgroup P of G. P is the semi-direct product of $Z(T)$ and U, where $Z(T)$ is the centralizer of a maximal F-split torus T of G, which has dimension 1 and U denotes the unipotent radical of P. Moreover $Z(T)$ is an almost-direct product of T and an anisotropic group M; it follows that $M_S\cdot T_S$ has finite index in $Z(T)_S$ since there is an isogeny of reductive groups from $M\times T$ on MT. Furthermore T_S^0 is contained in P_S^0, because the character group $\hat{P}(F)$ has finite index in $\hat{T}(F)$ and of course U_S and M_S are subgroups of P_S^0. According to Reidemeister–Schreier it is enough to prove compact presentability for the product $M_S T_S^0 U_S$ in order to prove it for P_S^0.

Before doing this in three different situations we can make two general remarks:

(1) The intersections $P_S^0 \cap (P')_S^0$ (for parabolic subgroups P and P') are always compactly generated: From the properties of a BN-pair ([**9**], 2.6) we have that $P \cap P' = Z(T')$, the centralizer of a suitable maximal F-split torus T'; now $T'(F_v)$ is compactly presented and the anisotropic part of $Z(T')$ is a reductive group—but we have

(2) For a reductive group H, defined over F, $H(F_v)$ and H_S are compactly presentable: Considering the action on the Bruhat–Tits-building we see, that $H(F_v)$ is the amalgamated sum of the stabilizers of the vertices of a fundamental chamber (by an amalgamated sum of a family of groups $\{G_i\}_{i \in I}$—where all G_i are contained in some group G—we understand the direct limit of these groups and their intersections $G_{ij} = G_i \cap G_j$ with respect to the injections $G_{ij} \hookrightarrow G_i$ for all $i, j \in I$).

6. The case $r=1$ and $s>1$

Now we have all the tools to prove the main theorem; in this section we shall settle the first two cases.

(a) $s=3$, $r=r_1=r_2=r_3=1$

From the last section we know that it is enough to show the compact presentability of $T_S^0 U_S$ (in this case M_S is even compact, since M remains anisotropic over $F_i := F_{v_i}$ for $i=1,2,3$).

We define three subgroups

$$H_i := T_S^0 \prod_{j \neq i} U(F_j) \subset T_s^0 U_S \quad \text{for} \quad i, j \in \{1,2,3\},$$

to which we shall apply Theorem G(b).

U is the semi-direct product $U_\alpha U_{2\alpha}$, where $\{\alpha\}$ or $\{\alpha, 2\alpha\}$ denotes the set of positive roots with respect to T ($U_{2\alpha} = \{1\}$ in the first case). Then T acts on U by inner automorphisms, which means by multiplication with $\alpha(t_i)$ or $\alpha^2(t_i)$ respectively on the F_i-vector space $U(F_i)$ for $t_i \in T(F_i)$. Therefore we can find an element $t = \prod_{i=1}^3 t_i \in T_S$, which induces contracting automorphisms on $U(F_1)$ and $U(F_2)$, if we choose $|\alpha(t_1)|_1 < 1$ and $|\alpha(t_2)|_2 < 1$ and we have even $t \in T_S^0$ by defining t_3 in such a way that $|\alpha(t_3)|_3 = (|\alpha(t_1)|_1 |\alpha(t_2)|_2)^{-1}$. So we have a contracting element for the group $H_3 = T_S^0 U(F_1) U(F_2)$ and we can do the same for the groups H_1 and H_2. Observe that the third component, where the automorphism would be expanding, is trivial! By Theorem G(b) we conclude that all three groups H_i are compactly presented and also that their intersections are compactly generated. Thus the amalgamated sum of H_1, H_2 and H_3 is compactly presented, but this product is nothing else than the whole group $T_S^0 U_S$, since all relations, which define this group as a semidirect product or give the commutability of the three factors $U(F_i)$ are relations in one of the three groups H_i.

(b) $s=2$, $r=1$, $r_1>1$ (without loss of generality)

In analogy with case (a) we define the following groups:

$$H_1:=M_S\cdot T_S^0\cdot U(F_1) \quad \text{and} \quad H_2:=M_S\cdot T_S^0\cdot U(F_2).$$

In the same way we obtain that $T_S^0\cdot U(F_1)$ and $T_S^0\cdot U(F_2)$ are compactly presentable; on account of the last remark of Section 5 this is also true for M_S and therefore by Theorem G-(a) for H_1 and H_2. But this time the amalgamated sum of H_1 and H_2 gives not P_S^0, since $U(F_1)$ and $U(F_2)$ do not commute in this sum. So we need further factors:

We take a maximal F_1-split torus T' in M, which is not trivial by the assumption $r_1>1$. T' normalizes U and therefore we can split up U into root-subgroups with respect to T' and the intersection of U with the centralizer of T' (cf. [**8**], 3.11). Let $R=R_+\cup R_-$ be the system of all roots with respect to T', then we have the following decomposition (defined over F_1): $U=U_+\cdot U_0\cdot U_-$, where U_+ is the product of all unipotent subgroups U_α with $\alpha\in R_+$, U_- the same for $\alpha\in R_-$ and $U_0=U\cap Z(T')$.

Now we consider the groups $H_+:=T'(F_1)\cdot U_+(F_1)$ and $H_-:=T'(F_1)\cdot U_-(F_1)$, which are compactly presentable, because we find contracting elements $t'\in T'(F_1)$, choosing $|\alpha(t')|_1<1$ for all $\alpha\in R_+$ or $\alpha\in R_-$ respectively—and are able to apply again Theorem G(b).

As a next step we define semi-direct products L_+ of H_+ and $T_S^0\cdot U(F_2)$ and L_- of H_- and $T_S^0\cdot U(F_2)$, H_+ and H_- being normal. According to Theorem G(a) these groups L_+ and L_- have compact presentations; inside L_+ the commutability of $U_+(F_1)$ and $U(F_2)$ is defined and the same is true for $U_-(F_1)$ and $U(F_2)$ within L_-. If we can show that this implies that even $U_0(F_1)$ commutes with $U(F_2)$, we are finished, because $M_S\cdot T_S^0\cdot U_S$ is the amalgamated sum of H_1, H_2, L_+ and L_-, which are all compactly presentable and it is easy to see that all their intersections are at least compactly generated.

For this purpose we choose a maximal F_1-split torus $\bar{T}$ in G, that contains T and T'—which is possible since $T\cap T'$ is finite; on the other hand $T\cdot T'$ is of finite index in $\bar{T}$. U_0 can be given as the product of groups U_a with roots a corresponding to $\bar{T}$ and obviously we have $a|T=\alpha$ or $a|T=2\alpha$ and $a|T'=0$. If there exists a root b (with respect to $\bar{T}$ and defined over F_1) with $b|T=0$, it is possible to select b in such a way that $a+b$ is also a root (here we have to suppose that G is absolutely almost-simple: for this statement and the following conclusions see [**6**], no. 15–18). Since $[\bar{T}:T\cdot T']$ is finite we must have $b|T'\neq 0$, which implies either $U_{a+b}(F_1)\subseteq L_+$ and $U_{-b}(F_1)\subseteq L_-$ or the converse, the same is true for all groups $U_{r(a+b)+s(-b)}(F_1)$ with $r,s\in\mathbb{N}$. Now we have to use Chevalley's commutator formula in order to compute $[U_{a+b},U_{-b}]$, which shows that each element in $U_a(F_1)$ is a product of elements contained either in $U_+(F_1)$ or $U_-(F_1)$; this is valid for all a and therefore also for $U_0(F_1)$. We know already that all these factors commute with $U(F_2)$ and conclude that $U_0(F_1)$ and $U(F_2)$ are commutable too.

But it may happen that there is no root b with $b|T=0$; looking at the classification tables in [**21**] we see that there is only one such case, namely the type ${}^2A_n$. This means that we have $G=SU_3(D,h)$ with a skew-field D and a hermitian from h with respect to an involution of the second kind of D. For this group we have the following decomposition (defined over F_1): $U=U_a\cdot U_b\cdot U_{a+b}$ with $a|T=b|T$ and $a|T'=(-b)|T'$, which implies $U_0=U_{a+b}=[U_a,U_b]$; since $U_a\subseteq U_+$ and $U_b\subseteq U_-$ we have the same argument as before.

7. The case $r=1$ and $s=1$

(a) For that case I am not able to give a unified proof; so we have to consider all groups G with global rank 1 and local rank at least 3 (i.e. $rk_F G=1$ and $rk_{F_v}G\geqq 3$ with $S=\{v\}$)—as before we assume G to be absolutely almost simple but not necessarily simply connected. In order to classify these groups we have to use the diagrams given in [**21**], but there is no specification for global fields of finite characteristic, but Professor Tits has communicated to me the following supplement:

The set of diagrams for global function fields is contained in the set for number fields, but the anisotropic part has to be of inner or outer type A_n. Thus we have the following list of types with global rank 1 and local rank $\geqq 3$ (written "global→local"):

${}^1A_n\to{}^1A_n$	$(n\geqq 3)$	${}^2D_n\to{}^1D_n$	$(n=3,4,5)$
${}^2A_n\to{}^1A_n$	$(n\geqq 3)$	${}^2D_n\to{}^2D_n$	$(n=4,5)$
${}^2A_n\to{}^2A_n$	$(n\geqq 3)$	${}^{3(6)}D^9_{4,1}\to{}^1D_4$ or ${}^2D_4$	
$C_3\to C_3$		${}^2E^{35}_{6,1}\to{}^2E^2_{6,4}$ or ${}^1E^0_{6,6}$	
${}^1D_n\to{}^1D_n$	$(n=4,5)$		

Since we have to prove the compact presentability of a group over F_v, we should start with $\bar G=G\otimes F_v$ and identify in such a group the proper parabolic subgroup P of G, defined over F, as well as the character χ which defines P^0. For this purpose we imbed the maximal F-split torus T of G into a maximal F_v-split torus $\bar T$ of $\bar G$ and extend χ to a character on $\bar T$, which gives us $\bar T^0$. Moreover let $\bar P$ be a minimal F_v-parabolic subgroup, which is contained in P and has the decomposition $\bar P=Z(\bar T)\cdot\bar U$ ($\bar U$ the unipotent radical of $\bar P$). Since the quotient $\bar G(F_v)/\bar P(F_v)$ is compact, the same is true for $P^0(F_v)/\bar P^0(F_v)$ and we can restrict ourselves to $\bar P^0$. $Z(\bar T)$ contains an anisotropic group $\bar M$, such that $\bar M(F_v)$ is compact, and the torus $\bar T\supseteq\bar T^0$. Thus it remains to prove that $\bar T^0(F_v)\cdot\bar U(F_v)$ has a compact presentation—using again Theorem G(a).

We decompose $\bar U$ into root groups with respect to $\bar T$ and look for elements in $\bar T^0(F_v)$ which provide us with contracting homomorphisms for appropriate subgroups of $\bar U(F_v)$, given as products of root groups $\bar U_a$. In this way we obtain compactly presented groups (by Theorem G(b)), which we amalgamate in order to show that $\bar T^0(F_v)\cdot\bar U(F_v)$ has a compact presentation. But in general $\bar U$ contains a subgroup $\bar U_0$, on which $\bar T^0$ acts trivially. For those root groups $\bar U_a\subseteq\bar U_0$ we have to show that the elements of $\bar U_a(F_v)$ can be written as commutators of elements which are already contained in "good" subgroups, described above. This definition as a commutator may not be unique; so we have to prove that different commutators give the same element which can be done using an identity due to Philip Hall. In the last step we have to check that all commutator relations in $\bar U$ are either valid in some good subgroup or follow from these relations, using a convenient definition for the elements in $\bar U_0(F_v)$ as a commutator.

(b) As an example we shall give the details for a group which is locally of type ${}^1D_4$ with rank 4 and comes from a global group of type ${}^1D_4$ or ${}^2D_4$ with rank 1. We may think of the local group as $G=SO_8$, corresponding to the direct sum of four hyperbolic planes, the global group being $G=SU_4(D,s)$ for an anti-hermitian form s over a quaternion algebra D with standard involution.

(i) We need the list of positive roots of $\bar{G}$ with respect to a maximal (split) torus $\bar{T}$ (cf. [**9**], planche IV):

$$a_1,\ a_2,\ a_3,\ a_4;\quad a_1+a_2,\ a_2+a_3,\ a_2+a_4;\quad a_1+a_2+a_3,\ a_1+a_2+a_4,\ a_2+a_3+a_4;$$
$$a_1+a_2+a_3+a_4;\quad a_1+2a_2+a_3+a_4.$$

The group G and its maximal split F-torus T is defined by $a_1|T=a_3|T=a_4|T=0$ (the unique positive F-root is $\alpha=a_2|T$), the proper parabolic F-subgroup of G is then given by its character $c:=a_1+2a_2+a_3+a_4$ (the second fundamental weight $\bar{\omega}_2\approx\varepsilon_1+\varepsilon_2$: cf. [**9**], loc. cit.), therefore $\bar{T}^0$ is defined by $c=0$.

(ii) We shall now construct subgroups H_i of $\bar{U}(F_v)$, which admit a contracting automorphism, induced by an element $t\in\bar{T}^0(F_v)$, such that $\bar{T}^0\cdot H_i$ is a compactly presented group. For such an element t we have to satisfy the condition $c=a_1+2a_2+a_3+a_4=0$. That is easy, if we have free choice for one root a_j: For each H_i we designate only the roots a for which $\bar{U}_a$ belongs to H_i.

$$H_1: a_1,\ a_2,\ a_3;\quad a_1+a_2,\ a_2+a_3;\quad a_1+a_2+a_3.$$

$$H_2: a_1,\ a_2,\ a_4;\quad a_1+a_2,\ a_2+a_4;\quad a_1+a_2+a_4.$$

$$H_3: a_2,\ a_3,\ a_4;\quad a_2+a_3,\ a_2+a_4;\quad a_2+a_3+a_4.$$

$$H_4: a_1,\ a_3,\ a_4.$$

We have to add further groups H_i, for which we have to fix all values $|\alpha_i(t)|_v$ for $i=1,\dots,4$; it is more convenient to use the normalized additive valuation $v(\ )=$ const. $\exp(-|\ \ |_v)$. By this means we define the groups H_i by denoting an integer-valued valuation vector with components $v(\alpha_i(t))$ for a contracting element t.

$$H_5\leftrightarrow(2,-3,2,2): a_1,\ a_3,\ a_4;\quad a_1+a_2+a_3,\ a_1+a_2+a_4,\ a_2+a_3+a_4;$$
$$a_1+a_2+a_3+a_4.$$

$$H_6\leftrightarrow(2,-1,0,0): a_1+a_2;\quad a_1+a_2+a_3+a_4;\quad \textit{et al.}$$

$$H_7\leftrightarrow(0,-1,2,0): a_2+a_3;\quad a_1+a_2+a_3+a_4;\quad \textit{et al.}$$

$$H_8\leftrightarrow(0,-1,0,2): a_2+a_4;\quad a_1+a_2+a_3+a_4;\quad \textit{et al.}$$

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It is easy to check that within these groups H_i $(i=1,\dots,8)$ all commutation relations between root groups are defined—with the exception of those commutators which produce an element in U_c.

(iii) Our next tool is P. Hall's identity for commutators $[x,y]=x^{-1}y^{-1}xy$ with $x^y:=y^{-1}xy$:

$$[x^y,[y,z]]\cdot[y^z,[z,x]]\cdot[z^x,[x,y]]=1,$$

from which we deduce

$$[[x,y],z]=[x,[y,z]] \quad \text{if} \quad [x,z]=1 \text{ and } [x,y] \text{ commutes with } x \text{ and } [y,z]. \qquad (*)$$

In our group $\bar{G}$ the following relations are valid:

$$\bar{U}_c=\bar{U}_{a_1+2a_2+a_3+a_4}=[\bar{U}_{a_2},\bar{U}_{a_1+a_2+a_3+a_4}] \qquad (1)$$

$$=[\bar{U}_{a_1+a_2},\bar{U}_{a_2+a_3+a_4}] \qquad (2)$$

$$=[\bar{U}_{a_1+a_2+a_3},\bar{U}_{a_2+a_4}] \qquad (3)$$

$$=[\bar{U}_{a_1+a_2+a_4},\bar{U}_{a_2+a_3}] \qquad (4)$$

For the required presentation of $\bar{T}^0(F_v)\cdot\bar{U}(F_v)$ we define an element of $\bar{U}_c$ by equation (2) and we obtain as a consequence of $(*)$ that the other formulas (1), (3) and (4) are also true.

(iv) To get the missing commutation relations for $\bar{U}_c$ we can now use the various descriptions (1)–(4) and we have to show that $\bar{U}_c$ is contained in the centre of $\bar{U}$; it is enough to prove that $\bar{U}_c$ commutes with $\bar{U}_{a_i}$ for $i=1,\dots,4$.

For $\bar{U}_{a_2}$ we use equation (2) and the fact that it commutes with $\bar{U}_{a_1+a_2}$ and $\bar{U}_{a_2+a_3+a_4}$, which takes place in H_1 and H_3 respectively.

For $\bar{U}_{a_i}$ with $i=1$, 3 or 4 we use equation (1) and have to compute e.g. for $i=1$ the following commutator:

$$[\bar{U}_{a_1},[\bar{U}_{a_2},\bar{U}_{a_1+a_2+a_3+a_4}]]\ni[x,[y,z]]$$

with $[x,z]=1$ and $[x,y]$ commuting with all other terms, thus we have:

$$[x,[y,z]]=x^{-1}y^{-1}z^{-1}yzxz^{-1}y^{-1}zy=x^{-1}y^{-1}z^{-1}(yxy^{-1}x^{-1})xzy$$

$$=x^{-1}y^{-1}xy(yxy^{-1}x^{-1})=[x,y][x^{-1},y^{-1}]^{-1}$$

but this product equals one according to the formulas $[\bar{u}_a(r),\bar{u}_b(s)]=\bar{u}_{a+b}(rs)$ and $\bar{u}_a(r)^{-1}=\bar{u}_a(-r)$ for arbitrary roots a,b and $a+b$ of $\bar{G}$ and $r,s\in F_v$.

Remarks. (1) I have to admit that this example is one of the easiest, in general the computations are much longer.

(2) The proof is modelled after Abels' example of a finitely presented solvable group with a non-finitely presented quotient, given in [3].

(3) If we could use Abels' main theorem in [4] (5.6.1 is proven only for characteristic 0 and formulated for Lie algebras), parts (ii) and (iv) from (b) were unnecessary.

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