Session D

Stellar evolution, nucleosynthesis and convective mixing

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Convection and mixing in stars: theory versus observations

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Abstract. We summarize the results of stellar evolution theory obtained over the past three decades and highlight the points of disagreement with the present-day observational information. Arguments are given to favour stellar models in which the classical treatment of convection and mixing is abandoned and more complex schemes are adopted.

Keywords. Convection, stars: structure, stars: evolution, Hertzsprung-Russell diagram

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1. Convection and mixing: the great uncertainties

Despite the great achievements of the stellar evolution theory, there are many points of disagreement between theory and observations which are ultimately related to our poor knowledge of the extension of convective regions (cores, shells and envelopes) and associated mixing. The local Schwarzschild criterion $\nabla_R = \nabla_A$ (SC) provides the simplest evaluation of the extension of convective regions and the Mixing Length Theory (MLT) simplifies the complicated pattern of motions therein by saying that full, and instantaneous mixing takes place. *The reality is, however, more complicated than this simple picture.* First of all, inconsistencies are known to develop at the border of convective regions which give rise to the so-called *H-semi-convection* during the core H-burning of high mass stars (HMS), *He-semi-convection* and *breathing convection* in the core He-burning phase of low (LMS) and intermediate mass stars (IMS). Secondly, convection and mixing are non-local phenomena that in principle can also affect (*overshoot*) into regions that are stable according to the SC. Finally, mixing requires a certain amount of time to occur, and may not be complete. Unfortunately as soon as the classical scheme is abandoned, things lose simplicity and a unique solution does not exist, thus making uncertain the effects of convection and mixing. Nevertheless, it is worth recalling that (i) the size of the central convective core affects the luminosity, effective temperature, and lifetime of the corresponding evolutionary phase; (ii) the inward extension of the external convection may affect the surface abundances; (iii) the size of the intermediate convective shells has less straightforward effects: in HMS it may affect the extension of the blue loops, in IMS and LMS it may alter the surface abundances during the late evolutionary stages (RGB and AGB).

Hydrogen semiconvection. During the core H-burning phase of HMS, radiation pressure and electron scattering opacity give rise to a large convective core surrounded by an H-rich region, which is potentially unstable to convection if the original gradient in chemical abundance is maintained, but stable if suitable mixing is allowed to take place so that neutrality is restored. Negligible energy flux is carried by the process. The chemical gradient depends on which neutrality condition is used, either Schwarzschild (1958) or Ledoux (1947). The former gives smoother chemical profiles and in some cases

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leads to the onset of an intermediate fully convective layer. The Ledoux criterion is a stronger condition favouring stability with respect to the SC. Similar instability occurs also during the early shell H-burning stages. The effects of H-semiconvection on the evolution of HMS have been summarized by Chiosi & Maeder (1986) and most recently by Chiosi *et al.* (1992a).

Helium Semiconvection. As He-burning proceeds in the convective core of stars of any mass, the C-rich mixture inside the core becomes more opaque than the C-poor material outside; therefore the radiative temperature gradient increases within the core. The resulting super-adiabaticity at the edge of the core leads to a progressive increase (local convective overshoot) in the size of the convective core during the early stages of He-burning, see Schwarzschild (1970), Paczynski (1971), Castellani *et al.* (1971a,b). Once the convective core exceeds a certain size, overshoot is no longer able to restore the neutrality condition at the border due to a characteristic turn-up of the radiative gradient. The core splits into an inner convective core and an outer convective shell. As further helium is captured by the convective shell, this latter tends to become stable, leaving behind a region of varying composition in which $\nabla_R = \nabla_A$. This type of mixing is called He-semiconvection, whose extension varies with the star mass, being important in LMS and IMS up to about 5 $M_\odot$, and negligible in more massive stars.

Helium Breathing Convection. In all computed models, when $Y_c \leqslant 0.1$, the convective core may undergo recurrent episodes of rapid increase followed by an equally rapid decrease until it engulfs the whole semiconvective region. Castellani *et al.* (1985) have named this phase "breathing pulses of convection". Semiconvection increases the core He-burning lifetime (by approximately a factor of two), whereas breathing convection increases the mass of the C-O core left over at the end of He-burning phase. This fact will greatly shorten the early AGB phase. Models with semiconvection and models with semiconvection plus breathing convection have different predictable effects on the expected ratio of the number of AGB stars to the number of HB stars in well studied globular clusters. Chiosi (1986) argued that breathing convection is most likely an artifact of the idealized algorithms used in describing mixing. Given that breathing convection is a consequence of the time-independent treatment of semiconvection, and that both are based on local descriptions of mixing, the question arises whether nonlocal, e.g. full convective overshoot (see below), and/or time-dependent mixing may overcome the above difficulties.

2. Does the classical theory fully agree with the observations?

Stellar models calculated with the above prescriptions are often named *the classical models*. They are the body of theoretical predictions to which observational data are compared. In the following we will review a number of observational facts likely suggesting that in stellar interiors the convective regions extend beyond the classical limit. As the list is long, we will limits ourselves to a few cases:

Widening of the Main Sequence. Long ago Maeder & Mermillod (1981) analyzing clusters like the Pleiades noticed that the main sequence extends to too bright a luminosity to fit standard models and suggested a certain amount of overshoot.

Intermediate Age Clusters. The young rich clusters of the Magellanic Clouds (MCs) are classical templates to which the results of stellar evolution theory are compared. Looking at the case of NGC 1866, (i) for the observed luminosity of the giants, there are too many stars above the predicted main sequence turn-off, whose number is a significant fraction of the number of giant stars; (ii) the predicted ratio of post main sequence stars to the main sequence stars was about four times the observed one. To cast light on this problem, Chiosi *et al.* (1989) introduced the concept of the integrated luminosity function of main sequence stars normalized to the number of giants (NILF), which simply reflects the ratio of core He- to H-burning lifetimes, and concluded that only models with substantial core overshoot could match the observed NILF. The analysis of other MC's clusters, like NGC 1831 and NGC 2164 (Vallenari *et al.* 1991, 1992), yielded similar results. This conclusion has been recently strengthened by Keller *et al.* (1998), who presented new HST photometry for several rich clusters of the MCs.

Old Open Clusters. The old open clusters (ages from 1 to 7-8 $\times 10^9$ yr) having turn-off masses between 1 $M_\odot$ and 2 $M_\odot$, probe the stellar structure in the mass range, in which on the main sequence the transition occurs from radiative to convective cores, from pp chain to CNO cycle, and from very bright to much less evident RGBs. Long ago Barbaro & Pigatto (1984) pointed out that the CMD of these clusters (e.g. NGC 2420, NGC 3680, IC 4651, M67, and others) does not agree with the predictions from classical models. The main signatures of disagreement were the detailed shape of the turn-off and RGB, the clump of red stars (most likely core He-burners), and the number of stars brighter than the main sequence at the beginning of the sub-giant branch with respect to the main sequence stars. All this suggested that a certain amount of convective overshoot during the core the H-burning phase ought to occur. Aparicio *et al.* (1990) first tried to establish the amount of required overshoot and how its onset should depend on the star mass. They suggested that in this mass interval convective overshoot should gradually increase with the star mass. Their recipe has been adopted in all stellar models of the Padua library (see Bertelli *et al.* 1994). Strong support to this idea came from the study of the CMD of IC 4651 by Bertelli *et al.* (1992) and the systematic analysis of several old open clusters by Carraro & Chiosi (1994). Recently, Rosvick & VandenBerg (1998) and Gim *et al.* (1998) have re-addressed this topic analyzing the CMD of NGC 6819 and NGC 7789, two rich open clusters whose age is 2.5 and $\geqslant 1.6 - 1.7$ Gyr and whose turn-off mass is about 1.6 and $1.8 M_\odot$ respectively. In both cases the morphology at the turn-of is such that *"only models with substantial core overshoot can explain"*. Another type of evidence comes from eclipsing binaries, for which good determinations of mass, radius, luminosity, and abundances are available for stars near the turnoff, see Andersen *et al.* (1990), Napiwotzki *et al.* (1991), Nordstrom *et al.* (1996), Pols *et al.* (1997). Since many stars fall beyond the limit for the core H-burning phase of classical models, a cooler turnoff seems to be required, which was attributed to substantial overshoot.

Star Counts in Globular Clusters. Old globular cluster are perhaps ideal laboratories for testing mixing theories, because the core H-burning phase is radiative and central mixing occurs only during the core He-burning stage. Furthermore, they are well populated so that star counts acquire statistical significance and the comparison with their theoretical counterparts (lifetimes) is meaningful. Finally, evolved stars in globular clusters are confined in a narrow range of initial masses so that tests are basically made at constant mass.

LF of RGB Stars: the bump. Observations of RGB stars in globular clusters (Fusi-Pecci *et al.* (1990) confirm the existence of a bump in the differential LF of these stars. Since the RGB-LF maps the H-profile established during the central H-burning phase (Renzini & Fusi-Pecci (1988), the luminosity of the bump identifies the mass coordinate of the bottom of the homogeneous envelope and in turn the maximum depth reached by the external convection. Fusi-Pecci *et al.* (1990), revising the relationship between M_V^{HB} and metallicity of HB stars, noticed that the observed M_V^{BRGB} was about 0.415 ± 0.07 fainter than predicted by standard models. Three possible causes were singled out: (i) higher opacity (0.2 mag), (ii) lower initial He content (0.2 mag if Y=0.20 instead of 0.235), (iii) deeper extension of the envelope convection. Opacity is unlikely because of the low

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metallicity in most globular clusters. Much lower He content has some drawbacks, i.e. much higher turn-masses, and lower ages in turn. Convective envelopes extending deeper by about about $0.7 \times H_P$ could remove the discrepancy (Alongi *et al.* (1991). However, Cassisi & Salaris (1997) have argued that the discrepancy goes away even with normal extension of the convective envelopes if updated input physics is used.

Relative Frequency of AGB Stars. The ratio of star counts $R_2 = N_{AGB}/N_{HB} = t_{AGB}/t_{HB}$ is known to depend on the kind of mixing once the mass of the HB stars is determined (initial He content). The comparison of theoretical predictions for ratio t_{AGB}/t_{HB} derived from different mixing schemes with $R_2 N_{AGB}/N_{HB}$ leads to the following results. In models calculated with the SC, the amount of mixed material is too low and in turn R_2 is too high. In models with semi-convection, the extension of the mixed region is too large and in turn R_2 is too low. Breathing convection makes things even worse because only half of the AGB stars can be accounted for. Finally, models with non local mixing are able to reconcile theory and observations if the overshoot region extends about $1 \times H_P$ across the Schwarzschild border, see e.g. Chiosi *et al.* (1987).

AGB & Carbon Stars. The TP-AGB phase of LMS and IMS is at base of our current understanding of the LF of AGB stars in general, the relative frequency of C-stars, the surface abundances of chemical elements (s-process included), the initial-final mass relationship, the stellar yields of several important elements, etc... This phase is quite complicated as it requires solid understanding of several phenomena such as (i) inter-shell nucleosynthesis, (ii) convective dredge-up, (iii) envelope burning in most massive stars ($M \geqslant 3 - 4M_\odot$), and (iv) mass loss by stellar winds. Items (ii) and (iv) determine the overall uncertainty. The most recent TP-AGB models are by Marigo *et al.* (1996, 1998, 1999), Marigo (1998), and Wagenhuber & Groenevegen (1998) to whom the reader should refer for all details and exhaustive referencing. Perhaps the most important consequence of envelope burning is the breakdown at high luminosity of the core mass-luminosity ($M_c - L$) relation. While the luminosity ascent of low-mass AGB stars ($M \leqslant 3M_\odot$) in which the H-burning shell is the dominant energy source is governed by this relation, massive AGB stars can rapidly reach much higher luminosity. This anticipates the ejection of the envelope. Current models can reasonably explain the LF of carbon stars, the distribution of these in the M_{bol} vs log(age) plane, and the surface abundances. Nevertheless there are a few debated points:

More mixing? With the standard efficiency of the 3^{rd} dredge-up mechanism too few if none at all low luminosity C-stars are formed in contrast with the observations. This hints that more efficient external mixing is required, see Hervig *et al.* (1997), Marigo *et al.* (1996, 1998, 1999).

Paucity of Bright C-Stars. Why there are so few bright C-stars? Three explanations are possible: (1) Envelope burning (fresh C is converted into N via CNO cycle); (2) Very efficient stellar wind that removes the envelope before the C-phase is started; (3) Convective overshoot that lowers M_{up} down to $5 - 6M_\odot$ instead of the $8 - 9M_\odot$ of the classical models.

Initial-Final Mass Relationship: IFMR . Establishing the relation between the final mass M_F, left at the end of the AGB phase, and the initial mass M_I of the progenitor is of paramount importance for various aspects related to the previous evolution, mixing being first in the list. Unfortunately, at present both the observational and theoretical determinations of the IFMR still suffer from great uncertainty because of its semi-empirical nature. M_F requires some theoretical support (mass-radius relation) which, however, is quite well established (Weidemann 1990; D'Antona & Mazzitelli 1990). M_I is more difficult to assess as it depends on the kind of mixing at work. The theoretical predictions show a clear disagreement with Weidemann (1990) for the solar vicinity, concerning both the shape of the relation and the value of M_{up}. A better accordance is found with the IFMR by Hervig *et al.* (1997) and the data by Jeffries (1997). These data seem to indicate the occurrence of convective overshoot during the previous phases (lower M_{up} and bigger M_{CO} cores and M_{WD} masses in turn at given M_I).

Anomalies in Chemical Abundances. Surface abundances of RGB stars are particularly useful probes of mixing taking place during the 1^{st} dredge-up, because mass loss by stellar wind is less of a problem. Canonical models predict precisely the changes in abundances that should occur during the 1^{st} dredge-up [see Iben & Renzini (1983)]: (i) substantial reduction of Li, Be, and B; (ii) decrease of C abundance by 30%; (iii) decrease in the $^{12}C/^{13}C$ ratio from 90 to about 20-30; (iv) increase of N abundance by a factor 0.44 $(C/N)_o$, where $(C/N)_o$ is the initial abundance ratio by number (net increase of about 80%); (v) modest increase in He, and decrease in H, and very little variation in O-abundance. As far as Li and Be are concerned, although there appears to be substantial agreement between classical predictions, there are indications that some extra-mixing may have occurred during the main sequence phase. In the case of $^{12}C/^{13}C$, there is only partial agreement with the observations. While the majority of stars appear to have $^{12}C/^{13}C$ ratios consistent with the theory, a sizable minority (30%) appear to have $^{12}C/^{13}C$ considerably lower. In the recent study by Charbonnel *et al.* (1998, and references therein) accurate data for Li, C, N and $^{12}C/^{13}C$ for five field giants with $[Fe/H] = -0.6$ and two stars in 47 Tuc are presented together with precise absolute magnitudes M_{bol} based on Hipparcos parallaxes. This allows the authors to establish an evolutionary sequence: $^{12}C/^{13}C$ drops from 20 to 7 for M_{bol} passing from 1 to 0.5, while Li disappears. The ratio $^{12}C/^{14}N$ decreases by 0.2 to 0.4 dex. The two stars in 47 Tuc show even lower $^{12}C/^{14}N$. These observations strongly indicate that extra-mixing occurs only when the stars reach the bump stage of the LF. The nature of this extra-mixing is still obscure (likely rotation according to the authors). Star to star abundance variations of C, N, O Na, and Al together with low $[Mg/Fe]$ and large $[Al/Fe]$ in globular cluster red giants have been reported (Shetrone 1996). Among other explanations, Denissenkov & Da Costa (1998) suggest deep extra-mixing may occur in RGB stars even if problems remain for the $[Mg/Fe]$ vs $[Al/Fe]$ anti-correlations. In addition to this, there are difficulties with the surface abundances of AGB stars and their descendants and the blue super-giant stars in the so-called Hertzsprung gap. Both topics are not addressed here.

The Mass Discrepancy of Cepheid Stars. It has long been debated whether the masses determined from stellar evolution theory agree with those derived from pulsation theory (Cox 1985). In general, pulsational masses (M_{pul}) are estimated to be 30 to 40% lower than evolutionary masses (M_{evol}) of the same luminosity. The mass discrepancy problem can be reduced to several causes, each of which is affecting the masses in question in a different way (Cox 1985). Among the various possibilities, we limit ourselves to consider the case of a bad determination of M_{evol}, which is customarily derived from a mass-luminosity relationship. This latter depends on the kind of mixing that occurred all over the past history of a Cepheid star. At given initial mass models with overshoot are brighter than classical models, so that they cross the instability strip at higher luminosity than the latter. Conversely, at any given luminosity the Cepheid mass of models with overshoot is significantly lower than that of the classical ones (Matraka *et al.* 1985; Bertelli *et al.* 1985). Once again, the star clusters of the LMC with Cepheids are the ideal workbench, because all the stars lie at the same distance, membership is less of a problem, and the evolutionary mass can be derived from the turn-off and giant stars. Chiosi *et al.* (1992b) first addressed this question using the Cepheid stars and CMD of NGC 2157 and NGC 1866. From the fit of the CMD with theoretical simulations based either on classical models or models incorporating core overshoot they determined the M_{evol}

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of the Cepheid stars, together with age, chemical composition and distance modulus. By means of a calibrated mass-period-luminosity-colour (MPLC) relationship for Cepheid stars (Chiosi *et al.* 1993) and the chemical composition suited to the cluster in question, they derived M_{pul} as function of the distance modulus. Full consistency among M_{evol}, M_{pul} and distance modulus obtained with the two methods was possible only for models with overshoot. The preference of this type of models in assigning the mass to Cepheids has been also suggested by Evans *et al.* (1998) and Wood *et al.* (1997). Particularly relevant is the study by Keller & Wood (2006) using the bump Cepheids of the LMC & SMC. In contrast Bono & Marconi (1997) looking at the distribution of the Cepheids of NGC 1866 in the Bailey diagram (V-amplitude versus period) conclude that classical models ought to be preferred.

Super-Giant and Wolf-Rayet Stars. The HRD for super-giant stars in the solar vicinity (Blaha & Humphreys 1989), and in the MCs (Fitzpatrick & Garmany 1990; Massey *et al.* 1995) together with the position and relative frequency of Wolf-Rayet (WR) stars have so far eluded precise quantitative explanations. The discussion below will be limited to the Milky Way and LMC and to stars brighter than $\mathrm{M_{bol}} = -7.5$, to minimize photometric incompleteness. Key features to consider are:

The missing blue gap. There is no evidence of the so-called blue Hertzsprung gap (BHG) between the core H- and He-burning phase. The gap is the observational counterpart of the very rapid evolution across the HRD following core H-exhaustion and prior to stationary core He-burning. In contrast, we observe a continuous distribution of stars all across this region. Many of the stars in the BHG show evidence of CNO processed and He-rich material at the surface (see Kudritzski (1998) and references) suggesting an advanced stage of evolution.

The relative frequencies of stars across the HRD. Star counts in the HRD of super-giant stars in the Milky Way and LMC performed in the luminosity interval $-7.5 \geqslant \mathrm{M_{bol}} \geqslant -9$ (roughly corresponding to main sequence stars in the mass interval 25 to 40 $M_{\odot}$), and the comparison with the evolutionary models calculated with mass loss, semi-convection, and overshoot (and modern input physics) reveals that there is a clear shortage of main sequence stars, which is more pronounced for the LMC sample. Indeed, recalling that the ratio $\mathrm{N_{MS}/N_{PMS}}$ cannot be too different from 10, it means that the group of stars supposedly in the main sequence phase should amount to about 80-90% of the total. The extension of the MS band towards the red depends on mass loss and mixing: it reaches the spectral type O9.5 in constant mass models with semi-convection and the spectral type B0.5 in models with mass loss and moderate convective overshoot. In this latter case, the percentage of stars falling in the MS band amount to about 80%. In reality the percentage is subjected to decrease if one takes WR into account for which post-MS stages are more appropriate (see the recent discussion by Chiosi 1998). If the above shortage is not due to strong photometric incompleteness, it seems as if MS band ought to extend to even lower temperature. Possible mechanisms making the MS band wider have already been discussed by Chiosi (1998) and will not be repeated here.

The ever puzzling Wolf-Rayet Stars. WR stars are commonly understood as the descendants of early type stars initially more massive than about $40M_{\odot}$ that owing to severe mass loss do not evolve to the red side of the HRD, but soon after central H-exhaustion or even in the middle of this phase reverse their path in the HRD towards higher effective temperatures, first at constant luminosity and later at decreasing luminosity, see Maeder & Conti (1994) and Chiosi (1998). During all these evolutionary stages, the stellar models (no matter whether with semi-convection, standard overshoot or diffusive overshoot) have the pattern of surface chemical abundances typical of the WR objects. However, while the surface abundances and the decrease in the luminosity passing from WNL to

WNE and WC stars indicated by the data (Hamann *et al.* (1993)) are matched by the models, their effective temperatures are much hotter than those possessed by real WR stars. It is often argued that the discrepancy in the effective temperature can be cured by applying the well known correction for an expanding atmosphere, see Bertelli *et al.* (1984), Maeder & Meynet (1987) and Hamann *et al.* (1993). However, among the galactic WN stars studied by Hamann *et al.* (1993), only WNE stars with strong emission lines show photospheric radii larger than the hydrostatic ones. So that the above correction does not apply to the majority of WR stars. Finally, since the minimum initial mass of the models able to reach the so-called WR configuration is greater than about $40M_{\odot}$, there is the additional difficulty that the relative number of WR stars with respect to the progenitor OB type stars exceeds the expectation based on the possible duration of the WR phase and initial mass function, see Chiosi *et al.* (1992a), Deng *et al.* (1996a,b) and Chiosi (1998) for details.

3. Convective overshoot and mixing in stellar models

H-semi-convection, He-semi-convection, breathing convection, and local overshoot are different attempts to cure the inconsistencies implicit in the classical scheme: simple Schwarzschild criterion and MLT. In a way or another they seek to answer the following questions:
(1) What determines the extension of the convectively unstable regions (either core or envelope or both) together with the extension of the surrounding regions formally stable but that in a way or another are affected by mixing? In other words how far convective elements can penetrate into formally stable regions?
(2) What is the thermodynamic structure of the unstable and potentially unstable regions?
(3) What is the time scale of mixing? Instantaneous or over a finite (long) period of time? What is the mechanism securing either full or partial homogenization of the unstable regions?
Attempts to answer the above questions have generated more complex schemes aimed at providing a sound model for overshoot and associated mixing. If the physical ground of convective overshoot is simple, its formulation and efficiency are much more uncertain giving rise to a variety of solutions and evolutionary models. This uncertainty is reflected in the variety of solutions both in local and non local formulations and associated evolutionary models that have been proposed over the years. As the body of literature is very large, we will limit ourselves the mention here the few that are relevant to this presentation Maeder (1975), Bressan *et al.* (1981), Bertelli *et al.* (1985), Bressan *et al.* (1986), Xiong (1983, 1986), Langer (1989a,b), Maeder & Meynet (1987, 1988, 1989, 1991), Xiong (1989, 1990), Aparicio *et al.* (1990), Alongi *et al.* (1991), Maeder (1990), Canuto & Mazzitelli (1991, 1992), Grossman *et al.* (1993), Grossman (1996), Grossman & Taam (1996), Canuto & Dubovikov (1998), Canuto (1998a), and Ventura *et al.* (1998).

Despite all this, the vast majority of stellar models (see the Padova and Geneva libraries) stand on the ballistic scheme proposed long ago by Maeder (1975) and Bressan *et al.* (1981) to evaluate the mass extension of the overshoot region. The scheme is non local but makes use of the MLT and hence contains the ML paramemeter. The differ only in the assumptions made for the extension and thermal structure of the overshoot region. Mixing is always assumed to instantaneous. The overshoot distance at the edge of the convective core has been proposed between nearly zero and about $2 \times H_P$. As many evolutionary results depend on the extension of the convective regions, this uncertainty is most critical. Because a generally accepted theory for overshoot is not yet available, most

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of those studies sought to constrain the efficiency of overshoot by comparing theoretical predictions with observations.

In addition to the convective core, overshoot may occur at the bottom of the convective envelope during the various phases in which this develops, such as on the RGB. The effect of envelope overshoot on stellar models of LMS and IMS has been studied by Alongi *et al.* (1991), whereas that for HMS by Chiosi *et al.* (1991).

Finally, much debated question is whether steep gradients in molecular weight may constitute almost insuperable barriers against the penetration of convective elements into stable radiative regions. According to Maeder (1975) and Deng *et al.* (1996a) the gradients in molecular weight can be eroded by overshoot. According to Canuto (1998b) overshoot is limited by gradients in molecular weight.

Diffusive overshoot and mixing. Special mention deserve the few cases in literature in which convective overshoot and associated mixing have been treated as a diffusive process taking into account the existence of many scale lengths, i.e. Deng *et al.* (1996a,b), Salasnich *et al.* (1999) and Ventura *et al.* (1998). The studies share several common aspects but differ in other specific points and also the mass range in which they they have applied.

The Deng et al (1996) model. The assumption of straight mixing is abandoned, the contribution of all possible scales from very small to the size of the whole mixed region is included by means of the concept of the characteristic scale most effective for mixing, and important properties of turbulence that are usually neglected, i.e. *intermittency* and *stirring* are taken into account. However, it still stands on the MLT as far as the derivation of the maximum extension L_{OV} of overshoot region and the velocities in the unstable regions are concerned. An estimate of the maximum overshoot distance L_{OV} is given by the sum of all possible scale lengths, i.e. $Lov \simeq 1/(1-f) \times l_0$ where l_0 is the maximum scale typical of the MLT or equivalently λH_P with λ the mixing length parameter, and f is the breaking factor between two successive generations of turbulent elements whose dimensions decrease by a factor of two ($f \simeq 0.5$). Furthermore the *energy transport* across the unstable region (full convection and overshoot) is assumed to be adiabatic. Finally, to describe both motions and mixing, at all scales the diffusion equation is used

$$\frac{dX}{dt} = \left(\frac{\partial X}{\partial t}\right)_{nucl} + \frac{\partial}{\partial m_r}\left[(4\pi r^2 \rho)^2 D \frac{\partial X}{\partial m_r}\right]$$

where X is the chemical abundance of the generic element consideration and D is the diffusion coefficient which expressed as $D = (1/3) F_i\, F_s\, v_d L$ where, v_d is the velocity of a suitable mean effective scale driving mixing, and L is the dimension of the region interested by diffusion. The characteristic velocity v_d is not known a priori. It must be derived from the physical conditions under which mixing occurs. The factor F_i accounts for intermittency and the factor F_s accounts for the *stirring efficiency* at the largest scale.

Intermittency. In brief, the small scale elements become less and less able to fill the whole volume or in other words mixing becomes lees efficient. Using the so-called β-model of intermittency by Frisch (1977), Deng *et al.* (1996a) derive $F_i = (l_d/l_0)^{3/2}$.

Stirring. The largest eddy in a turbulence field works as a rigid stick stirring the material in a mixer and inducing smaller scale motions. However, if the stick (largest eddy) is comparable in size with the dimension of the total region to be mixed, the net mixing efficiency turns out to be much less. In order to take it into account we correct the diffusion coefficient by the factor $F_s = (L - l_0/l_0)^3 = (L/l_0 - 1)^3$ for $l_0 \leqslant L$. This correction turns out to be important only in the convective envelope because it extends over several pressure scale heights.

Diffusion Coefficient and the Most Effective Scale l_d. The diffusion coefficient may change with the convective region under consideration, i.e core, intermediate shell (not always present) and external envelope each of which in turn is surrounded by its overshoot zone. Suitable expressions are proposed by Deng *et al.* (1996a) for each region. In a turbulent medium, all scales from the dimension of the unstable zone itself down to that of the dissipative processes, are possible, however there will be a typical scale l_d at which mixing is most efficient. The minimum scale is the Kolmogorov micro-scale $l_K = (\nu^3/\bar{\epsilon})^{1/4}$ where ν is the kinematic viscosity and $\bar{\epsilon}$ is the kinetic energy flux injected into the turbulence field. If $l_d = l_K$ no mixing would occur. In contrast, if $l_d = l_0$ almost instantaneous mixing takes place. Numerical experiments yield $l_d = P_{dif} \times 10^{-5} l_0$ where l_0 is in units of H_p and P_{dif} is a fine tuning parameter of the order of unity. By slightly changing P_{dif} all existing evolutionary schemes are recovered, going from the semiconvective type of models (Langer 1989a,b) to the fully homogenized overshoot models (Bressan *et al.* 1981; Bertelli *et al.* 1985; Alongi *et al.* 1993).

The Ventura et al (1998) model. The Deng et al (1996) model has been further refined by Ventura *et al.* (1998) who considered (i) Full Spectrum of Turbulence (FST, billions of eddy scales are considered) with the appropriate distribution of convective fluxes taken from Canuto *et al.* (1996); (ii) Full coupling of nuclear evolution and turbulent transport by means of a diffusive scheme; (iii) Convective overshoot described by assuming that the turbulent velocity exponentially vanishes outside the formally convective region according to an e-folding free parameter, tuned to fit observations. They also provide a more physically consistent definition of the scale length ζ at the boundary of a convective region (unknown quantity). In the MLT this is simply approximated to $\Lambda = \alpha H_P$, where α is a free parameter. Depending on the micro-physics $1.5 < \alpha < 2.2$. In the FST the definition is as follows. Far from the boundaries, it has to approach the hydrostatic scale length H_P. At each layer of a convective region ζ_{up} and ζ_{low} are derived and their harmonic mean $< \zeta >$ is used. Close to the boundaries $\zeta \to \zeta_{up}$ or ζ_{low}. In deep layers, recalling that a convective structure has a polytropic index $n = 1.5$ and $\zeta = (1+n)H_P$ (Lamb 1932), one gets $\zeta = 2.5 \times H_P$ and $< \zeta >= z_{up}/2 = \zeta_{low}/2 = 1.25 \times H_P$. In the FST, Λ should also include overshoot ($\Lambda = \zeta + L_{OV}$). Therefore it is assumed that $L_{OV} = \beta H_{P,top}$ (or $L_{OV} = \beta H_{P,bot}$), where β is a fine tuning parameter. With the fluxes of Canuto *et al.* (1996) the solar fit requires $\beta = 0.1$. Finally, the diffusion coefficient is $D = 16\pi^2 R^4 \rho^2 \tau^{-1}$, where the diffusion time scale τ is given by the one-point density -radial velocity relationship $< \rho' u' >= -\tau \partial\rho/\partial r$ which unfortunately not known. The diffusion coefficient is approximated to $D = (1/3) u\, l_d$, where u is the average turbulent velocity, and $l_d = \Lambda$. The velocity is given by $u = u_b\, exp[\pm(1/\chi f_{thick})\, ln(P/P_b))]$ where u_b and P_b are the velocity and pressure at the boundary, χ is a parameter and f_{thick} is the thickness of the convective layer in units of H_ρ. In many respects this scheme resembles the one proposed by Salasnich *et al.* (1999). Both contain adjustable parameters.

3.1. *Stellar Models with Overshoot*

From the body of literature on this subject the following common results can be singled out:

(1) The core H-burning phase of all stars more massive than $M \geqslant M_L$ runs at higher luminosities, stretches to lower T_{eff}, and lasts longer as compared to classical models.

(2) In the mass range $M_L \leqslant M \leqslant 2M_\odot$, the onset of the convective core and associated overshoot takes place gradually as required by the turn-off morphology of old clusters.

(3) The over-luminosity caused by overshoot during the core H-burning phase still re-

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mains during the shell H- and core He-burning phases. As a consequence of it, the lifetime of the He-burning phase (t_{He}) of IMS and HMS gets shorter in spite of the larger mass of the convective core. Therefore the ratio t_{He}/t_H gets a factor of 2 lower than in classical models.

(4) As the helium cores of all LMS have nearly the same mass, the inclusion of convective overshoot leads to results similar to those obtained with the classical semi-convective scheme (Bressan *et al.* 1986).

(5) Due to the larger masses of the He and C-O cores left over at the end of core H- and He-burning phases, respectively, the critical masses M_{up} and M_{HeF} are about 30% smaller than in classical models.

(6) *In models with straight overshoot (no diffusion) the loops of IMS and HMS tend, however, to be less extended than in classical models.*

(7) The evolution of HMS ($M \geqslant 30M_\odot$) is complicated by the presence of mass loss all over their evolutionary history so that well behaved loops are destroyed. Nevertheless, models may evolve according the general scheme blue - red - blue and may appear as WR stars of different morphological type.

(8) Models with diffusive overshoot share the same properties of models with straight overshoot. They differ only in some quantitative details, for instance the wider loops of IMS stars and the higher τ_{He}/τ_H ratio Deng *et al.* (1996a), Deng *et al.* (1996b).

(9) Models of HMS with straight and diffusive overshoot and standard mass loss rates are still unable to fill the BHG and solve the problems with the formation of WR stars, see the discussion in Deng *et al.* (1996b).

(10) The situation is much better with the new models by Salasnich *et al.* (1999), in which the ballistic model is abandoned and the decay model of overshoot by Grossman (1996) is applied, and perhaps more important the mass-loss rates during the red super-giant phase are deeply revised. These are given by the relation $log(\dot{M}) = 2.1 \times log(L/L_\odot) - 14.5$ which yields values that are significantly higher than those by de Jager *et al.* (1988). See Salasnich *et al.* (1999) for all details.

(11) The new models possess very extended blue loops which may reach the main sequence region. Since in this phase the models spend $\sim 50\%$ of the He-burning lifetime, *they are able to solve for the first time the mystery of the missing BHG.*

(12) Finally, they offer a new channel for the formation of faint WR. The progenitors of these stars are not the massive objects ($M \geqslant 60M_\odot$) evolving *vertically* in the HRD, but the less massive ones (in the range $15 - 20M_\odot$) evolving *horizontally* in HRD and undergoing substantial mass loss as RSG.

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Discussion

Brun: Through 3D global simulations of core convection we have shown that $d_{\rm ov} \leqslant 0.2H_p$. You showed models that use a value of 0.75. Don't you think that this high number for overshoot comes from the fact that the models lack other physical processes like rotational mixing ?

Chiosi: I agree. The stellar models I have presented neglect rotation and mixing induced by rotation. Indeed, my intention was to touch this topic as well, but I run out of time. In any case, I would like at least to mention the series of papers by Maeder & Meynet on stellar evolution with rotation. However, let me remind that often the disagreement among different authors on $d_{\rm ov}$ is a matter of definition. For instance Bressan *et al.* (1981) defined the overshoot distance by means of the scale length across the Schwarzschild border (half below and half above it), whereas many other authors define it simply from the Schwarzschild border (a factor of two goes away).