## **BOOK REVIEWS**

ready form, Springer must bear some responsibility for allowing it to mar the appearance of their celebrated series.

C. J. SMYTH

CHAMPENEY, D. C., A handbook of Fourier theorems (Cambridge University Press 1987) xii + 185 pp. 0 521 26503 7, £25.

This handbook is intended to assist postgraduates and research workers in the physical sciences, particularly communications and electronic engineering, who have met Fourier analysis and its applications in a non-rigorous way, and wish to find out the exact conditions under which particular results can be used. Its major part therefore consists of rigorous statements of important results in Fourier theory, together with explanatory comments and examples. This is preceded by chapters which introduce necessary mathematical ideas, for example Lebesgue integration, the inequalities of Hölder and Minkowski, and notions such as absolute and uniform continuity, and dominated and mean convergence. No proofs are given in the book, nor precise references for proofs of individual theorems, but there is a comprehensive bibliography accompanied by a summary detailing those books which cover the results of particular chapters.

As one would expect, the coverage of material is selective. There are a great many results on ideas associated with convolutions and products, and on convergence of Fourier series and integrals; the latter include results on convergence in  $L^p$ , and on (C, 1) and Abel summability, together with many standard counterexamples. Fourier integrals are treated over some nine chapters, which include one on power spectra and Wiener's theorem, and three on generalised functions and their transforms; these are followed by two chapters listing analogous results for Fourier series and generalised Fourier series.

I have one or two minor reservations over notation, for example it seems perverse to use \* for complex conjugate as well as convolution, and I feel that a direct definition of measure would have been more illuminating than one defining it as the integral of a characteristic function, provided this function is in L, when no properties of functions in L have been proved. However let me stress the virtues, an excellent commentary throughout, with cross references and a good index, very good coverage in its chosen areas, and glimpses of other topics such as the Hilbert transform and almost periodic functions. As a reference book written in modern style, this handbook will have considerable value to a mathematician as well as to an engineer. It is beautifully printed and I found very few misprints.

## PHILIP HEYWOOD

HARTE, R., Invertibility and singularity for bounded linear operators (Marcel Dekker Inc., New York and Basel, 1987) xii + 590 pp. 0 8247 7754 9, \$119.50.

This textbook provides not only a most comprehensive introduction to functional analysis but, in addition, contains accounts of Fredholm theory, multiparameter spectral theory and the complicated ideas of Joseph Taylor.

The various kinds of "singularity" which prevent an operator from being invertible are studied in detail. The text concentrates on two major theorems: namely, the open mapping theorem and the Hahn-Banach theorem. Completeness is introduced and several types of nonsingularity are studied. In particular their algebraic and topological characteristics are looked at and the transmission of these to subspaces, quotients and products. The Hahn-Banach theorem is followed by the dual space construction. It is indicated how nonsingularity behaves under the process of taking the dual. Two major constructions are described: namely, the extension of classical complex analysis to the vector-valued case and the "enlargement" of a normed space.

334

## **BOOK REVIEWS**

The latter is applied to the study of "ordinary" invertibility as well as "algebraic" invertibility in Banach algebras, and "essential" invertibility in the context of compact operators and Fredholm theory.

This book contains a wealth of material. There is a comprehensive set of exercises, carefully chosen to illustrate points in the main text and a large bibliography. It should prove invaluable both to the beginning graduate student in functional analysis and to the more experienced researcher looking for a readable account of the more specialised topics mentioned above.

## H. R. DOWSON

KAROUBI, M. and LERUSTE, C., Algebraic topology via differential geometry (London Mathematical Society Lecture Note Series 99, Cambridge University Press 1987) 363 pp. 0 521 31714 2, £15. (Originally published in French as Méthodes de géométrie différentielle en topologie algébrique, Paris 1982.)

There are a number of topics in algebraic topology one might present in a first course, homology and cohomology of simplicial/CW complexes and/or manifolds being a particularly attractive option. Usually the approach is via simplicial or singular homology: each has its advantages and problems. The volume under review takes a third path; it is a presentation of the de Rham cohomology (DRC) of smooth manifolds.

This path too has its own strengths and weaknesses. On the positive side the algebraic formalism required (tensor calculus, exterior algebra) is fairly minimal and of general geometric interest anyway, a good investment for the prospective student. The resulting cohomology group has a *natural* multiplicative structure furnished by the wedge product. Most importantly this setting allows a fruitful interplay between topological and differential-geometric ideas. This interplay has been valued and used by many of the great geometers, Poincaré, E. Cartan, de Rham, H. Cartan, Weil, Thom and Whitney amongst them. However, from the mid-fifties until more recent times, differential forms were set aside in favour of more topological methods. Now, work of Quillen and especially Sullivan in the seventies has led to a new interest in the use of differential forms in topology.

The DRC also has its disadvantages. To my mind differential forms are a good deal less intuitive than simplicial (or even singular) chains, and of course, torsion phenomena are undetected by these cohomology groups, although the authors note that the modern presentation of DRC following Sullivan and others yields this richer structure.

Now to the book itself. As an introduction to DRC it is quite excellent. The treatment is selfcontained and gives full details, full proofs—one could happily use this as a reading course for first-year postgraduate students. Prerequisites are very few; the book develops all the exterior algebra from scratch in the first chapter (30 pages), discusses differential forms on open subsets of  $\mathbb{R}^n$  next (40 pages), and then gives a nice introduction to differentiable manifolds, producing a large number of significant examples: the spheres, projective spaces, classical groups, Stiefel and Grassmann manifolds (50 pages). So we are a third of the way through the book before meeting DRC, but I find this all to the good; the knowledgeable can skip these sections but they will prove invaluable to a beginning postgraduate.

Chapter IV gives the formal definitions and basic properties of DRC and Chapter V shows how one can compute these groups (computability is the key advantage of homology/cohomology). Computation takes place via the Mayer-Vietoris sequence, a simple version of the Künneth theorem is given and the DRC of spheres and complex projective spaces is computed. Fairly standard applications follow—Brouwer's fixed-point theorem and invariance of domain, Hopf's theorem concerning H-spaces.

Chapter VI deals with Poincaré duality for oriented manifolds; duality here is set up by integrating the wedge product of forms of complementary dimension. Various applications are given (including a determination of the multiplicative structure of the cohomology of complex