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Dynamo coefficients from the Tayler instability

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Abstract. Current-driven instabilities in stellar radiation zones, to which we refer as Tayler instabilities, can lead to complex nonlinear evolutions. It is of fundamental interest whether magnetically driven turbulence can lead to dynamo action in these radiative zones. We investigate initial-value simulations in a 3D spherical shell including differential rotation. The Tayler instability is connected with a very weak kinetic helicity, stronger current helicity, and a positive $\alpha_{\phi\phi}$ in the northern hemisphere. The amplitudes are small compared to the effect of the tangential cylinder producing an eddy with negative kinetic helicity and negative $\alpha_{\phi\phi}$ in the northern hemisphere. The $\alpha_{\phi\phi}$ from the Tayler instability reaches about 1% of the rms velocity.

Keywords. Keyword1, keyword2, keyword3, etc.

1. Introduction

Stellar radiation zones are often hosting magnetic fields. The majority of solar dynamo models include the presence of strong toroidal magnetic fields in the tachocline, the transition from differential to uniform rotation below the convection zone. Also a fair fraction of intermediate-mass stars shows strong magnetic fields hosted by their radiative envelopes and are called magnetic Ap stars.

These magnetic fields may become unstable because they are connected with currents (Vandakurov 1972; Tayler 1973). We will use the term Tayler instability for this class of current-driven instabilities. Rotation and differential rotation alter the stability limits of the Tayler instability.

The limiting magnetic fields strengths have been determined under various conditions and in various contexts in a series of studies, e.g., Pitts & Tayler (1985), Gilman & Fox (1997), Cally (2000), Dikpati *et al.* (2004), Braithwaite (2006b), Arlt *et al.* (2007), Rüdiger & Kitchatinov (2010). The linear stability of magnetic fields is fairly well understood. Non-axisymmetric, large-scale modes are typically the consequence of the Tayler instability.

The nonlinear development of the instability may lead to turbulence as well as enhanced diffusivity and angular-momentum transport in radiative stellar zones. Another issue is the existence of a sustained dynamo, if the magnetic field which becomes unstable has a source of replenishment (Spruit 2002). While the large-scale non-axisymmetric unstable mode will only be able to produce a non-axisymmetric magnetic field, a turbulent state may imply the generation of a substantial axisymmetric part. The non-axisymmetry is then hidden in the turbulent field as to comply with Cowling's theorem. Mixed results have been obtained in attempts to show sustained dynamo action in simulations (e.g. Braithwaite 2006a; Brun & Zahn 2006; Gellert *et al.* 2008).

Our paper deals with the possible dynamo-effect from the Tayler instability by measuring the mean-field dynamo coefficients during the nonlinear evolution of the system.

Zahn *et al.* (2007) pointed out that a turbulent electromotive force (EMF) is the only way of regenerating the large-scale magnetic field from the non-axisymmetric instability. Measuring the EMF appears to be a suitable approach, even though we do not have an energy source in the system which would be necessary for sustained generation of magnetic fields from a continuously excited instability.

2. Numerical setup

We consider a spherical shell extending from the normalized radii $r_{\rm i} = 0.5$ to $r_{\rm o} = 1$. The colatitude θ and the azimuth ϕ are covered in their full extent. The spectral spherical MHD code in Boussinesq approximation by Hollerbach (2000) is employed for the simulations. In this study, the buoyancy term and the equation for temperature fluctuations are dropped for the sake of simplicity. The remaining, normalized equations are

$$\frac{\partial \boldsymbol{u}}{\partial t} = -(\boldsymbol{u}\cdot\nabla)\boldsymbol{u} + (\nabla\times\boldsymbol{B})\times\boldsymbol{B} - \nabla p + {\rm Pm}\triangle\boldsymbol{u}, \qquad (2.1)$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla\times(\boldsymbol{u}\times\boldsymbol{B}) + \triangle\boldsymbol{B}, \qquad (2.2)$$

where $\boldsymbol{u}$ and $\boldsymbol{B}$ are the velocity and magnetic fields and p is the pressure. Additionally, the relations $\nabla\cdot\boldsymbol{u} = 0$ and $\nabla\cdot\boldsymbol{b} = 0$ hold. The equations are normalized by the magnetic diffusivity η, whence the magnetic Prandtl number ${\rm Pm} = \nu/\eta$ in the Navier-Stokes equation, where ν is the kinematic viscosity. The magnetic permeability and the density are set to unity and do not appear in this system of equations.

The initial velocity profile is that of a differential rotation with the angular velocity Ω depending on the axis distance, $s = r\sin\theta$,

$$\Omega(s) = \frac{\rm Rm}{\sqrt{1+s^{\,q}}}, \qquad (2.3)$$

where we start with ${\rm Rm} = 20\,000$ and $q = 2$. The initial magnetic field is purely poloidal and confined to the computational domain. The parameter Rm is the second dimensionless parameter entering the system in the initial conditions:

$${\rm Rm} = \frac{R^2\Omega_*}{\eta}, \qquad (2.4)$$

where Ω_* is the angular velocity of the star and R is its radius. By comparing the Alfvén velocity of the magnetic field with the velocity of the fluid, one can convert the dimensionless magnetic fields of the simulations into physical units,

$$B_{\rm phys} = \sqrt{\mu\rho}\,\Omega_* R \frac{B}{\rm Rm} \qquad (2.5)$$

where μ is the permeability and ρ is the bulk density of the fluid, now in physical units.

The spectral truncations for these simulations were typically at 40 Chebyshev polynomials for the radial decomposition and 60 Legendre polynomials for the latitudinal decomposition. For each Legendre degree l, the spherical harmonics were running from $m = -l$ to $m = l$ accordingly.

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3. Results

The simulations are initially entirely axisymmetric, but apply a non-axisymmetric perturbation after a given time $t_{\rm pert}$. As long as no perturbation has been applied, the differential rotation winds up toroidal magnetic fields from the initial, purely poloidal one. Lorentz forces, however, start to act on the differential rotation and reduce it, while the total angular momentum of the spherical shell is preserved in the simulations. This scenario alone leads to uniformly rotating radiative zones.

During the amplification of toroidal fields, the stability properties of the magnetic field changes. The Tayler instability is suppressed by rotation, and non-axisymmetric instabilities are even further suppressed if differential rotation is present. Now, the differential rotation is gradually reduced while toroidal fields grow – the system will turn supercritical for the Tayler instability quite suddenly after some time. This is the time when the non-axisymmetric perturbation is injected into the system. In the case described here, $t_{\rm pert} = 0.003$ diffusion times.

The instability now develops a fairly complex pattern with higher modes being excited through nonlinear coupling, and some energy dropping back into the $m = 0$ mode as well. The latitudinal structure exhibits a very steep energy spectrum of $E_l \sim l^{-3.7}$ at $t - t_{\rm pert} = 0.001$ diffusion times. The system does not develop a fully turbulent state in these simulations. The azimuthal spectrum is even steeper with $E_m \sim m^{-6.7}$.

Here, we are interested in the question whether the averaged electromotive force can be expressed in terms of an (axisymmetric) α-effect. In other words, can the Tayler instability – through nonlinearities – act like a mean-field dynamo.

Assuming that the turbulent electromotive force **EMF** only depends on the mean magnetic field $\overline{\boldsymbol{B}}$ and its first spatial derivatives, one can write in general

$$\mathbf{EMF} = \quad \boldsymbol{\alpha}\overline{\boldsymbol{B}} + \boldsymbol{\gamma} \times \overline{\boldsymbol{B}} - \boldsymbol{\beta}(\nabla \times \overline{\boldsymbol{B}}) - \boldsymbol{\delta} \times (\nabla \times \overline{\boldsymbol{B}}) - \boldsymbol{\kappa}(\nabla\overline{\boldsymbol{B}})^{(\rm sym)}, \qquad (3.1)$$

with the symmetric $\boldsymbol{\alpha}$- and $\boldsymbol{\beta}$-tensors, the vectors $\boldsymbol{\gamma}$ and $\boldsymbol{\delta}$, and the third-rank tensor $\boldsymbol{\kappa}$ acting on the symmetric part of the tensor gradient of $\overline{\boldsymbol{B}}$.

We employ the test-field method developed by Schrinner et al. (2007) to measure the components of the above mentioned tensors. While running the actual simulation, an additional set of 27 test equations is integrated delivering the various components of the mean-field tensors and vectors simultaneously as functions of the meridional location and time. The spatial distribution of the components of the symmetric part of the $\boldsymbol{\alpha}$-tensor, averaged over 0.0005 diffusion times or 1.6 rotation periods, is shown in Fig. 1.

The strongest effect comes from the inner cylinder which tends to produce an eddy of helical motion giving rise to an α-effect which is not primarily caused by the Tayler instability. The actual Tayler-α is the positive $\alpha_{\phi\phi}$ (alpha_pp) in the bulk of the northern hemisphere at $r > 0.6$. Note that the component $\alpha_{\phi\phi}$ – the one which generates the poloidal field from the toroidal one – is the smallest among the diagonal components of the tensor. The rms velocity fluctuations in the computational domain are around $0.005 R\Omega_*$. The $\alpha_{\phi\phi}$ in the Tayler unstable region of the domain is roughly 1% of this rms value or $5 \cdot 10^{-5} R\Omega_*$. The tangent-cylinder effect delivers an $\alpha_{\phi\phi}$ of about 7% of the rms velocity.

The velocity fluctuations are mostly horizontal, even in this unstratified setup. Radial velocity fluctuations are about five times smaller than horizontal motions. The positive $\alpha_{\phi\phi}$ which we think results from the Tayler instability is associated with a positive current helicity and very weak, negative kinetic helicity. This is certainly an indication for the magnetic nature of the instability and the raising turbulence which is different from convection. However, this actually means the test-field method will probably underesti-

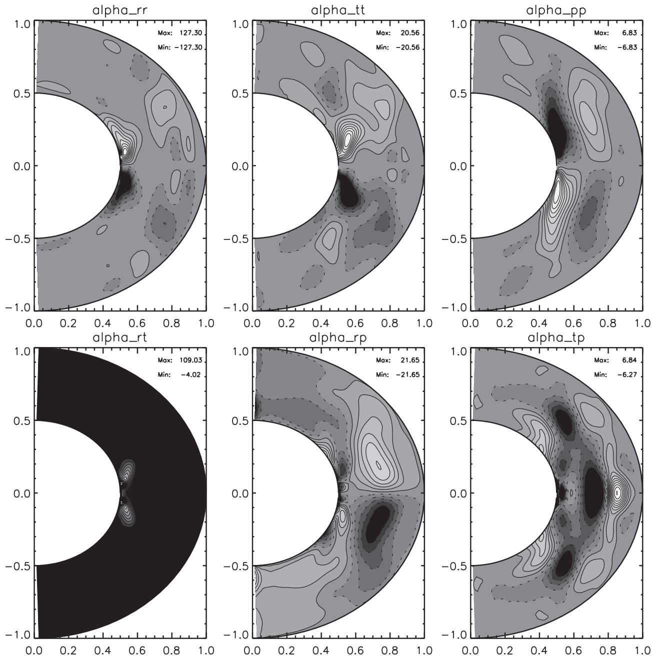

Figure 1. Distributions of α_{rr}, $\alpha_{\theta\theta}$, and $\alpha_{\phi\phi}$ in the upper row, and $\alpha_{r\theta}$, $\alpha_{r\phi}$, and $\alpha_{\theta\phi}$ in the lower row, averaged from a period which is 0.0005 to 0.0010 diffusion times after the perturbation was injected. Light areas (solid lines) represent positive α_{ij} while dark areas (dashed lines) represent negative values.

mate the values of $\boldsymbol{\alpha}$. The magnetic-field fluctuations are by a factor of 3–4 larger than the velocity fluctuations, so $\alpha_{\phi\phi}$ could also be about 3–4% of the rms velocity instead of only 1%.

4. Outlook

While the simulations show that the Tayler instability may act like a mean-field dynamo, the α is very small. There is no energy source in these simulations, sustained dynamo action is thus not possible. Future simulations with some sort of energy source will tell whether a continuous dynamo is possible and feasible for stellar radiation zones. Just as for convection-driven dynamos, the question also matters of how long it takes for the large-scale magnetic field to grow. If this happens on a resistive time-scale, one again has to be puzzled with the slow generation of large-scale magnetic fields from a small-scale dynamo, as it was seen in forced and convective turbulence (see Brandenburg & Subramanian 2005 for a review, and e.g. Käpylä *et al.* 2010 for a solution).

References

Arlt, R., Sule, A., & Rüdiger, G. 2007, *A&A*, 461, 295
Brandenburg, A. & Subramanian, K. 2005, *Physics Reports*, 417, 1
Braithwaite, J. 2006a, *A&A*, 449, 451
Braithwaite, J. 2006b, *A&A*, 453, 687
Brun, A. S. & Zahn, J.-P. 2006 *A&A*, 457, 665
Cally, P. S. 2000 *Sol. Phys.*, 194, 189
Dikpati, M., Cally, P. S., & Gilman, P. A. 2004, *ApJ*, 610, 597

https://doi.org/10.1017/S1743921311017637 Published online by Cambridge University Press

Gellert, M., Rüdiger, G., & Elstner, D. 2008, *A&A*, 479, L33
Gilman, P. A. & Fox, P. A. 1997, *ApJ*, 484, 439
Hollerbach, R. 2000, *Int. J. Num. Meth. Fluids*, 32, 773
Käpylä, P. J., Korpi, M. J., & Brandenburg, A. 2010, *A&A*, 518, A22
Pitts, E. & Tayler, R. J. 1985, *MNRAS*, 216, 139
Rüdiger, G. & Kitchatinov, L. L. 2010, *Geophys. Astrophys. Fluid Dyn.*, in press
Schrinner, M., Rädler, K.-H., Schmitt, D., Rheinhardt, M., & Christensen, U. R. 2007, *Geophys. Astrophys. Fluid Dyn.*, 101, 81
Spruit, H. 2002, *A&A*, 381, 923
Tayler, R. J. 1973, *MNRAS*, 161, 365
Vandakurov, Yu. V. 1972, *SvA*, 16, 265
Zahn, J.-P., Brun, A. S., & Mathis, S. 2007, *A&A*, 474, 145

Discussion

Donati: Ap stars are unlikely no more dynamo-type fields as this would imply some kind of positive correlation between field strength and rotation rate that we don't see. It would also be difficult to explain why only 9–10% of massive stars are magnetic. However, this exotic dynamo may be responsible for the weak field cutoff of the Ap star histogram. And for the weak fields recently in normal A, B and O stars.

Arlt: The measurements of the dynamo coefficients were not particularly addressing Ap stars, but of general nature. The simulations indicate that dynamo action without a sharp inner boundary or with strongly dominating toroidal fields (like in the solar tachocline) will be very weak.

Brun: Do you find a genuine dynamo action in your simulation of stellar radiative interior of a massive star, or just a transitory growth of field due to Tayler's instability? If dynamo action is present, at which magnetic Reynolds number is the onset?

Arlt: The simulations lack an energy source, so are not capable of showing a genuine dynamo. All we could do at this stage is measure the mean-field coefficients generated by the nonlinear evolution of the instability. Differential rotation and fields all decay at large times.