A whole epoch in the theory of probability has ended with the death of Boris Vladimirovich Gnedenko on the 27th of December 1995. His name now belongs to the ages.

A short biographical outline of B. V. Gnedenko’s life would include the following facts. He was born on the 1st of January 1912 in Simbirsk, on the Volga River, in the centre of Russia. In 1927, he entered Saratov University and graduated from it in 1930. He then worked for several years in Ivanovo as an Assistant Professor. From 1934 until 1937, he followed a post-graduate course at Moscow State University, and on completing his studies he worked at the Mathematical Institute of this University. In 1945, Gnedenko was elected to the Ukrainian Academy of Sciences, and moved first to Lvov and then to Kiev where he held the position of Director of the Mathematical Institute. From 1960 until the end of his life he held the Headship of the Department of Probability Theory at Moscow State University (in the Faculty of Mathematics and Mechanics).

But this account gives only a superficial impression of Gnedenko’s life: this was full of scientific achievements and dramatic events, happiness and losses.

Gnedenko finished his schooling at the age of fifteen, and was at first not allowed to enter Saratov University because of his youth. But owing to his persistence, he managed to receive special permission from the Ministry of Education and was finally allowed to enter the university in spite of his age. According to his own account, his choice of specialization was considerably influenced by the October Revolution of 1917: mathe-
matics as an abstract science was beyond the understanding of Communist Party functionaries, thus remaining out of their control. He therefore selected mathematics as his life study. Nevertheless, there is little doubt that mathematicians (like other people) were put under observation, and this affected Gnedenko’s life dramatically.

Not only his entry to the university, but also his graduation from it was unconventional. At that time in 1930, the government made the decision that students, instead of following the full five-year training programme, would finish their courses in three years. Thus, their studies were subjected to great confusion and stress; moreover, final examinations were organized by the so-called ‘brigadier method’. All students were divided into groups (brigades), and the brigade-leader took the exam for the whole group, his mark being assigned to all students in his brigade. Gnedenko was the brigade-leader of such a group; although his answers earned all students in his group high marks, he was left with a feeling of deep humiliation because of this form of collectivism.

After graduating from the University, Gnedenko was invited to join the Textile Institute in the small town of Ivanovo which was one of the centres of the textile industry. There he faced the problems of machine maintenance, which stimulated his interest in the theory of probability and statistics. His first scientific papers devoted to applied problems in this area are dated 1933.

Upon attending a seminar series on probability, sponsored by A. N. Kolmogorov and A. Ya. Khinchine, Gnedenko decided to continue his studies as a post-graduate student at the Mathematical Institute of Moscow University. It was there that his lifelong enthusiasm for the theory of probability began. At that time, the Moscow Mathematical School was very strong, and Gnedenko met many famous mathematicians at the University, among them P. Alexandrov, N. Bari, S. Bernstein, V. Glivenko, D. Men’shov, I. Petrovski, E. Slutski, N. Smirnov, S. Sobolev, V. Stepanov. Khinchine and Kolmogorov were his real teachers. He cared for them dearly and held them in the highest affection all his life. It is worth mentioning that, in general, friendly relations between teachers and students were not restricted to classes and seminars. Very often, they continued beyond the university walls, especially as the difference in their ages was not so great. Teachers and students often took walks together, listened to music, discussed cultural events, books, and of course, constantly discussed mathematical problems. Gnedenko often mentioned that Kolmogorov was a connoisseur of art, and they talked at length about ancient Russian icons and architecture, poetry and history. Such close communication with cultured teachers gave much to their students; Gnedenko, aware of the benefits of such interaction, tried to follow the same path during his own professional life.

In June 1937, Gnedenko defended his PhD thesis; in it he examined various aspects of infinitely divisible distributions. After this, he began to work at the mathematical Institute of Moscow State University (now the Steklov Mathematical Institute). However, he had to interrupt his work there because of the need to join the army. According to the law, he had to do a year’s military service. The following little known but significant episode, which greatly influenced his future, occurred just after the beginning of Gnedenko’s military service. Following a false denunciation, he was arrested and imprisoned. This was not an extraordinary event at the time; what was extraordinary was Gnedenko’
behaviour. Every day, during six months, he was interrogated for about 16 hours. His interrogators insisted that he give them the names of the ‘enemies of the People’ among the mathematicians working at Moscow University. This demand Gnedenko categorically refused to satisfy, and no such enemy was unmasked. After this, the torturers set Gnedenko free. Anyone acquainted with the history of the Soviet Gulag will appreciate Gnedenko’s fortitude.

However, it was not only Gnedenko who displayed bravery in the circumstances. Kolmogorov insisted on the renewal of Gnedenko’s employment at the Mathematical Institute. This also demanded great courage. As a rule, people branded with a ‘black mark’ could not occupy research positions at institutes or universities, and could suffer other forms of disfavour. It is sufficient to mention that in 1941, when the German Army was approaching Moscow, Gnedenko was not allowed to join the Soviet Armed Forces because of this ‘black mark’.

The years prior to World War II were extremely productive for Gnedenko. Exactly at that time he obtained his classical results on the summation of independent random variables. In particular, he introduced the so-called accompanying infinitely divisible laws which allowed him to examine various problems through a unified approach. Using this, he found necessary and sufficient conditions in the limit theorems for:

(i) sums of independent infinitesimal summands (approximation by infinitely divisible laws);
(ii) normed sums of independent identically distributed random variables (approximation by stable laws);
(iii) maximum terms of sequences of independent random variables.

It is remarkable that these results of Gnedenko were derived at much the same time as the similar results of W. Doeblin. This was an extension of the rivalry existing between their famous teachers A. Ya. Khinchine and P. Levy in the examination of infinitely divisible laws. Such friendly competition resulted in new successes in the theory of probability.

The results of the research in the area of limit theorems were summed up in the book *Limiting Distributions for Sums of Independent Random Variables*, written together with Kolmogorov and published in 1949. The deep contents of the research reported made it a handbook for several generations of mathematicians working in the field of probability theory. Almost every specialist in the theory of probability regards this book among those at the very basis of the theory.

Gnedenko’s disciples throughout the world have obtained a variety of interesting results in this area, both generalizing earlier assertions and discovering completely new facts. But not only did he give continual advice and supervision to numerous researchers working on limit theorems, Gnedenko himself succeeded in deriving a number of new and profound results. In the 1960s, he proved the so-called transfer theorems revealing the structure of limiting laws for random sums of independent random variables. It is interesting to note that he arrived at this topic not only because of his desire to generalize his earlier results, but also by considering some applied problems in reliability. He was very proud of the fact that his theory had immediate non-trivial applications. Random summation was of permanent interest to Gnedenko until the last years of his life. The
book *Random Summation: Limit Theorems and Applications*, written together with V. Yu. Korolev, which is due to be published by the CRC Press in 1996, contains his newest results in this direction.

For 15 years after 1945, Gnedenko lived in Ukraine. The reason for this was his election as Academician of the Ukrainian Academy of Sciences. First, he worked at Lvov University. There he met S. Banach, not long before his death. Judging from his memoirs, he retained strong impressions of this meeting all his life. In 1949, he moved to Kiev where he was appointed to the position of Director of the Mathematical Institute. Simultaneously, he worked as a professor at Kiev University.

It was at that time that he proposed a simple proof to a local limit theorem for sums of i.i.d. random variables having a lattice distribution. Later on, this proof was generalized to the multidimensional case. Kolmogorov called this result one of the most useful in probability theory, having a variety of applications. Gnedenko’s basic interest, during his stay in Ukraine, was mathematical statistics. He found explicit results for the distributions of the maximum difference between real and empirical distributions, of which the limiting distribution had been derived by Kolmogorov and N. V. Smirnov. The results obtained not only enabled him to give elementary proofs of known limit theorems, but were significant for applied statistical investigations. They also resulted in a series of works on asymptotic expansions in limit theorems.

Let us mention here one more aspect of Gnedenko’s activity, perhaps somewhat unexpected. In the Kiev Institute of Energetics, a programming laboratory existed. During the mid 1950s, however, this laboratory was about to be closed due to the interference of Communist Party functionaries. Gnedenko succeeded in having the laboratory moved to the Mathematical Institute. He created normal working conditions for its staff and took part in the research himself. They succeeded in solving several medical diagnostic problems, which opened new possibilities for medicine at that time. It was a particular pleasure for Gnedenko to tell colleagues how physicians who specialized in different areas (oculists, psychologists, cardiologists, etc.), and did not believe that computers could do anything useful, came to ask him for further cooperation after this demonstration. This episode resulted in the book *Elements of Programming* (1960), written in cooperation with V. S. Korolyuk and E. L. Yushchenko.

It is no exaggeration to say that the flourishing schools of probability in Eastern Germany, Bulgaria, Hungary, Lithuania and Ukraine are very much indebted to Gnedenko. Mathematicians from these countries studied in the Department of Probability Theory (headed by him) and submitted their theses in it. As a rule, contacts in the scientific field grew into friendships. And these proved to be even stronger than the state system of the visitors’ countries. Many papers, books and articles were written by Gnedenko jointly with his foreign colleagues and friends. Let us mention here a single ambitious project realized together with D. König — the two volumes of *Handbuch der Bedienungstheorie*, published in 1983 by the Akademie-Verlag, Berlin. A large team of specialists in queueing theory from the USSR and Eastern Germany gathered to write this fundamental handbook.

Gnedenko supervised and edited many translations into Russian of books written by his colleagues. One particularly popular book, *Trilogy on Mathematics* written by
A. Renyi deserves special mention. It seems that this book contained many features appreciated by Gnedenko: it was devoted to the history of the theory of probability; it contained deep mathematical reasoning; it was well written; it could be recommended not only to students but to any person who wanted to learn something new about mathematics and history. But besides this popular book, there were many translations of ‘serious’ books on probability, queueing and reliability.

Gnedenko was a welcome guest in Bulgaria, Lithuania, Hungary, Germany, Australia, Canada, the USA, Israel, Greece, and other countries. While visiting these countries, Gnedenko pursued interests beyond the strictly scientific. He liked travelling and enjoyed talking about his impressions. He appreciated history and art, music and books. When visiting a country, he tried to see as many places of interest as possible. He was fond of museums, bought many books and records. He was interested in local habits and different ways of life. For example, when visiting Australia, he was much impressed by the Australian technique of sheep-shearing and repeatedly mentioned this process as an example of well-organized work. His dream was to visit three countries which he considered to be the pillars of European civilization: Greece, Italy and Israel. He succeeded in travelling to Greece and Israel only at the age of eighty when he was already failing in health; he was fortunate in visiting Italy at an earlier age. But despite his disabilities he brought back many lively impressions from these two countries, and it was an evident pleasure for him to speak about them.

Constant interest in applied problems was typical of Gnedenko. His early work on machine maintenance and his investigation of the organization of Moscow’s anti-aircraft system during World War II led Gnedenko to queueing problems. No doubt his interest was maintained by his teacher, Khinchine, who contributed a great deal in the theory of queues. Khinchine’s book *Mathematical Methods in the Theory of Mass Service* published by the Steklov Mathematical Institute in 1955 was perhaps the first book in Russian devoted to this topic. After Khinchine’s death, Gnedenko collected all the works of Khinchine on queueing theory and published them as the book *Works on the Mathematical Theory of Mass Service* (1963) providing it with his addendum, which gave a survey of the field and outlined new queueing models. It was thanks to Gnedenko that research in queueing theory was developed in the USSR in the early 1960s. No expert in this field could miss his specialized seminar at Moscow State University. This weekly seminar named ‘Probability Methods in Technics’ was sponsored by Gnedenko together with Yu. K. Belyaev and A. D. Soloviev for about 30 years. During that period, several generations of researchers in applied probability were influenced by it. The book *An Introduction to Queueing Theory*, written by Gnedenko in cooperation with his disciple I. N. Kovalenko in 1966, whose second edition appeared in 1987, gave an impetus to a large number of researchers throughout the world.

In the 1960s–1970s, he organized several representative scientific conferences on the theory of queues. At a time when participation in scientific meetings throughout the world was not always easy for scientists in the Communist countries, these conferences contributed much to raising the research level in both the USSR and Eastern Europe. We have already mentioned the handbook on queueing written by German and
Soviet mathematicians. To a large extent, this project was made possible by these conferences.

The same situation arose in the theory of reliability. Gnedenko became the father of this applied discipline in the USSR. Just like A. Ya. Khinchine, Gnedenko had many contacts with engineers in several branches of industry and agriculture. Speaking with them, he realized the significance of reliability problems. And he started persuading managers, engineers and mathematicians of the necessity of carrying out investigations in the reliability area. In his opinion, every manufacturing process should have a reliability service, and owing to his enormous energy, such services were created.

He organized a ‘reliability office’ in Moscow where every engineer could get a highly skilled consultation and tutorial. Simultaneously, Gnedenko collected a group of qualified mathematicians to solve reliability problems. The book Mathematical Methods in the Theory of Reliability (1965), written together with Yu. K. Belyaev and A. D. Soloviev, raised the understanding and standards of engineers working in this field. It also played an important role in drawing the attention of mathematicians to a new set of applied problems.

Gnedenko was a great teacher. If one requires confirmation of this, it is sufficient to list only a few of his numerous disciples: A. V. Skorohod, I. N. Kovalenko, V. S. Korolyuk, Yu. K. Belyaev, P. Franken. During his entire life, he paid close attention to teaching. More exactly, he put his whole soul into teaching. It was due to his enlivening lectures and his regular informal contacts with students that Gnedenko gained the authority and affection that he enjoyed. His textbook on the theory of probability still remains one of the best. It was first published in 1949 and the 6th revised edition appeared in 1988. The textbook has been translated into all the main languages. But his activity in the field of teaching was not restricted to the theory of probability. He took part in numerous discussions on how best to teach students. His basic idea was that future mathematicians must learn to formulate problems, not only solve them. For this, it is necessary to know the history of the subject, to be a widely educated person, and of course to be obsessed by the subject.

But not only did he declare these ideas, he was persistent in putting them into practice. He was open for discussions and talks. He was fond of speaking before students about his reminiscences, telling them interesting historical events, and explaining ‘how to solve it’. His ideas and tutorial experience were reflected in numerous papers and articles published in pedagogical journals and newspapers. He placed strong emphasis on the clarity of presentation of mathematical ideas. This can be traced back to his famous textbook. But perhaps his first experience in the popularization of mathematics is the book Elementary Introduction to the Theory of Probability (1947), written together with A. Ya. Khinchine. This book was a real bestseller that ran into many editions; it was translated in many countries. He always promoted projects directed to the wide spread of education, and was strongly against both snobbish and ignorant projects in education.

Gnedenko felt keenly the present disastrous situation of education in Russia. Sometimes, he gave way to despair about the impossibility of improving the situation. But complaining was not in his character; his rule was to act.
He had a broad outlook. During his long life, he kept an eye on trends in science to make timely corrections in university programmes. Just to give an example, three years ago he initiated an actuarial specialization in the Faculty of Mechanics and Mathematics at Moscow State University. This was quite out of character for this faculty, but very appropriate at the time. It took him many efforts to organize such a specialization and the corresponding teaching programme. But he did this despite his increasing ill-health.

Generally speaking, Gnedenko liked acting unconventionally. In his textbook, there were several chapters on the history of the theory of probability. This was quite innovative. At most, standard textbooks contained a chapter devoted to historical questions. But more commonly, they contained a few lines on such issues. It was a basic proposition with Gnedenko that every mathematician must make himself feel a part of the mathematical community and see himself as ‘standing on the shoulders of giants’. Only then will his ideas correlate with the needs of science, even if these ideas are fairly new. It was unconventional to have a position on the history of mathematics belonging to the Department of Probability Theory, but Gnedenko had. Right from the time of his postgraduate course, Gnedenko was enthusiastic about the history of mathematics. In fact, he was the first to begin investigating the history of mathematics in Russia. In 1946, he published a small book on this topic, based on his student work in 1937. He wrote many works devoted to the life and investigations of mathematicians such as P. Chebyshev, L. Euler, N. Lobachevski, A. Lyapunov, A. Markov, M. Ostrogradski, and to the development of mathematics, basically in Russia and Ukraine. During his last years, he wrote memoirs recording many interesting episodes of his life, his thoughts about mathematics and teaching, about the applications of mathematics and about the numerous interesting people he had met. Many pages are devoted to his teachers and colleagues A. Ya. Khinchine and A. N. Kolmogorov. Reading this manuscript is a real pleasure, and one hopes that these memoirs will be published in the near future.

Gnedenko realized clearly that it is useless to improve university educational programmes without improving the system of school education. He had close contacts with schoolteachers and paid careful attention to their work. One need hardly mention that he wrote many works devoted to this important problem. His opinion was appreciated by teachers because it was based on his own experience. Together with Kolmogorov, he organized a mathematical school in Moscow. He was one of the organizers of numerous mathematical competitions both in Russia and in other countries. But when he felt that something was wrong in education, as for example when he disagreed with a programme in mathematics or the method used in learning some subject, he did his best to improve the situation immediately. He would write articles in popular newspapers, talk at various meetings and conferences, and persist in his objections until his voice was heard.

No memories of Gnedenko are complete without mention of his remarkable hospitality. Those who had a chance of knowing him will never forget this. Usually, events occurred as follows. If one had a problem and wanted Gnedenko’s opinion or advice, he or she received an invitation to visit Gnedenko’s home, no matter what kind of problem it was. It could be a particular mathematical task, or perhaps one wanted to tell him one’s impression about a trip. Or the problem could consist of a difficulty with teaching.
Obituary

Whichever it was, the consequence was an invitation. Gnedenko lived in a building belonging to Moscow University. He opened the door himself and invited you to his study full of books, works of art, and papers. He never seemed to be in a hurry, and his visitors found it easy to discuss their problems with him. If you discussed a mathematical problem, Gnedenko’s remarks were deep and sometimes unexpected. If you needed his advice, he willingly gave it. In all cases, he was extremely attentive and well-disposed. However, there were exceptions to this rule; on the rare occasions when Gnedenko saw that the purpose of his visitor was dishonourable, he expressed his displeasure directly.

After your problem had been discussed, the talk usually turned to other topics. Gnedenko’s favourites were: the interrelation of mathematics and its fields of application, problems of education, history, books, poetry, art, and many others. Speaking about any topic, Gnedenko did not insist on his insights although he did not hide them. He knew many interesting stories and told them with a deep sense of humour. Gnedenko enjoyed classical music and had a large collection of records. Sometimes, when he was especially proud of some new purchase, he would propose that you listen to it. His taste in music was traditional and he was not afraid to confess that he could not understand some of the modern composers. On other occasions, he would ask you to look through a new album of paintings of old masters. This was, in fact, the method of communication typical of the older generations of Russian intellectuals.

Sometimes, his wife Natalia Konstantinovna joined the company and, in this case, talk continued at the tea table. The relations between Boris Vladimirovich and Natalia Konstantinovna were such that everybody could feel their love and deep respect for each other (once, Gnedenko is even said to have beaten up a scoundrel who said outrageous words about his wife). It was impossible to imagine them alone. Nevertheless, such a time was to come. The death of Natalia Konstantinovna in 1987 was the greatest loss for Gnedenko. His health deteriorated significantly after then.

He understood this, and decided to finish those pieces of work which he considered the most important: to prepare a new edition of his textbook Theory of Probability, to write a new book on random summation, to prepare his memoirs, to revise his classical book with A. N. Kolmogorov. He fulfilled this plan.

In fact, he was very active during his last years. Besides writing, he headed his department, he organized a new actuarial speciality in the faculty, he travelled abroad to the USA, Israel and Greece, and he participated in the scientific life of his country and of the world. He worked hard until his last hours. He wrote: ‘In science, both in its theoretical and applied fields, in history, mathematics, philosophy, and education, everywhere talented personalities are required in order to bring forth something new, in order to inspire people, in order for people to find their ideal.’ He was such a personality. The memory of B. V. Gnedenko will live as long as these words are true.

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