(1.) If a line through the origin cut the conic U in A, and V in B, to find the equation of the locus of a point P which divides AB in the ratio $\lambda : \mu$, $(\lambda + \mu = 1)$.

Here the required equation is

$$\begin{vmatrix} r^4 & r^3 & r^2 & r & 1 \\ 2(\lambda a_1 + \mu\beta_1), & 1 & 0 & 0 & 0 \\ 2(\lambda^2 a_2 + \lambda\mu a_1\beta_1 + \mu^2\beta_2), & s_1 & 2 & 0 & 0 \\ 2\lambda^3 a_3 + 3\lambda^2\mu a_2\beta_1 + 3\lambda\mu^2 a_1\beta_2 + 2\mu^3\beta_3, & s_2 & s_1 & 3 & 0 \\ 2\lambda^4 a_4 + 4\lambda^3\mu a_3\beta_1 + 6\lambda^2\mu^2 a_2\beta_2 + 4\lambda\mu^3 a_1\beta_3 + 2\lambda^4\beta_4, & s_3 & s_2 & s_1 & 4 \end{vmatrix}$$

This, when finally reduced, gives

$$\begin{aligned} & u_2^2(v_2 + \mu v_1 + \mu^2 v_0)^2 + \lambda^2 u_1^2 v_2(v_2 + \mu v_1 + \mu^2 v_0) \\ & + \lambda^2 u_0 u_2(2v_2^2 + 2\mu v_1 v_2 - 2\mu^2 v_0 v_2 + \mu^2 v_1^2) \\ & + \lambda u_1 u_2(\mu v_1 + 2v_2)(v_2 + \mu v_1 + \mu^2 v_0) \\ & + \lambda^3 u_0 u_1 v_2(\mu v_1 + 2v_2) \\ & + \lambda^4 u_0^2 v_2^2 = 0. \end{aligned}$$

(2.) If $OP^2 = OA . OB$, we get by the same method,

$$\begin{aligned} & u_2^2 v_2^2(u_2 v_2 - u_0 v_0)^2 - u_1 v_1 u_2^2 v_2^2(u_2 v_2 + u_0 v_0) \\ & + (u_2^2 v_2^2 - 4u_0 v_0 u_2 v_2)(u_1^2 v_0 v_2 + u_0 u_2 v_1^2) \\ & + u_1^4 v_0^2 v_2^2 + u_0^2 u_2^2 v_1^4 = 0. \end{aligned}$$

(3.) Finally, if P be the harmonic conjugate of O with respect to A and B, we get for the equation to the locus of P,

$$\begin{aligned} & (u_2 v_0 - u_0 v_2)^2 + (u_2 v_1 + u_1 v_2)(u_1 v_0 + u_0 v_1) \\ & + (4u_2 v_0 + u_1 v_1 + 4u_0 v_2)(u_1 v_0 + u_0 v_1) \\ & + 4(u_1 v_0 + u_0 v_1)^2 + 4u_0 v_0 u_1 v_1 \\ & + 8u_0 v_0(u_2 v_0 + u_0 v_2) \\ & + 16u_0 v_0(u_1 v_0 + u_0 v_1) \\ & + 16u_0^2 v_0^2 = 0. \end{aligned}$$

A Trigonometrical Note.

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