Olver, P. J. *Equivalence, invariants, and symmetry* (Cambridge University Press, Cambridge, 1995), xvi+525 pp., 0 521 47811 1 (hardback), £24.95.

In recent years there has been tremendous progress in the qualitative understanding of solution sets of equations, including differential equations, in a neighbourhood of their singular points by studying more tractable, equivalent problems. Singularity Theory concerns notions of equivalence for finite-dimensional equations in a neighbourhood of points where the implicit function theorem fails to give a local description of the solution set. For ordinary differential equations singularities are fixed points and the normal form approach to equivalence, which goes back to H. Poincaré, A. Lyapunov and D. G. Birkhoff, is to recognise when the trajectories of one equation are the same as (orbitally equivalent to) those of a simpler system whose features are accessible. The theory of invariant manifolds for dynamical systems, and alternative methods more generally for infinite-dimensional problems, have the same aim of providing qualitative information from studies of equivalent problems.

Equivalence in a different context and of a more precise, classical flavour is to propose that two equations are equivalent if one can be obtained from the other by a smooth change of independent and dependent variables locally in a neighbourhood of a non-singular point. An underlying theme is to find the transformation which effects the equivalence. (For motivation consider the advantage in knowing how an equation is equivalent to one where there is already a well-developed theory for finding solutions; for example, linear equations or integrable Hamiltonian systems.) Other general questions, such as when two equations of the same type (Euler–Lagrange equations, for example) are equivalent, are as important nowadays as when the problem of how to tell when two polynomials are equivalent spawned Classical Invariant Theory.

An invariant function is one whose value for a given equation is independent of the choice of coordinates and hence has the same value for all equivalent systems. The discovery of a complete set of invariants is a primary goal and in favourable circumstances leads to a solution of the equivalence problem. Every equation has a symmetry group, though in the worst cases it is trivial. If two systems are equivalent then they must have isomorphic (under conjugation by equivalence transformations) symmetry groups and hence symmetry groups are invariant (though they are not invariant functions) under equivalence. Looked at another way, a complete solution of the equivalence problem will yield the symmetries of an equation since the equation is equivalent to itself under transformation by every element of its symmetry group. Thus a study of the symmetries of two equations may reveal that they cannot be equivalent; but equally importantly in significant cases with substantial symmetry it is possible to characterise equivalence through symmetry. Pursuing this theme, the invariant functions must be invariant under the action of the symmetry group and a classification of functions invariant under group actions assumes a central role in the general theory. A complete classification of these invariant functions may also lead to a solution of the equivalence problem. Finally, canonical forms are characteristic members of a class of equivalent equations. (The Jordan forms of square matrices are good examples of canonical forms.)

All this, and a great deal more, one learns from Professor Olver's book. There are many *ad hoc* approaches to the question of equivalence for particular equations. However he focuses on the foundations of a systematic, algorithmic approach to a wide class of equivalence questions. In particular he espouses a method due to Elie Cartan which, although potentially very complex in its execution, he admires because of its depth and generality and because of its potential with the aid of modern computer algebra to offer complete and explicit solutions for large classes of problems.

The apparently algebraic nature of the question of equivalence notwithstanding, its solution is to be found following reformulation as a question in differential geometry. In the case of Cartan's method all questions of equivalence are regarded as special cases of the general question of whether two coframes are equivalent following a diffeomorphism of a manifold. (A coframe is an ordered set of one-forms which at each point of a manifold form a basis of the cotangent space.) Professor Olver's book is an advanced monograph which offers to the mature reader a balance between an overview of basic geometry, albeit from this particular viewpoint, with copious references and some proofs and an exposition of the method in many explicit examples and through the inclusion of a large number of exercises involving symmetry groups and complete sets of invariants. Although it claims to require only a background in multilinear calculus, tensor and exterior algebra, and some group theory, the reader will, I think, find the many allusions to a huge variety of topics quite daunting, though no doubt rewarding as well. The style is lively and stimulating and, while making considerable demands on both the commitment and mathematical background of the reader, in 480 detail-packed pages it manages to serve up in digestible form a great deal of material of interest far beyond the equivalence problem.

Whether this monograph is for you will be determined by your need to know the material and the extent to which your basic knowledge coincides with what the author considers basic. You may also be swayed by the prodigious nature of some of the calculations which emerge in the solutions of equivalence problems. But there is no room for doubt about the author's authority in the subject. As a definitive work at its price every mathematics research library should have a copy.

J. F. TOLAND

MITRINOVIĆ, D. S., SÁNDOR, J. and CRSTICI, B. *Handbook of Number Theory* (Mathematics and its Applications Vol. 351, Kluwer Academic Publishers, Dordrecht, 1995), xxvi + 622 pp., 0 7923 3823 5, £179.

What is a handbook? The dictionary describes it as "a small guidebook, or book of instructions." This handbook is, at 622 pages, not small, nor is it a book of instructions. Perhaps it should not be called a handbook at all! The only other "Handbook" on my shelf is the *Handbook of Mathematics* by Bronshtein and Semendyayev (English edition: Verlag Harri Deutsch, 1985), which cost me a bargain £10 in 1989. It is not small either! At 972 pages it gives a masterly compact summary of much useful mathematics – no number theory though.

But what of the *Handbook of Number Theory*? What is its scope and how is it organised? It consists of a long list of results in analytic and prime number theory largely. The majority of results are either inequalities or estimates, for example for the size of sets or sums. The results are organised into themes with the major historical results preceding the latest ones. It is manifestly a massive work of painstaking scholarship, referring where possible to original sources, and well-organised. Writing this review just after the death of Paul Erdös, I realise that one way of describing the scope of the book is to say that it is almost entirely about (some of) the topics in number theory which interested him, for references to his work appear in every chapter except one.

The book has sixteen chapters, the first seven of which are devoted to the classical arithmetical functions $\phi(n)$, $d(n)$, $\sigma(n)$, $\mu(n)$ and so on. The other chapters concern primes in arithmetical progressions, additive and diophantine problems concerning primes, exponential sums (the only chapter without a reference to Erdös – a mistake, surely?!), character sums, binomial coefficients and consecutive integers, partitions, congruences, additive and multiplicative functions. Surprisingly there is also an interesting chapter entitled "Estimates for finite groups and semi-simple rings". A typical result of this chapter is the asymptotic order of the number of solutions of the equation $x^k = e$ in the symmetric group on n symbols.

The broad description given above is perhaps enough to see that the choice of topics covered is, as the authors acknowledge, a personal one and no attempt has been made to cover all areas of number theory. So forms and diophantine equations appear mostly in the context of problems involving primes! There is no algebraic number theory. So, while it is not comprehensive enough in many areas of number theory to be called a "handbook", it is hard to think of a title which adequately describes its contents.

My main criticism of the book relates to the practicalities of finding and using the results in it. The authors state in the preface that the book is intended for use by non-specialists