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RADIAL PULSATION IN VARIABLE STARS WITH MASS LOSS

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ABSTRACT It is possible to show that cool giants with very large photospheric scale heights do not perfectly reflect pulsational waves at the photosphere. This means that for these stars the classical formulation of pulsation in which the outer boundary for the resonance cavity is assumed to be perfectly reflecting is not valid. This can have significant consequences for the eigenfrequencies of the pulsation of Long Period Variables such as Mira type variables as well as for the stability of their pulsation.

There are various problems connected with the modelling of pulsation of cool giant stars on the asymptotic giant branch. These were recently discussed by Wood (1990). Two problems are highlighted here :

1) The observed values of the pulsation constant Q do not match theoretical Q-values very well either for the fundamental or first overtone pulsation.
2) The growth rate of the fundamental mode is too large, and it has to be reduced artificially to produce stably pulsating models.

The first of these problems may be caused by inaccuracies in the observational determination of Q which is rather sensitive to the determination of the effective temperature. The spectra of these cool stars are dominated by absorbtion in molecular bands which greatly complicates determination of their effective temperature. The second problem may be due to the lack of a good time dependent theory of convection. A stable pulsation can only be obtained by introducing an artificial damping into the calculation (Wood, 1990). An order of magnitude estimate of this damping can be made from some of the results of Wood (1974). Model calculations take about 10 oscillations before a shell of some $10^{-2}\ \mathcal{M}_\odot$ is ejected. The excess energy input, E_{ex}, into the pulsations can then be calculated and is $\approx\ 5\ 10^{28}$ W. The mass loss of some $10^{-6}\mathcal{M}_\odot/yr$ carries away some energy but this energy loss term is much smaller than E_{ex}. Most of the excess energy still has to be radiated away in the wind, or carried away by the waves generated by the pulsation. At the base of the wind, the energy flux in waves is :

$$E_w \ = \ 4\pi R_*^2 \frac{1}{2}\rho_0 \langle \delta v^2 \rangle (a + v_0) \qquad (1)$$

Here v_0 and ρ_0 are the outflow velocity and the density at the base of the wind and a is the sound speed. $\langle \delta v^2 \rangle$ is the mean square of the velocity amplitude of

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the waves. By equating E_w and E_{ex} and using a mass loss rate of $10^{-6}\mathcal{M}_\odot/yr$ an upper limit for the wave amplitude at the stellar surface can be obtained :

$$\langle \delta v^2 \rangle^{\frac{1}{2}} < 4 \text{ km/s} \tag{2}$$

The speed of sound in these cool stellar atmospheres is around 5 km/s. The excess energy in the pulsation can therefore be carried to infinity with shock waves that have a velocity amplitude which is of the order of the speed of sound.

The discrepancies 1) and 2) between observed stars and theoretical models may result from the modelling assumption that the pulsational waves are reflected perfectly at the surface of the star. This perfect reflection makes the model a closed cavity resonator whereas the real star is not a closed cavity.

For most stars the assumption that the stellar atmospheric background is hydrostatic holds quite well. The situation changes when the star has a significant stellar wind. In some cases the gas pressure may not decrease sufficiently rapidly in the stellar wind. The outer boundary is then not a pressure node for the pulsation because $p \neq 0$ and it transmits pulsational waves : the resonant cavity is open at one end. This causes damping of the pulsation and a change in frequency of the eigenmodes. The difference is very similar to the difference between open and closed organ pipes. Increasing the mass loss rate for a given stellar model is equivalent to drilling a hole of increasing size in the organ pipe.

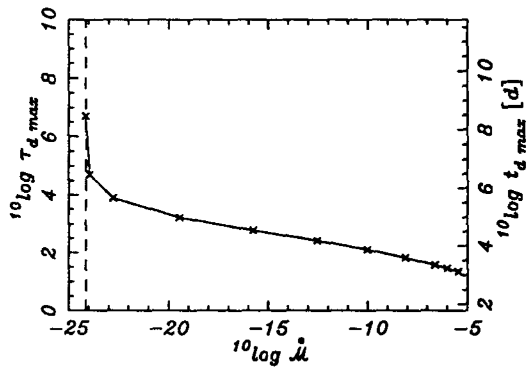

Fig. 1. The maximum damping time as function of mass loss rate. The crosses are the results for stellar polytropic models combined with a shock wave driven wind (Pijpers and Habing, 1989).

It can be shown (Pijpers, 1992) that there is a maximum value of the characteristic damping time t_d determined by the same parameters that determine the mass loss rate. Figure 1 shows this damping time for the shock wave driven wind model of Pijpers and Habing (1989). Now the sonic point of the wind is a boundary for information carried by sound waves. Outgoing waves will simply pass through the sonic point but incoming sound waves will never reach it. This provides the boundary condition for linear pulsation in a star with a stellar wind. The pulsational wave must be a purely outgoing sound wave at the sonic point. The energy of the pulsation carried away by the sound waves through the sonic point causes the pulsation to be damped.

The system of differential equations of the LAWE can be written in such a way that the background is expressed in a single complex potential function Λ. The evaluation of Λ inside the star requires the solution of the equations of stellar structure. The essentials of the core-mantle structure that is typical of AGB stars (Keeley, 1970) are represented by using a combination of polytropes. The wind is modelled using the shock wave driven wind model of Pijpers and Habing (1989).

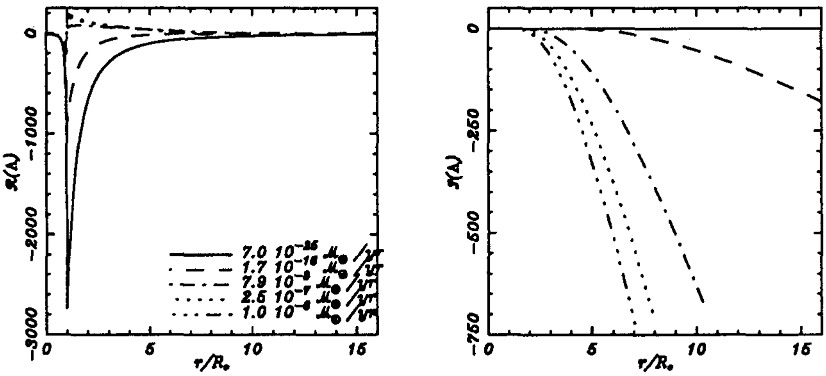

Fig. 2. The real and imaginary part of the Λ-function for various mass loss rates. The line coding is the same in both panels.

In figure 2, the real and imaginary parts of the Λ-function are shown for varous mass loss rates. The sharp dip in the real part of Λ is the stellar surface, where for a classical stellar model $\Re(\Lambda)$ would go to $-\infty$. This implies that no information could then pass through the stellar surface in either direction and it would serve as a reflective boundary for the LAWE.

The leakage of pulsation energy out of the star caused by significant mass loss may stabilize the pulsation modes of Miras and other cool giants and supergiants. The frequency of the eigenmodes in stars that have a well developed stellar wind is different from the eigenfrequencies in classical pulsation models. This may explain the discrepancy between values of the pulsation constant Q found observationally and those calculated from standard models without mass loss. The mass loss rate will act as a governing parameter for the eigenfrequencies and therefore for the pulsation constant Q.

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