COOL WHITE DWARFS : COOLING THEORY AND GALACTIC IMPLICATIONS

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1. Introduction

The understanding of the physics of cool white dwarfs (WD) bears important consequences for Galactic evolution and cosmological implications. The observed cutoff in the disk WD luminosity function (WDLF) yields the determination of the age of the Galactic disk, as suggested initially by Winget et al. (1987). The recent microlensing observations toward the LMC (Alcock et al., 1996) suggest that WDs might provide a substantial fraction of the halo dark matter (Chabrier, Segretain & Méra, 1996; Adams & Laughlin, 1996). The correct analysis of these applications implies a correct WD cooling theory and reliable photometric predictions, which in turn require accurate interior and atmosphere models. Important improvement in this latter domain has been accomplished recently by Bergeron, Saumon & Wesemael (1995) and Bergeron, Wesemael & Beauchamp (1995), which yields the determination of photometric color indices and bolometric corrections down to 4000 K (see Leggett, these proceedings). In this paper, we review the most recent improvement in WD interior and cooling theory.

2. Internal structure. Cooling theory

The theory of WD cooling was first outlined by Mestel & Ruderman (1967) who identified the dichotomic properties of WDs, where degenerate electrons provide the pressure support but do not contribute significantly to the heat capacity ($c_V/k_B = \pi^2/2(T/T_F)$ for a fermion gas, where c_V is the specific heat per unit mass and $T_F \sim 10^9$ K is the electron Fermi temperature for WD central densities) while the ions contribute negligible pressure but

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provide most of the thermal energy. Abrikosov (1960) and Salpeter (1961) independently pointed out the possible onset of crystallization in cool WDs. VanHorn (1968) first developed a consistent theory of WD crystallization and Lamb & VanHorn (1975) first calculated the evolution of a pure carbon crystallizing WD. These calculations have been extended to C/O mixtures by Wood (1992) who examined extensively the importance of the various parameters entering WD evolution, e.g. the core composition and the atmosphere structure. A further significant breakthrough in WD cooling theory is due to Stevenson (1980) who first pointed out the importance of the crystallization diagram in a *two-component* (e.g. C/O) plasma and of the difference of chemical composition in the fluid and in the solid phase. This motivated numerous calculations for the characterization of the phase diagram (Barrat, Hansen & Mochkovitch, 1988; Ichimaru, Iyetomi & Ogata, 1988), the effect on WD cooling (Mochkovitch, 1983) and on the WDLF (García-Berro, Hernanz, Mochkovitch & Isern, 1988). More recently Segretain & Chabrier (1993) characterized the evolution of the crystallization diagram of stellar plasmas for arbitrary binary mixtures as a function of the charge ratio. The chemical differentiation at crystallization calculated with these diagrams was shown to produce an extra source of energy ΔE in the WD (Segretain et al., 1994), which in turn leads to a substantial increase in the age of crystallized WDs for a given luminosity ($\Delta t \propto \Delta E/L$), an important issue for a correct determination of the age of the Galactic disk (Hernanz al., 1994).

In this section, we derive an analytical theory for the evolution of cool, crystallizing WDs, based on first principles of thermodynamics (Landau & Lifschitz, 1980), aimed at describing the main physical effects in terms of simple physics (see also Isern et al., 1997). These calculations include all the afore-mentioned processes and yield a reasonable estimate of the gravitational energy release and time delay induced by chemical fractionation at crystallization, a question of strong debate among the community. This provides useful guidelines to verify the validity of complete, numerical calculations.

The first laws of thermodynamics yield for white dwarf cooling

$$L = -\int_0^M \frac{dq}{dt} dm - \int_0^M \epsilon_\nu dm = -\int_0^M \frac{du + d\Omega}{dt} dm - \int_0^M \epsilon_\nu dm \qquad (1)$$

where ϵ_ν is the neutrino rate and dq/dt is the heat rate per unit mass along the change of an equilibrium state. du and $d\Omega$ are the change of specific internal energy and gravitational energy, respectively. The first one reads :

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$$du = c_V dT + ((T\frac{dP}{dT})_v - P)dv + \Delta u_{crys} + [\Sigma_i(\mu_i dN_i)_l + \Sigma_i(\mu_i dN_i)_s]_{v,T}$$
$$= Tds - Pdv + T\Delta s + [\Sigma_i(\mu_i dN_i)_l + \Sigma_i(\mu_i dN_i)_s]_{v,T} \quad (2)$$

The μ_i denote the chemical potentials, s the specific entropy and $v = 1/\rho$ is the volume per unit mass. The dN_i are the variations of carbon and oxygen nuclei in the fluid and the solid phase due to the change of composition at crystallization. They can be calculated with the thermodynamics lever rule. When the central crystal grows[1], there is a thin C-enriched surrounding fluid layer where *locally* the variation of nuclei $d(\delta N_{i_l})$ is *not* equal to $-dN_{i_s}$ (no *local* mass conservation) [2]. How this carbon excess in the fluid is redistributed homogeneously will be examined below. No variation of composition yields of course $\delta N_i = 0$ and thus no extra internal energy.

The first two terms on the r.h.s. of (2) have been recognized originally by Mestel & Ruderman (1967), the third term was first introduced by Van Horn (1968) and is the crystallization latent heat $l = -\Delta u_{crys} = T(s_{sol} - s_{liq})$.

The condition of hydrostatic equilibrium yields for the variation of gravitational energy :

$$\delta\Omega = \int_0^M P\delta v\delta m \approx < \frac{P_e}{\rho} > M \quad (3)$$

since the electron pressure largely dominates the ionic pressure.

Eqns. (1)-(3) can be rewritten :

$$L + L_\nu = -\int_0^M c_V \frac{dT}{dt}dm - \int_0^M (T\frac{dP}{dT})_v \frac{dv}{dt}dm + l\frac{dm_S}{dt} + \delta u\frac{dm_S}{dt} \quad (4)$$

where $\delta u = \Sigma_i(\int \mu_i d(\delta N_i))$ and m_S is the mass crystallized. Note that the last two terms in eqns. (2) and (4) are evaluated at *constant volume* and thus *do not stem from a contraction work, but from the change of composition at crystallization.*

[1]Note that this fractionation process is described sometimes in the literature as drowning O-*flakes*. This is an erroneous picture, based on Stevenson's old *eutectic* diagram, which corresponds to an *inhomogeneous* solid. The correct *spindle* diagram yields a homogeneous solid. It is easy to verify that the solid is denser than the liquid, so that the crystal grows at the center of the star. Were WDs made of water, the story would be different because of the volume expansion at crystallization

[2]This is similar to e.g. silicium deposition for semi-conductor devices in a silicium+impurity liquid, which yields a silicium concentration gradient and eventually a silicium-rich layer. We have in fact a distillation process in the WD.

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The contribution of the first three terms of eqn. (4) can be estimated easily, as done initially by Mestel & Ruderman (1967) and Lamb & Van Horn (1975) :

$$\begin{aligned} \delta U_{th} &= \int_0^M c_V dT\, dm \approx \delta\Omega \text{ from the virial theorem} \\ \delta U_{grav} &= \int_0^M T\frac{dP}{dT}\, dvdm \approx \int_0^M [T\frac{dP_i}{dT} + o(T/T_F)^2]dvdm \approx < \frac{P_i^{th}}{\rho} > M \end{aligned} \tag{5}$$

where $P_i^{th} = \rho \mathcal{R} T/\mu_0$ (where μ_0 is the mean ionic molecular weigth) is the thermal (non electrostatic) ionic pressure.

Equations (3) and (5) yield $\delta U_{grav} \sim \frac{<P_i^{th}>}{<P_e>}\delta\Omega$. WD characteristic central density $\rho \sim 10^6$ g.cm^{-3} and central temperature $T \sim 10^6$ K yield $P_i^{th}/P_e \sim 10^{-3}/\bar{Z}$. Thus only a negligible fraction of the energy due to gravitational contraction is radiated. Most of the work is expended in raising the electron Fermi energy, as first noted by Lamb & Van Horn (1975). The latent heat contribution can be estimated from the differences between the solid and liquid ionic entropies (VanHorn, 1968) : $l \sim -kT_c/AH$, where T_c is the crystallization temperature ($\sim 3 \times 10^6$ for C/O, see Segretain & Chabrier, 1993), A is the mean atomic mass and $H = 1$ a.m.u.$= 1.66\times10^{-24}$ g. This yields an energy release $U_{latent\,heat} \sim 10^{47}$ erg $\sim 10^{-2}\,\Omega$, where $\Omega = GM^2/R \sim 10^{49}$ erg is the WD gravitational energy. The negative sign indicates that the energy is *emitted* at crystallization.

The last term can be estimated as follows :

$$\delta u = (\frac{\partial u}{\partial X})_{v,T}\delta X \approx (\frac{\partial u_i}{\partial X})_{v,T}\Delta X \approx \Delta u_i \tag{6}$$

where X is the mass fraction of one of the components (say carbon), $\Delta X = X_l - X_s$ and $\Delta u_i = u_{i_l} - u_{i_s}$ is the difference of Madelung energy in the C/O plasma between the fluid and the solid phase[3]. The Madelung energy (per unit mass) of the mixture reads :

$$u_i/kT = \alpha\Sigma_k \frac{X_k}{A_k}\Gamma_k = \alpha\Gamma_e\{X\frac{Z_1^{5/3}}{A_1} + (1-X)\frac{Z_2^{5/3}}{A_2}\} \tag{7}$$

where α denotes the Madelung constant in the fluid or in the solid phase ($\alpha_s = -0.9, \alpha_l = -0.899$), $\Gamma_e = e^2/a_e kT$ (a_e is the mean inter-electronic distance) and the index k denotes each ionic species (C^{6+}, O^{8+}). This yields:

[3]In fact δu is only a fraction of Δu_i since it stems from the difference w.r.t. to the average energy over the C-enriched layer. This is not consequential for the present estimate

$$\Delta u_i/kT \approx -0.9\Gamma_e \Delta X (Z_1^{5/3}/A_1 - Z_2^{5/3}/A_2) \quad (8)$$

The virial theorem $P_i/\rho = \frac{1}{3}u_i$ yields :

$$\delta U/\delta\Omega = \frac{\int \delta u \, dm_s}{\delta\Omega} \sim \frac{<\Delta P_i>}{<P_e>}\frac{M_s}{M} = \frac{\Delta P_i}{P_i}\frac{P_i}{P_e}\frac{M_s}{M} \quad (9)$$

Note that P_i is now the ionic electrostatic pressure. For $Z_1 = 6$, $Z_2 = 8$, $x_1{=}x_2{=}1/2$, we get $P_i/P_e \sim 10^{-2}{-}10^{-1}$, $\Delta P_i/P_i \sim 2\Delta x\,(Z_1^{5/3}{-}Z_2^{5/3})/(Z_1^{5/3}{+}Z_2^{5/3}) \sim -0.5 \times \Delta x$, where $\Delta x \sim 0.1 - 0.3$ is the difference of carbon number concentration between the solid and the fluid phase at crystallization (Segretain & Chabrier, 1993). With $M_s/M \sim 0.1$, this yields:

$$\delta U/\delta\Omega \sim 10^{-4} - 10^{-3} \quad (10)$$

in agreement with the detailed numerical calculations (see Figure 5 of Segretain et al., 1994).

Chemical differentiation at crystallization thus provides an additional source of energy wich remains much smaller than the gravitational energy. But, as shown below, the release of this quantity at a low-luminosity phase of the evolution has a significant effect upon the lifetime of the star at these stages.

$$\Delta t = \int_0^M \frac{\delta u(T)}{L(T)} dm \approx \frac{\delta U(T)}{L(T)} \approx \frac{\Delta P_i}{P_i}\frac{P_i}{P_e}\frac{M_S}{M}\frac{\Omega}{L} \quad (11)$$

With the afore-mentioned values we get $\Delta t \sim 5 \times 10^8$ yr *at the begining of crystallization*, $M_s/M \sim 10\%$ and $L = 10^{-3.5}\,L_\odot$ and $\Delta t \sim 2 \times 10^9$ yr at $L = 10^{-4.5}\,L_\odot$, $M_s/M \sim 80\%$, the observed cutoff luminosity. These simple calculations show the importance of the time delay induced by chemical fractionation at crystallization for cool (faint) WDs, even though the corresponding energy is small compared to the binding energy.

An other issue concerns the redistribution of the excess of carbon in the fluid at crystallization since the solid core is O-enriched, as obtained from the phase diagram. Since the fluid C-enriched layer around the crystal is lighter than the surrounding medium, a Rayleigh-Taylor instability develops locally, due to the variation of molecular weight. This problem has been considered in detail by Mochkovitch (1983) who showed that the typical crystallization time (for a $0.6M_\odot$ WD, the crystallization velocity at

the begining of crystallization is $v_c \sim 10^{-2}\, M_{WD}/7 \times 10^7\, yr \sim 10^{-8}$ cm.s^{-1}) is significantly larger than the convection time so that the liquid is likely to be rehomogeneized rapidly as crystallization goes on.

3. Galactic implications

The first application of these calculations concerns the age of the faintest WD ever observed, $ESO\,439-26$ (Ruiz et al., 1995). The trigonometric-parallax determination of this object yields an absolute magnitude $M_V = 17.4 \pm 0.3$, $M_{bol} = 17.1 \pm 0.1$ (Bergeron et al., 1997) about 1 mag faintward of the observed cut-off of the WDLF of the Galactic disk (Liebert et al., 1988). The location of this object in the M_V vs $V-I$ diagram, and a comparison with the photometric sequences of cool white dwarfs recently derived by Bergeron et al. (1995) yields the interpretation that it is a cool ($T_{eff} \sim 4500$ K), massive WD ($m \sim 1.2\, M_\odot$). Comparison with (pure carbon) evolutionary models of Wood (1992) yields an age determination for this WD, $t \sim 6.5$ Gyr, substantially below the lower limit for the age of the Galactic disk determined by detailed WD cooling theory (Hernanz et al., 1994). The Wood sequences do not include the afore-mentioned release of gravitational energy due to C/O differentiation at crystallization. Figure 1 displays several isochrones of a C/O WD for several masses (WDs more massive than $1.2\, M_\odot$ have a different O/Ne/Mg internal composition), obtained with a WD cooling theory including (solid line) and neglecting (dashed line) the fractionation process. The dotted lines display the luminosity of ESO 439-26 and its mass, assuming either a H-rich ($\sim 1.1\, M_\odot$) or a He-rich ($\sim 1.2\, M_\odot$) atmosphere (see Ruiz et al., 1995). As shown, ESO 439-26 is compatible with an age t$\sim$ 10 Gyr, in good agreement with the most recent determination of the age of the disk, whereas neglecting differentiation would yield $t < 9$ Gyr.

An other important galactic application is the age of the disk inferred from the comparison between the observed and the theoretical WDLF. As shown by Hernanz et al. (1994) and by the present analysis, chemical fractionation yields 1 to 2 Gyr older ages (i.e. 10-20% increase), depending on the initial C/O profile, w.r.t. estimates which do not include this process. Preliminary calculations along the present lines based on the recently observed WDLF (Oswalt et al., 1996) yield an estimate for the age of the cutoff at $\log L/L_\odot = -4.5$, i.e. an age for the Galactic disk, $t_D \sim 12$ Gyr.

4. Uncertainties in the theory

An important uncertainty in the models is the initial C/O composition and stratification. If the initial WD is already stratified, with oxygen accumulated near the center, as suggested by Mazzitelli & D'Antona (1986),

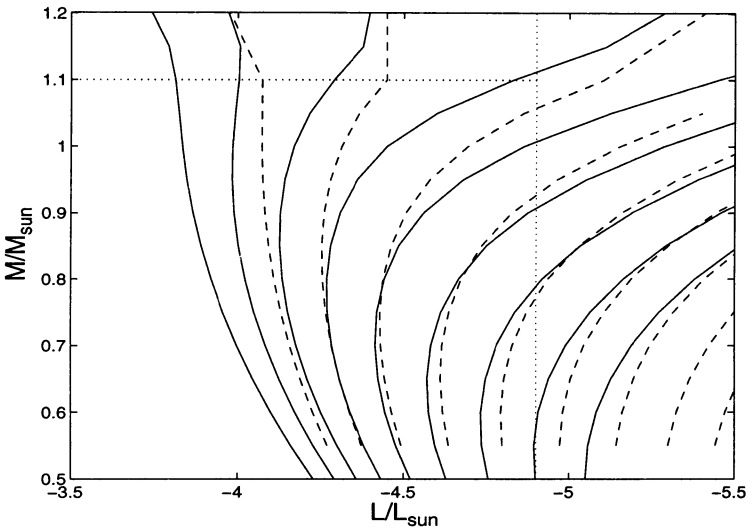

Figure 1. Isochrones for different masses of crystallized WDs ($\log L/L_{\odot} < -3.5$) in a mass-luminosity diagram. The solid lines include C/O differentiation at crystallization whereas the dashed lines do not. The isochrones are the *same in both cases* and go from $t = 7$ Gyr to $t = 15$ Gyr from left to right. The vertical dotted line is the inferred luminosity for *ESO* $439-26$ (Bergeron et al., 1997), while the horizontal dotted lines are the inferred mass for a H-rich ($1.1\ M_{\odot}$) and He-rich ($1.2\ M_{\odot}$) atmosphere. Crystallization starts earlier for massive (and thus rapidly evolving) WDs, which then enter the rapid Debye cooling regime, which causes the bending in the isochrones.

the energy release by differentiation, and the related time delay are reduced (see Segretain et al, 1994, Tables 3-4). The other main source of uncertainty concerns the model atmospheres and most importantly for the age of the disk, the thickness of the helium layer wich regulates the emergent heat flux. The thicker the layer the younger the age (Wood, 1992). These uncertainties translate in a $\sim 1-2$ Gyr uncertainty on the age of the disk. Futher progress in this direction is certainly essential for a better determination of the cooling and the observational properties of cool WDs.

5. Conclusion

We have shown in this review that substantial improvement in the theory of cool WDs has been accomplished within the past few years for the atmosphere as well as for the internal structure. We have shown that chemical fractionation at crystallization, although it liberates a negligible amount of gravitational energy, modifies substantially the cooling history of the star and yields an important time-delay for faint WDs. This process cannot be ignored in accurate WD cooling theory. As noted by Mochkovitch (pri-

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vate communication), although the importance of crystallization of alloys in stellar plasmas (WDs in particular) is still strongly debated in the astrophysical community, it has been recognized long ago in geophysics (see e.g. Loper, 1984; Buffett et al., 1992). Although the nature of the plasma is different, the *physics* of the process (thermodynamics and energy transport) is exactly the same. Important uncertainties remain essentially in the exact determination of the initial C/O profile in the star and in the structure of the hydrogen and/or helium outer envelope.

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DISCUSSION

ROBERT KURUCZ: What about the effect of rotation on solidification?

GILLES CHABRIER: It is not clear how rotation will affect the crystal growth, but there is no reason for it to *prevent* crystallization, which must occur below a certain temperature. The effect of rotation on the C-redistribution in the surrounding layer has been considered in detail by Mochkovitch (1983).

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