



# Correction to “Infinite Dimensional DeWitt Supergroups and Their Bodies”

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*Abstract.* We provide a correction to Theorem 3.5 in the article entitled “Infinite Dimensional DeWitt Supergroups and their Bodies” (Canad. Math. Bull. 57(2014), no. 2, 283–288).

This note provides a correction to Theorem 3.5 in [1]. Only part (iii) of that theorem requires a correction, as the original proof failed to separate the proof of (ii) from the proof of (iii). The proof of [1, Theorem 3.5(ii)] was complete once it was established that  $ad_a$  is quasi-nilpotent for each  $a$ , since it follows from this that  $K$  is quasi-nilpotent. The proof of (iii) was not complete as stated in the original article. The correction consists of a revised part (iii), along with its proof, in the following version.

**Theorem 3.5** *Let  $G$  be a DeWitt super Lie group such that there is an induced group structure on  $BG$ . Let  $\beta: G \rightarrow BG$  denote the induced group homomorphism and let  $K$  be its kernel.*

- (i)  *$K$  is a Banach Lie group whose Lie algebra  $\kappa$  is a freely finitely generated  $\Lambda^0$  left module.*
- (ii) *The Lie module  $\kappa$  is quasi-nilpotent, and consequently the Baker–Campbell–Hausdorff formula holds globally on it.*
- (iii) *There is a group operation  $\diamond$  on  $\kappa$  relative to which  $\kappa$  is a simply connected, global, Banach Lie group such that  $\exp(a)\exp(b) = \exp(a \diamond b)$  for all  $a, b \in \kappa$ . Moreover  $K$  is simply connected and consequently is diffeomorphic to the Banach space  $\kappa$  and is isomorphic as a Banach Lie group to  $(\kappa, \diamond)$ .*

**Proof of (iii)** By a Theorem of Wojtynski [2], the global Baker–Campbell–Hausdorff formula holds for  $\kappa$ . Moreover, it has long been known that there is a local Lie group operation  $\diamond$  on  $\kappa$  such that  $\exp(a)\exp(b) = \exp(a \diamond b)$  for all  $a, b \in \kappa$ . Wojtynski’s important contribution was to show that when  $\kappa$  is quasi-nilpotent, this operation is globally defined on  $\kappa$ , and that  $\kappa$  is, in fact, a global Banach Lie group relative to  $\diamond$ . We show that  $\exp$  is a bijection. Note that  $I = \exp(\kappa)$  is a subgroup of  $\kappa$ . It is an open subgroup, since  $\exp$  is a local diffeomorphism that takes 0 to the identity of  $K$ . Also,  $I$  is connected as is  $K$ . An open connected subgroup of a connected group must be the entire group, so  $\exp(\kappa) = K$ . Since  $\exp$  is a surjective homomorphism,  $K$  is isomorphic to  $\kappa$  modulo a discrete subgroup. Since there is a global chart from  $K$

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onto  $\widetilde{\mathbb{R}^{p|q}}$ ,  $K$  is simply connected. But both  $\mathcal{K}$  and  $\kappa$  are simply connected, and so the kernel of  $\exp$  must be trivial. Thus,  $\kappa$  and  $K$  are diffeomorphic and isomorphic. ■

## References

- [1] R. Fulp, *Infinite dimensional DeWitt supergroups and their bodies*. *Canad. Math. Bull.* **57**(2014), no. 2, 283–288. <http://dx.doi.org/10.4153/CMB-2013-025-6>
- [2] W. Wojtynski, *Quasi-nilpotent Banach-Lie algebras are Baker-Campbell-Hausdorff*. *J. Funct. Anal.* **153**(1998), no. 2, 405–413. <http://dx.doi.org/10.1006/jfan.1997.3202>

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