OPTIMUM EXPOSURE TIME AND FILTER BANDWIDTH IN SPECKLE INTERFEROMETRY

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ABSTRACT

The dependence of the signal to noise ratio on the exposure parameters in speckle interferometry is analysed. The choice of the optimum exposure parameters, which are found to depend on certain statistical functions of the image intensity, is discussed. Preliminary results of measurements of these statistical functions are given.

1. INTRODUCTION

Stellar speckle interferometry[1,2,3], in which the autocorrelation or, equivalently, the power spectrum of the image intensity is estimated, is a technique for obtaining diffraction limited resolution of stellar objects despite atmospheric turbulence. The result is a statistical property of the image intensity which is normally estimated by averaging over a large number of short exposures. The technique has been applied to studies of binary stars[4,5,6,7,8,9], giant stars[8,10,11] and other more complex objects[8,12]

The faintest binary that has so far been resolved by applying the speckle technique is of 9th magnitude whereas theoretical predictions[13,14] indicate that much fainter (up to 18th magnitude) binaries should be resolvable with the technique.

The problem of approaching the limiting magnitude of the speckle technique is one of signal to noise ratio. At high light levels each short exposure image is a random intensity distribution (speckle pattern) and the noise is a consequence of the randomness of the speckle. At low light levels the short exposure images contain only a few photons and are dominated by photon noise. The signal to noise ratio associated with a particular observation depends primarily on two factors. Firstly the number of exposures that are used and secondly the signal to noise ratio of the

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information contained in each exposure.

Traditionally the observation technique has been to record a large number, N, of short exposure images, which are later analysed by one or both of two equivalent methods. The first method is to sum the squared moduli of the Fourier transforms of the images using a coherent optical system[15]. The second method is to digitise the images using a microdensitometer and then to use a digital computer to process the data[15].

The recording of a large number of images causes data handling and storage problems which impose practical limitations on N. The first method of analysis is limited to $N < 1000$ due to the noise introduced by the light scattered from the film surfaces[15]. The second method requires large amounts of both microdensitometer and computer time[15].

To overcome these restrictions on N a number of on line data reduction schemes have been proposed and implemented[8,16,17]. At present the major effort is towards systems based on a two dimensional array of photon detectors linked to a digital vector autocorrelator. With an on line system N is restricted only by the length of the observation period and the computational power of the system.

For a given object the signal to noise ratio of the information contained in a single exposure may depend on a large number of factors including atmospheric conditions, telescope size and quality, the detection efficiency of the system, the size of each detector, the format of the array, the exposure time, and the passband of the filter. The first three factors are outside the control of an observer wishing to resolve a certain object with a given telescope and system. The spacing of the detectors in the array defines the resolution capability of the system and should be somewhat less than the size of the Airy disc of the telescope if diffraction limited performance is required. To maximise the detection efficiency the central wavenumber of the filter should be at the peak of the product of the spectral emissivity of the object and the spectral response of the detector. The exposure time and filter bandwidth, however, are variables which the observer may choose in an attempt to maximise the signal to noise ratio attainable within a given time and so reduce the observation period

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needed to resolve a given object. Clearly this is most important in terms of making efficient use of telescope time for faint objects for which observation times are necessarily lengthy.

In this paper the dependence of the signal to noise ratio at low light levels on the exposure parameters is analysed. The form of this dependence is found to be affected by a number of factors including two statistical functions of the intensity in the point spread function (image of an unresolved star) of the telescope. These functions are the temporal autocorrelation and the spectral crosscorrelation of the intensity at a point in the image. For a given telescope the form of these functions depends on the atmospheric conditions. Computational examples, using analytical forms for the two functions, are given to demonstrate the functional dependence of the signal to noise ratio on the exposure parameters.

Methods of estimating these functions are indicated and preliminary results given. Choice of the optimum parameters for speckle interferometry is discussed.

2. SIGNAL TO NOISE RATIO

Several estimates of the signal to noise ratio in speckle interferometry have been given[1,13,14,18,19,20]. The one most useful for the present purpose is that due to the author.[19] This estimate expresses the signal to noise ratio, SNR, in terms of the output of the sort of digital autocorrelator system discussed in the introduction. The expression is valid in the relevant case of low light levels (photon noise limit) and can be written as

$$\mathrm{SNR} = \frac{N^{\frac{1}{2}}\, n_{ph}\, A_d^{\frac{1}{2}}}{2^{\frac{1}{2}} A^{\frac{1}{2}}} \; \frac{f}{1 + 2f + f^2} \; . \; \frac{2}{A} \int_{-\infty}^{\infty} \frac{\sigma_E^2(x)}{\langle E\rangle^2} \, e^{-\frac{2r^2}{\alpha^2}} \, dx \qquad (1)$$

where

f is the ratio of the brightnesses of the components of the binary,

N is the number of exposures used,

n_{ph} is the average number of photons detected in each exposure,

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A_d is the area of the individual detectors which are assumed to be square and adjacent in an array large enough to contain the whole seeing disc,

A is the effective area of the seeing profile, which is assumed to be a gaussian of width 2α at the 1/e points, $A = \pi\alpha^2$,

$\frac{\sigma_E^2}{\langle E \rangle^2}(x)$ is the mean square contrast of the intensity measured by the detector at x in an exposure time T and in a bandwidth Δk when an unresolved star is imaged on the array. Only one spatial dimension is used for simplicity.

r is the distance from the instantaneous centre of the star image.

We now examine the three factors in equation (1) which have a dependence on the exposure time, T, and filter bandwidth, Δk.

2.1 The average photon count, n_{ph}

The average number of detected photons per exposure is equal to the detection rate multiplied by the exposure time. So we can write

$$n_{ph} \propto T \tag{2}$$

The detection rate is proportional to the total intensity in the image so we can write

$$n_{ph} \propto \int_0^\infty e(k)s(k)d(k)dk \tag{3}$$

where

e(k) is the total spectral emissivity of the source,

s(k) is the spectral distribution of the filter,

d(k) is the quantum efficiency of the detector at wavenumber k.

In practice e(k) and d(k) are usually slowly varying compared with s(k) and, to a good approximation, can be removed from the integral to give

$$n_{ph} \propto \int_0^\infty s(k)dk \tag{4}$$

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For simplicity we consider a rectangular spectrum

$$s(k) = s_{\Delta k} \qquad k_a < k < k_b \qquad (5)$$
$$= 0 \qquad \text{otherwise}$$

which gives

$$n_{ph} \propto s_{\Delta k}\,\Delta k \qquad (6)$$

where $\Delta k = k_b - k_a$ is the bandwidth of the filter and $s_{\Delta k}$ its efficiency.

2.2 The number of exposures, N

The maximum number of exposures which can be made and analysed in an observation time To is given by

$$N = \frac{To}{T + T_d} \qquad (7)$$

where T_d is the length of any pause between exposures. A pause between exposures is necessary if the computation time per exposure exceeds the exposure time. The computation time per exposure depends on the number of detected photons per exposure[8,17]. In an ideal system the computation time would be less than the exposure time for any object brightness. In practice this condition can only be achieved for faint objects[14,17], in which case we can write

$$N \propto 1/T \qquad (8)$$

For faint objects, then, N has no dependence on the bandwidth. For moderately faint objects it may have a dependence; this is discussed in section 4.2.

2.3 The mean square contrast $\frac{\sigma_E^2}{<E>^2}$

The mean square contrast of the measured intensity, E, at position x in the pattern, is defined as

$$\frac{\sigma_E^2}{<E>^2}(x) = \frac{<E^2(x)> - <E(x)>^2}{<E(x)>^2} \quad (9)$$

where <.> indicates an ensemble average. The measured intensity can be written as

$$E(x) = \int_t^{t+T} \int_{\substack{\text{area } A_d \\ \text{at } x}} \int_{k_a}^{k_b} I(t,k,x)\,dk\,dx\,dt \quad (10)$$

where I(t,k,x) is the intensity at time t, wavenumber k and position x. Assuming that the size of the detectors in the array is significantly less than the size of the Airy disc of the telescope (a necessary condition for diffraction limited performance) E(x) can be rewritten, to a good approximation, as

$$E(x) = \int_t^{t+T} \int_{k_a}^{k_b} I(t,k,x)\,dk\,dt \,.\, A_d \quad (11)$$

Substituting equation (11) into (9) and making the reasonable assumption that the mean intensity is slowly varying with k compared with s and Ω, gives

$$\frac{\sigma_E^2}{<E>^2} = \frac{1}{T^2 \Delta k^2} \iint_t^{t+T} \iint_{k_a}^{k_b} \Omega(t_1,t_2,k_1,k_2,x)\,dk_1\,dk_2\,dt_1\,dt_2 \quad (12)$$

where Ω is the fourth order correlation function defined by

$$\Omega(t_1,t_2,k_1,k_2,x) = \frac{<I(t_1,k_1x)I(t_2,k_2,x)>}{<I(t_1,k_1,x)><I(t_2,k_2,x)>} - 1$$

Recent experiments[21] have shown that the temporal autocorrelation function $\gamma(t_1,t_2)$ is slowly varying with wavenumber compared with $s(k)$ and $\Gamma(k_1,k_2)$, so to a good approximation $\Omega(t_1,t_2,k_1,k_2,x)$ may be factorized into temporal and wavenumber components,

$$\frac{\sigma_E^2}{\langle E\rangle^2}(x) = \frac{1}{T^2}\iint_t^{t+T} \gamma(t_1,t_2)dt_1dt_2 \cdot \frac{1}{\Delta k^2}\iint_{k_a}^{k_b} \Gamma(k_1,k_2,x)dk_1dk_2 \qquad (13)$$

where $\Gamma(k_1,k_2,x)$ is the spectral crosscorrelation function at position x defined by

$$\Gamma(k_1,k_2,x) = \frac{\langle I(k_1,x)I(k_2,x)\rangle}{\langle I(k_1,x)\rangle\langle I(k_2,x)\rangle} - 1 \qquad (14)$$

and $\gamma(t_1,t_2)$ is the temporal autocorrelation function defined by

$$\gamma(t_1,t_2) = \frac{\langle I(t_1)I(t_2)\rangle}{\langle I(t)\rangle^2} - 1 \qquad (15)$$

and assumed to be invariant to position.

2.4 Signal to noise ratio as a function of the exposure parameters

By making use of equations (1), (2), (6), (7) and (13) we now express the dependence of the signal to noise ratio on the two exposure parameters as

$$\mathrm{SNR} \propto \frac{T}{(T + T_d)^{\frac{1}{2}}} \cdot \frac{1}{T^2}\iint_t^{t+T} \gamma(t_1,t_2)dt_1dt_2 \qquad (16)$$

and

$$\mathrm{SNR} \propto S_{\Delta k}\Delta k . \frac{1}{\Delta k^2}\int_{-\infty}^{\infty} e^{-\frac{2r^2}{\alpha^2}} \iint_{k_a}^{k_b} \Gamma(k_1,k_2,x)dk_1,dk_2,dx \qquad (17)$$

In the next two sections equations (16) and (17) are computed using some analytical forms for $\gamma(t_1,t_2)$ and $\Gamma(k_1,k_2,x)$. The right hand side of equation

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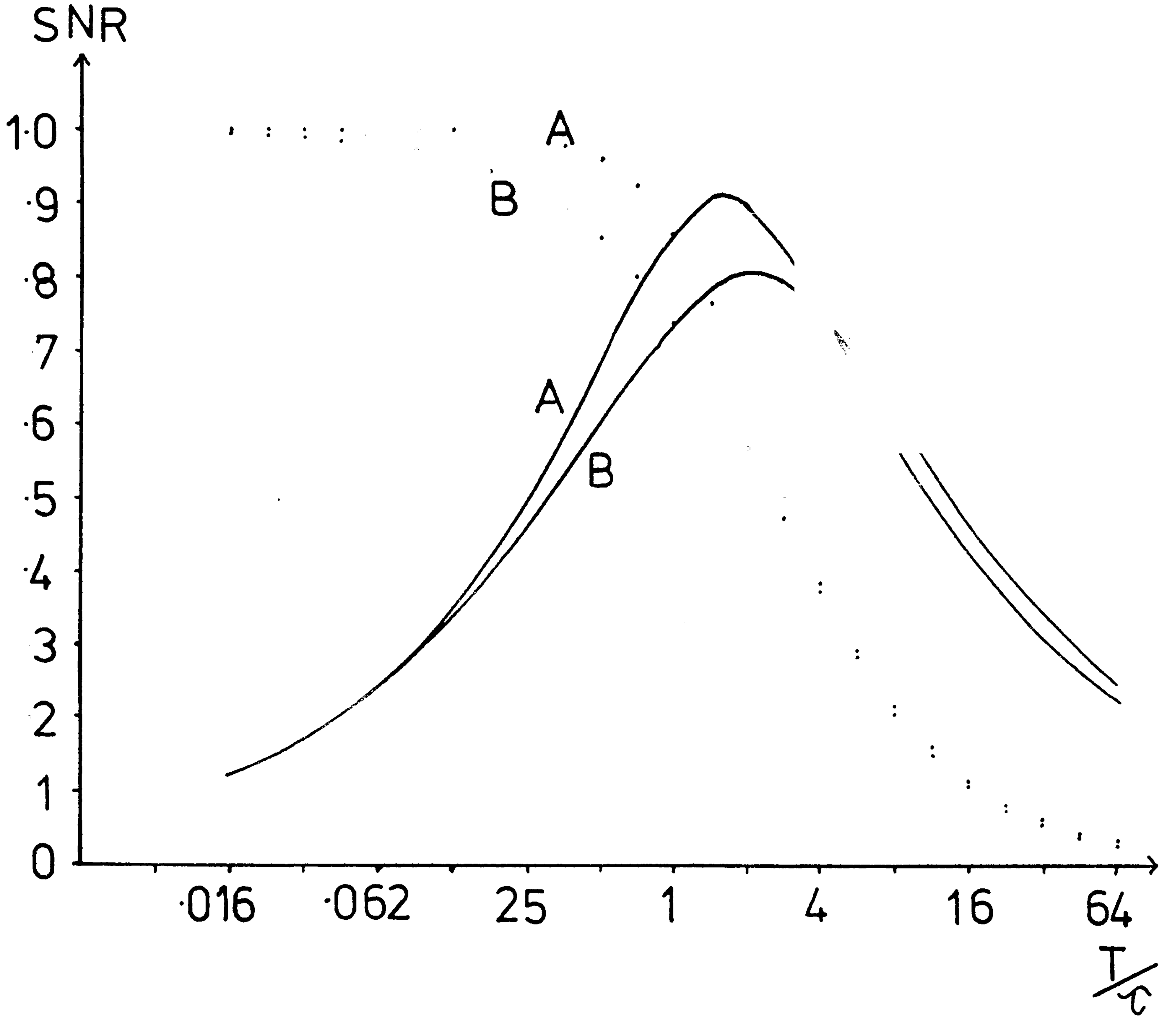

Figure 1 Signal to noise ratio as a function of the exposure time, T, for A gaussian and B negative exponential forms of the temporal autocorrelation function, γ(t) τ is a measure of the typical lifetime of the speckles within the image. The dotted curves show the reduction in contrast due to the finite exposure time. The value of T at the maximum of the SNR curves is the optimum exposure time.

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(16) generally has a maximum and the value of T at which this occurs is the optimum exposure time. The right hand side of equation (17) tends asymptotically towards a maximum with increasing Δk. Certain effects not treated in the theory are considered and the choice of the optimum bandwidth is discussed.

3. EXPOSURE TIME

Assuming I(t) to be a stationary ergodic process allows us to write $\gamma(t_1,t_2) = \gamma(t_1 - t_2)$. By transforming to sum and difference coordinates and using the fact that $\gamma(t)$ is an even function, the integral in equation (16) can be simplified giving

$$\mathrm{SNR} \propto \frac{T}{(T + T_d)^{\frac{1}{2}}} \cdot \frac{2}{T^2} \int_0^T (T - t)\gamma(t)\,dt \qquad (18)$$

Before considering how $\gamma(t)$ might be estimated in a practical case and hence the optimum exposure time determined, it will be useful to consider the dependence of SNR on T for two given analytical forms of $\gamma(t)$.

For

$$\gamma(t) = e^{-\frac{t^2}{\tau^2}} \qquad (19)$$

$$\mathrm{SNR}_A \propto \frac{T}{(T + T_d)^{\frac{1}{2}}} \cdot \left\{ \frac{\sqrt{\pi}\,\tau}{T} \operatorname{erf}\left(\frac{T}{\tau}\right) + \frac{\tau^2}{T^2}\left(e^{-\frac{T^2}{\tau^2}} - 1\right)\right\} \qquad (20)$$

For

$$\gamma(t) = e^{-\frac{t}{\tau}} \qquad (21)$$

$$\mathrm{SNR}_B \propto \frac{T}{(T + T_d)^{\frac{1}{2}}} \cdot 2\left\{\frac{\tau}{T} + \frac{\tau^2}{T^2}\left(e^{-\frac{T}{\tau}} - 1\right)\right\} \qquad (22)$$

Plots of SNR_A and SNR_B, for $T_d = 0$, are shown in figure 1, also shown is the contrast. For both forms of $\gamma(t)$ shown the signal to noise ratio shows a maximum; the value of T at this point is the optimum exposure time. τ is a measure of the "typical lifetime" of the speckles in the image.

Most previous analyses of speckle interferometry have assumed that the exposure time is sufficiently short to "freeze the atmosphere", so that the contrast is not significantly decreased by the finite exposure time. That is, they have assumed $T \ll \tau$. The curves of figure 1 show that "freezing" the atmosphere does not in fact lead to the optimum condition which might be described as occurring when the atmosphere is "partially frozen".

The form of $\gamma(t)$ may depend on a number of factors including telescope size and quality, zenith angle, and atmospheric conditions. So to determine the optimum T for speckle interferometry an estimate of $\gamma(t)$ should be made prior to making the exposures, preferably using a star in the same region of sky as the object to be resolved.

4. FILTER BANDWIDTH

Expressions for the spectral crosscorrelation function $\Gamma(k_1,k_2,x)$ have been derived by a number of authors[22-26]. For typical seeing conditions and a large telescope the phase in the pupil is uniformly distributed in the range $-\pi$ to $+\pi$ and there are a large number of seeing correlation areas in the pupil. Making these assumptions we can write

$$\Gamma(k,r) = \Gamma_o(k)\,\frac{2J_1^2(ak\frac{r}{R})}{(ak\frac{r}{R})^2} \tag{23}$$

where $k = k_2 - k_1$, a is the radius of the circular aperture and R the focal length of the telescope, and $\Gamma_o(k)$ is an even function dependent on the distribution of the phase and amplitude variations across the pupil of the telescope. As $\Gamma(k,r)$ is an even function of k and depends only on r and not on θ, equation (17) can be simplified giving

$$\mathrm{SNR} \propto S_{\Delta k}\Delta k.\int_o^\infty 2\pi r e^{-\frac{2r^2}{\alpha^2}}\,\frac{2}{\Delta k^2}\int_o^{\Delta k}(\Delta k - k)\Gamma(k,r)\,dk\,dr \tag{24}$$

so the functional dependence of SNR on bandwidth is affected by the size of the telescope and the "seeing" as well as $\Gamma_o(k)$. Before considering how

$\Gamma_o(k)$ might be estimated, we consider the dependence of SNR on bandwidth for a given analytical form of $\Gamma_o(k)$

$$\Gamma_o(k) = e^{-\sigma^2 k^2} \tag{25}$$

Plots of SNR against Δk for a = 2 m, $S_{\Delta k}$ = 1, one arc second seeing and various values of σ are shown in figure 2. The values were computed using a standard numerical integration routine. Initially SNR increases, almost linearly with Δk and then tends asymptotically to a constant maximum value. These curves differ from those of figure 1 in that there is no maximum which can be immediately related to an optimum bandwidth. This is true for any realistic form of $\Gamma_o(k)$.

The theory developed so far implies that speckle interferometry might be performed optimally in white light. In fact the use of a very wide bandwidth filter has some disadvantages which arise from certain effects not taken into account by the assumptions and simplifications made in the theory. We now discuss some of these effects.

4.1 Telescope dispersion

Speckle patterns formed in polychromatic light have a radial structure[27], speckles towards the edge of the pattern being elongated in the radial direction. The degree to which this radial structure is present depends on the atmospheric conditions and increases with the bandwidth of the light. The result of this is that the average "size" of the speckles in the pattern is increased with increasing bandwidth and hence the resolution of the technique is reduced.

4.2 Computational power of the system

The computation time per exposure is approximately proportional to the square of the number of photons detected in the exposure[17], so we can write

$$\begin{aligned} T_d &= 0 & T &> cT^2\Delta k^2 \\ T_d &= cT^2\Delta k^2 - T & T &< cT^2\Delta k^2 \end{aligned} \tag{26}$$

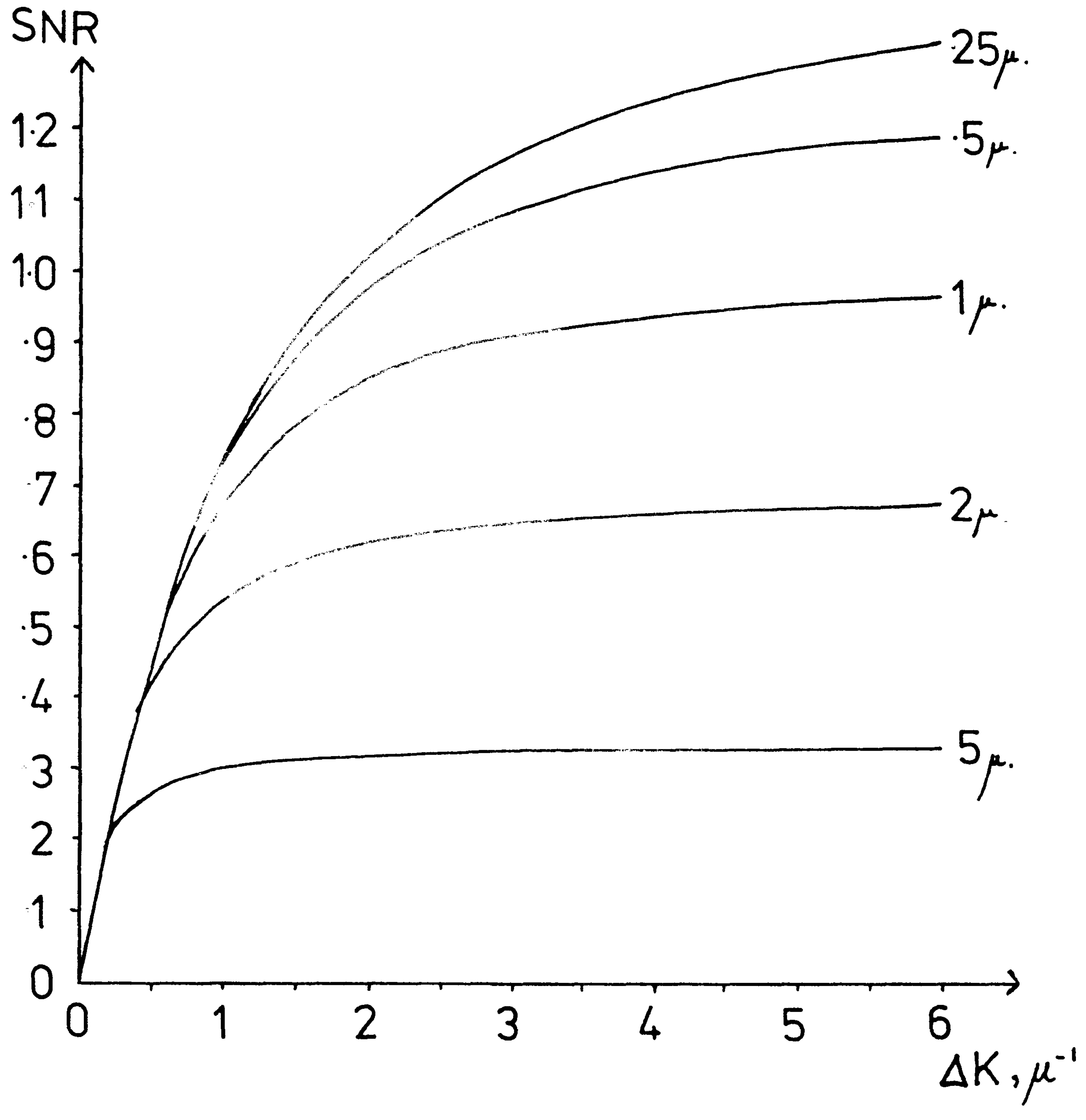

Figure 2 Signal to noise ratio as a function of filter bandwidth, Δk, for various values of σ

where c is a constant which depends on the total intensity in the image and the computational power of the system. The curves in figures 1 and 2 were plotted for $T_d = 0$ for all T and Δk values as would be the case for a very faint object and/or a very powerful computational system. In general we must use equation (26) substituted into (7) when calculating SNR. If the computation time exceeds the exposure time then a pause between exposures becomes necessary with a resultant wastage of information and consequent reduction of signal to noise ratio.

4.3 Binaries with components of different spectral types

The signal to noise ratio (equation (1)) was derived assuming that the intensity in the speckle pattern due to a binary can be written as

$$I_{BINARY}(x) = I(x) + f\, I(x - a) \tag{27}$$

where I(x) is the speckle pattern due to a single star and a is the separation of the binary. This assumption is valid if the binary separation is less than the size of the isoplanatic patch[28,29,30] and if

$$e_A(k)s(k)d(k) \propto e_B(k)s(k)d(k) \tag{28}$$

where e_A and e_B are the spectral emissivities of the two component stars. This latter condition is true for any e_A and e_B when s(k) defines a narrow bandwidth but is not generally true for a wide bandwidth. The difference in form of the two halves of (28) depends on the bandwidth and the difference between the surface temperatures of the two stars. If the two halves of (28) are significantly different in form then the "double speckle" effect will be reduced with a corresponding drop in signal to noise ratio.

4.4 Atmospheric dispersion

For objects not at zenith, atmospheric dispersion causes the speckles in a polychromatic image to be elongated in one direction. However, this is not a serious problem as atmospheric dispersion can be compensated by using

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for example, a pair of prisms[15].

4.5 Discussion

Clearly then the choice of filter bandwidth in speckle interferometry is a complex problem depending not only on the factors affecting Γ_0 and the telescope size and the seeing, but also on the desired resolution, the object and the computational speed of the data reduction system. All these factors should, if possible, be taken into account when deciding on the bandwidth to be used. If the details of the object are not available then a reasonable choice of bandwidth might be that value of Δk at which SNR reaches, say, 80% of its maximum value, provided this does not break the inequality $T > cT^2\Delta k^2$.

5. ESTIMATING THE TEMPORAL AUTOCORRELATION FUNCTION, $\gamma(t)$

In the absence of image movement, or quiver, an estimate of $\gamma(t)$ is given by $\hat{C}_N(t)$. $\hat{C}_N(t)$ is the autocorrelation of the photon counts recorded using the apparatus shown in figure 3.

$$\hat{C}_N(t) = \frac{\frac{1}{N}\sum_{i=1}^{N} n(t_i,\delta t)n(t_i + t,\delta t)}{\{\frac{1}{N}\sum_{i=1}^{N} n(t_i,\delta t)\}^2} - 1 \qquad t \neq 0 \qquad (29)$$

where $n(t,\delta t)$ is the number of photons detected in the short time interval δt at t

Image movement does not affect stellar speckle interferometry as the autocorrelation or the squared modulus of the Fourier transform of an image intensity distribution is independent of its lateral position. Image movement does, however, affect determinations of $\hat{C}_N(t)$. As the instantaneous centre of the stellar image moves relative to the detector aperture (see fig. 3) the mean photon count becomes non-stationary in time and a low frequency component appears in the power spectrum. The form of $\hat{C}_N(t)$ for a typical set of four experiments performed over a twenty minute period using the 91 cm telescope of the Royal Greenwich Observatory are shown in figure 4. Two distinct time scales are apparent; a rapid decorrelation (3 - 4 ms)

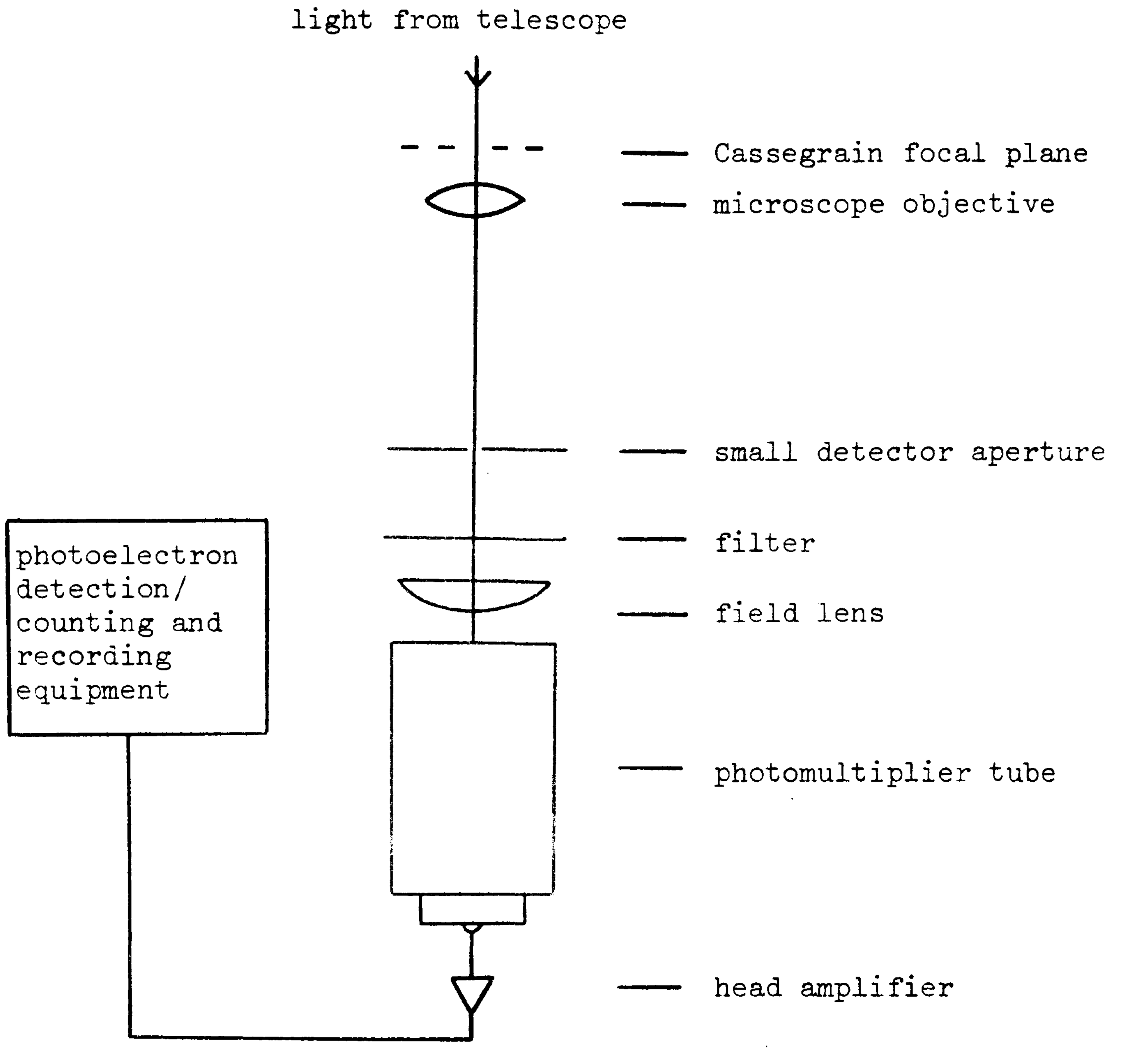

Figure 3 Schematic of apparatus used to record the photon counts which are then digitally autocorrelated to give $\hat{C}_N(t)$

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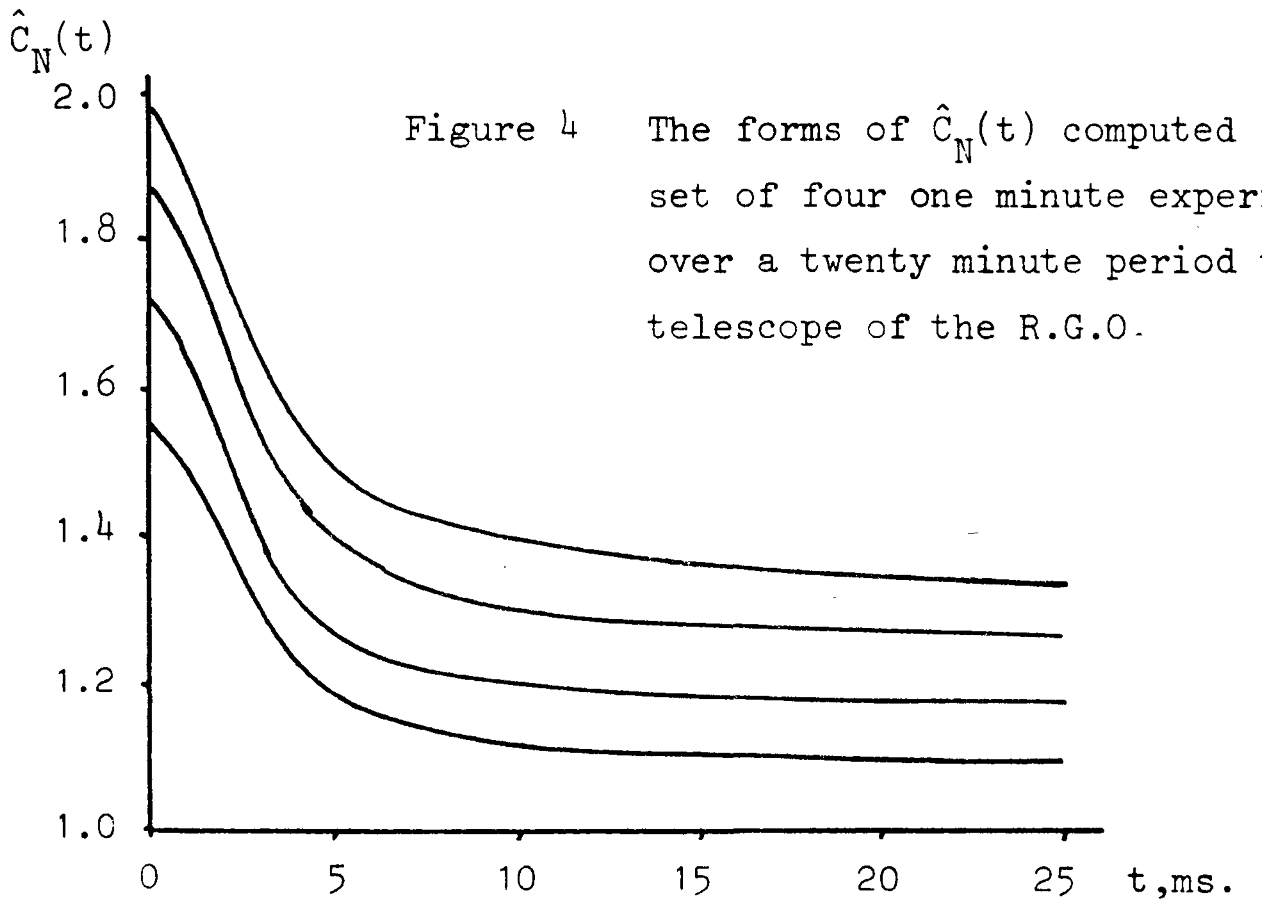

Figure 4 The forms of $\hat{C}_N(t)$ computed from a typical set of four one minute experiments performed over a twenty minute period using the 91 cm telescope of the R.G.O.

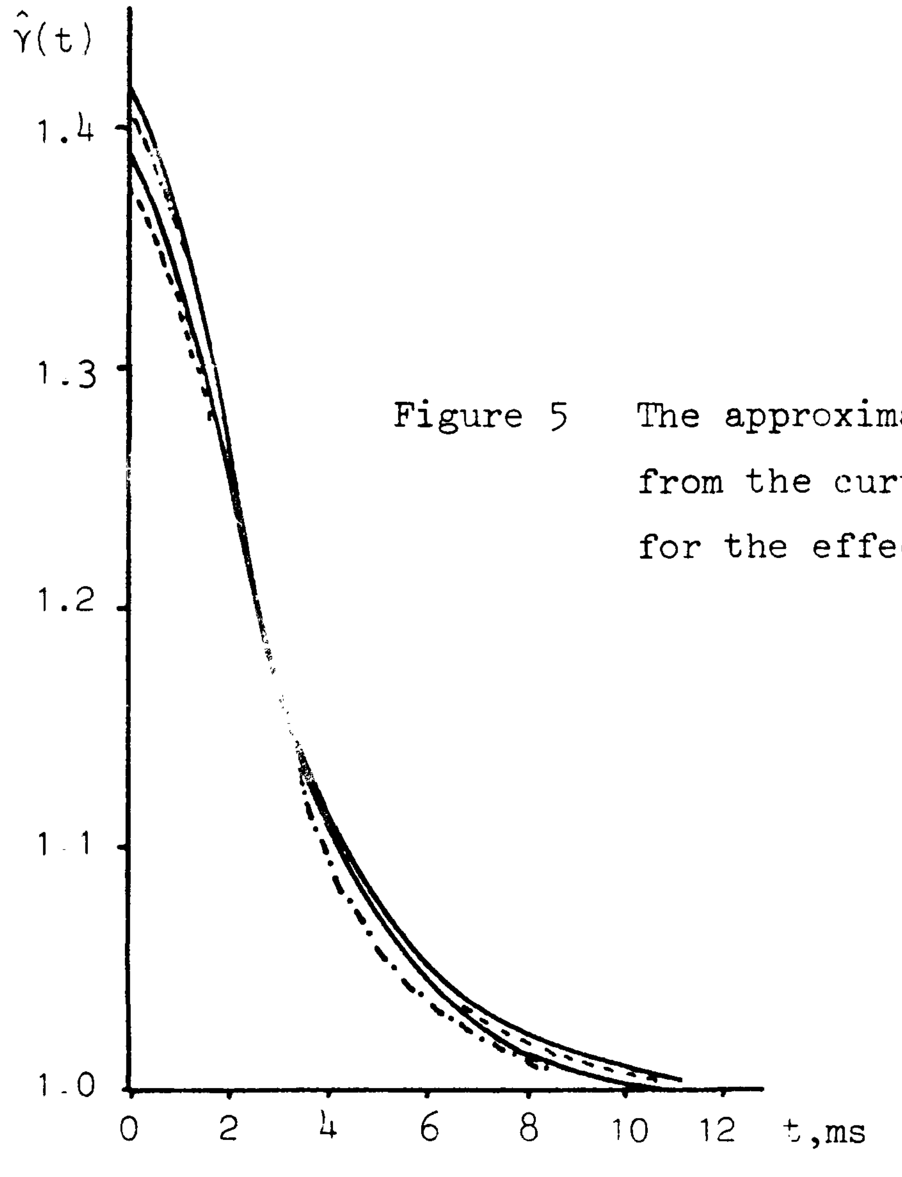

Figure 5 The approximate forms of $\gamma(t)$ obtained from the curves of figure 4 by compensating for the effect of image movement

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associated with the "lifetime" of the speckles in the image and a slow decorrelation ($\sim$1 s) associated with the image movement. The large differences between the heights of the slow decorrelation curves is possibly due to the recentring of the detector aperture within the image between the experiments. The dependence of $\hat{C}_N(t)$ on atmospheric conditions and variables such as defocus, zenith angle, size of detector aperture, etc. is discussed in another paper[21]. For the present purpose only the rapid decorrelation is of interest. Assuming that the "boiling" of the speckles and the image movement are independent, we can write

$$C_N(t) + 1 = (\gamma(t) + 1)(\beta(t) + 1) \tag{30}$$

where $\beta(t)$ is the correlation function associated with the image movement. Assuming $\beta(t)$ to be essentially linear over the range of the rapid decorrelation and by extrapolating the tail of the curve and then dividing this into $\hat{C}_N(t)$ we obtain an approximate form for $\gamma(t)$. The approximate forms of $\gamma(t)$ obtained in this way from the curves of figure 4 are shown in figure 5.

This method has been used to estimate $\gamma(t)$ from measurements of $\hat{C}_N(t)$ made on a number of observing nights using the 91 cm telescope of the Royal Greenwich Observatory, Herstmonceux. Values of $t_{1/e}$, the width of $\gamma(t)$ at its 1/e point were found to be typically in the range 3 to 4 ms. The lowest value of $t_{1/e}$ measured was 2.2 and the highest 8.6 ms. $t_{1/e}$ was found to be fairly stationary over periods of an hour and more. It should be noted that these results apply only to the telescope used and should not be regarded as necessarily typical of other telescopes and sites. The R.G.O. site is only 34 m above sea level and at a latitude where jet stream activity is a common feature.

The method of estimating $\gamma(t)$ outlined above is obviously somewhat time-consuming and the extrapolation of the tail of the curve is a probable source of errors. Ideally image movement would be reduced to negligible proportions, using a fast response time star tracker, while the determinations of $\hat{C}_N(t)$ were made. No such device was available when these observations were made.

6. ESTIMATING THE SPECTRAL CROSSCORRELATION FUNCTION, $\Gamma_o(k)$

In the absence of image movement an estimate of $\Gamma_o(k)$ is given by $\hat{S}_N(k)$ the crosscorrelation of the photon counts recorded using the apparatus shown in figure 6.

$$\hat{S}_N(k) = \frac{\frac{1}{N}\sum_{i=1}^{N} n(k_1,t_i,\delta t)\; n(k_2,t_i,\delta t)}{\frac{1}{N}\sum_{i=1}^{N} n(k_1,t_i,\delta t)\;\frac{1}{N}\sum_{i=1}^{N} n(k_2,t_i,\delta t)} - 1 \qquad (31)$$

where $n(k,t,\delta t)$ is the number of photons detected in the short time interval δt at t in a narrow bandwidth centred on k. Image movement affects $\hat{S}_N(k)$ in two ways. Firstly, the recorded mean photon counts in the two channels become non stationary and a correlation not present in a short exposure is introduced in a similar way to the effect of image movement on measurements of $C_N(t)$. Secondly, as $\Gamma(k,r)$ is non stationary in r if the detector aperture is not held at $r = 0$, the result is an estimate of $\Gamma(k,r)$ averaged over some distributions of r, which depends on the form of the image movement. The first effect can be compensated for by a method similar to that used to estimate $\gamma(t)$ from $\hat{C}_N(t)$. To compensate for the second effect a distribution of r must be assumed taking an estimate of the amount of image movement into account.

The normalised estimates of $\Gamma_o(k)$ at six k values are shown in figure 7 These estimates were obtained from measurements of $\hat{S}_N(k)$ made using the 91 cm telescope of the R.G.O. The measurements are compensated for the non stationary effect but the image movement was estimated to be sufficiently small for the second effect to be negligible. The dotted line in figure 7 is a gaussian function with $\sigma = .425\mu$

Estimates of σ made in a similar way on several nights had values ranging from .28 to .55 μ These values may only be typical of the telescope used; significantly different results might be obtained on another telescope or at another site.

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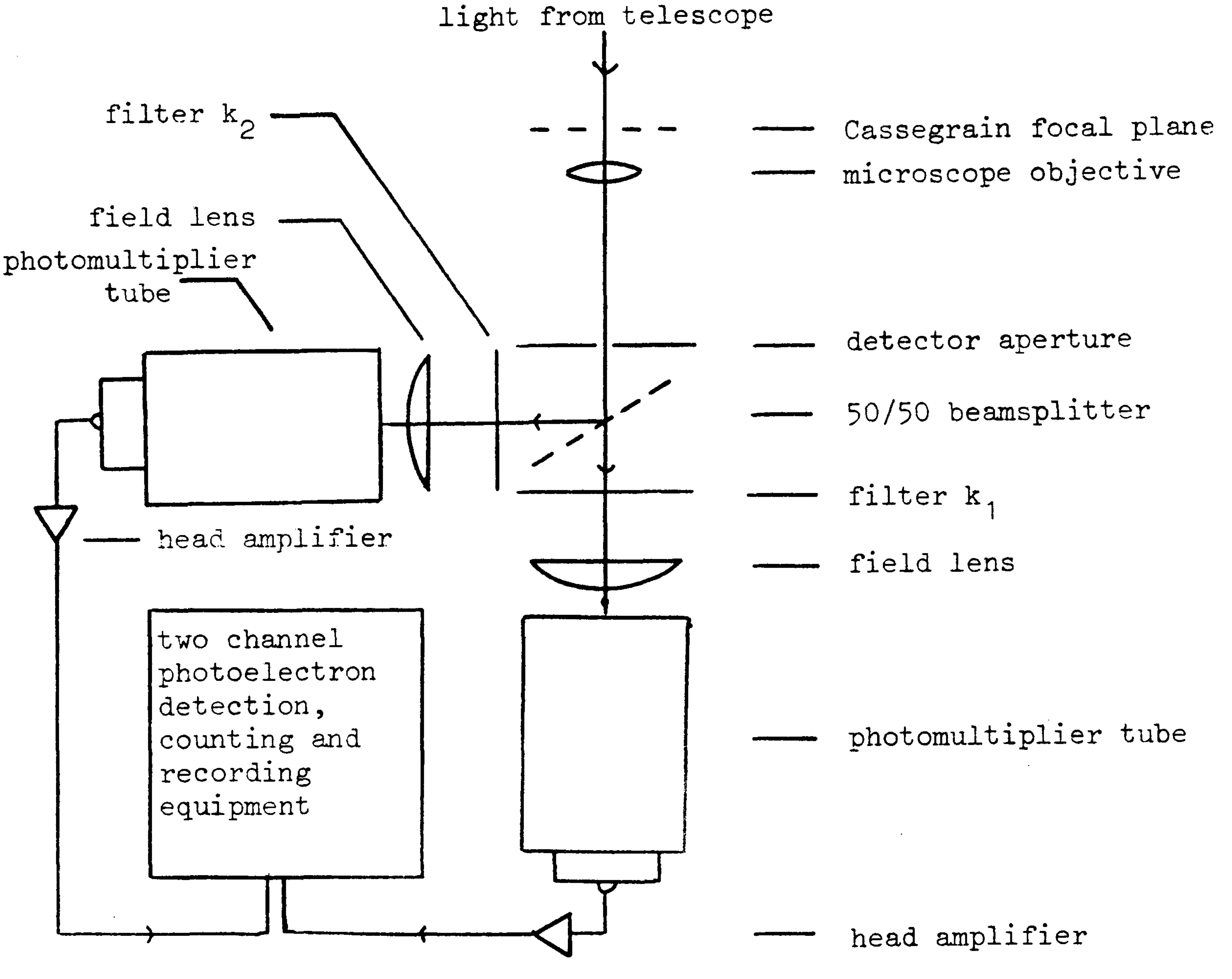

Figure 6 Schematic of apparatus used to record the photon counts which are then digitally crosscorrelated to give $\hat{S}_N(k)$

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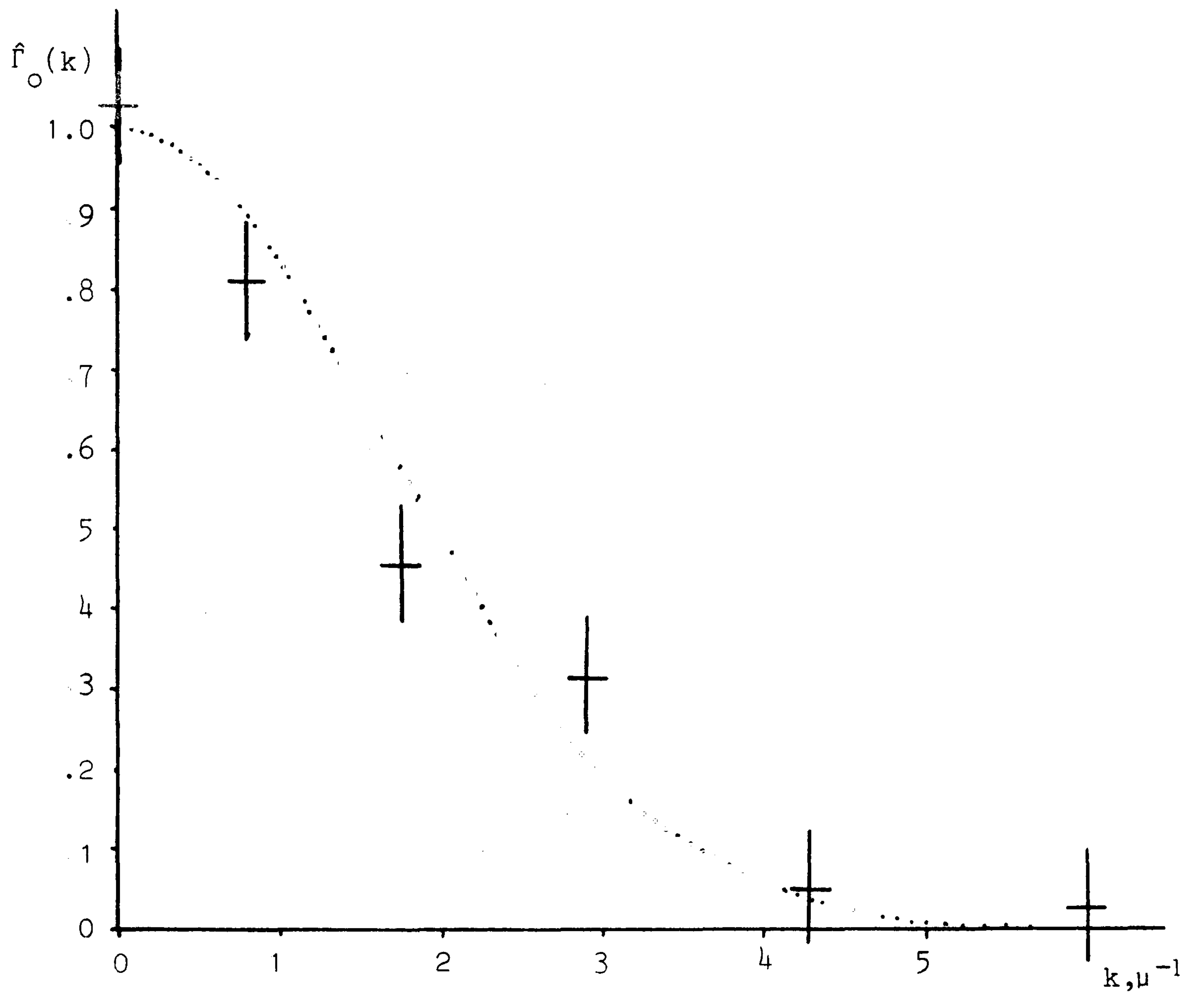

Figure 7 Normalised estimates of $\Gamma_o(k)$ obtained from measurements of $\hat{S}_N(k)$ made using the 91 cm telescope of the R.G.O. The dotted line is a gaussian with $\sigma = .425\ \mu$

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7. CONCLUSIONS AND POSSIBLE FUTURE DEVELOPMENTS

The dependence of the signal to noise ratio at low light levels on the exposure parameters, T and Δk, has been analysed and discussed. Detailed analysis has been restricted to the most relevant case of very faint objects and/or very powerful computational systems for which the dependence of SNR on T and Δk can be treated independently. The form of the dependence of SNR on T and Δk in the more general case has also been given.

The optimum exposure time can be computed from an estimate of the temporal autocorrelation function $\gamma(t)$. The optimum exposure time does not occur when the atmosphere is "frozen" but is better described as occurring when the atmosphere is "partially frozen". This, in part, explains the large difference between the magnitude of the faintest binary so far resolved by the speckle technique and the limiting magnitudes predicted by theories which assumed a "frozen atmosphere".

The dependence of the signal to noise ratio on filter bandwidth has been analysed and found to depend on the spectral crosscorrelation function $\Gamma_o(k)$. A number of factors not treated in the analysis, which must sometimes be considered when choosing the bandwidth to be used in speckle interferometry, have been discussed. The analysis applies strictly only for binary stars, however, the results concerning the exposure time can also be applied to other objects.

Methods of estimating the functions $\gamma(t)$ and $\Gamma_o(k)$ have been outlined and the problems caused by image movement discussed. Preliminary results using one telescope on several nights have been given.

A knowledge of the typical forms of $\gamma(t)$ and $\Gamma_o(k)$ for a particular telescope would be useful in assessing its potential for speckle interferometry. Further experiments, preferably with a system capable of reducing image movement, are needed to establish these results.

If $\gamma(t)$ and/or $\Gamma_o(k)$ were to be estimated prior to performing the interferometry, it would obviously be convenient for the required apparatus to be incorporated into the speckle interferometer. In the case of estimating $\gamma(t)$ this might be achieved, quite simply, by using one of the

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photon detectors in the array in place of the detector aperture, field lens, photomultiplier arrangement shown in figure 3 and by using the on line auto-correlator to calculate the temporal rather than the spatial correlation.

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DISCUSSION

L. Mertz: The photon counters mentioned so far are not pure in the sense that they jumble the order of arrival of photons within each frame. A pure two-dimensional photon counter would permit a posteriori definition and optimization of frame boundary times, e. g. pairing sequential photons or allowing overlapping (running) frames that permit phase tracking for unwrapping the phases of the spatial Fourier components to give images.

J. G. Walker: This is a possibility for future systems. At present all the speckle systems in use and currently being developed use the sampling scheme I have described. The scheme you mention would be very difficult to apply in real time.

C. H. Townes: Can someone inform us what is the faintest magnitude that has so far been successfully studied by speckle interferometry and what is the nature of the present practical limitations?

S. P. Worden: Boksenberg has observed objects as faint as +15 with about 10 photons per frame. These data are now being processed. I have successfully derived diameters of 13th magnitude asteroids from some of his data. The Arizona/A.F.G.L. digital camera has been used to observe 13th magnitude objects with about 40 photons per frame. We have reduced +11.5 magnitude data on Saturn's moons. I will describe these results in my paper.*

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*See page 28-1.