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RECURSIVE PROPERTIES OF ISOMORPHISM TYPES

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A structure on a recursive universe is said to be *recursive* if the satisfaction predicate restricted to quantifier-free formulae is recursive. Different enumerations of the universe of a recursive structure produce structures that, although classically isomorphic, may possess different recursive properties. In this dissertation we consider conditions under which certain recursive properties are common to all recursive structures in a particular classical isomorphism type. The dissertation is in two parts. In the first part we present a general framework and obtain some general results. In the second part we use these general results to study the situation in the particular case of linear orders.

Part 1

For Γ a recursively-enumerable set of formulae, a structure $\underline{\underline{A}}$ on a recursive universe is defined to be Γ-*recursively enumerable* if the satisfaction predicate restricted to Γ is recursively enumerable. For recursively enumerable sets $\Gamma_1 \subseteq \Gamma_2$ of formulae we shall, under certain conditions, characterize structures $\underline{\underline{A}}$ with the following properties.

(1) Every isomorphism from $\underline{\underline{A}}$ to a Γ_1-recursively enumerable structure is a recursive isomorphism.

(2) Every Γ_1-recursively enumerable structure isomorphic to $\underline{\underline{A}}$ is

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recursively isomorphic to $\underline{\underline{A}}$.

(3) Every Γ_1-recursively enumerable structure isomorphic to $\underline{\underline{A}}$ is Γ_2-recursively enumerable.

These results have as corollaries some well-known results of Ash and Nerode [1], Goncharov [2] and Nurtazin [3]; as well as some results new to the literature.

The above-mentioned results all assume a certain 'extra decidability' of the structure $\underline{\underline{A}}$. In the final section of part one we present a result not requiring this assumption. The result provides a condition sufficient for a structure not to satisfy property (3). This condition enables us to characterize 'relatively recursive' structures satisfying property (3) in the case when Γ_1 is finite.

Part 2

The *successivity* relation $S(x, y)$ and the *block* relation $B(x, y)$ on linear orders are defined by the following formulae:

$$S(x, y) \equiv x < y \wedge \forall z \neg(x < z < y) ;$$

$$B(x, y) \equiv \bigvee_{n=2,3,\ldots} \exists z_1, \ldots, z_n \left(x = z_1 \wedge y = z_n \wedge \bigwedge_{i=1}^{n-1} S(z_i, z_{i+1}) \right) .$$

In this part we consider conditions under which linear orders satisfy properties (1), (2) and (3) for three choices of Γ_1 :

(i) the set of quantifier-free formulae;

(ii) the set of universal formulae;

(iii) the set of quantifier-free formulae together with the formulae $B(x, y)$ and $\neg B(x, y)$.

A recursive linear order is said to be 1-*recursive* if the satisfaction predicate restricted to universal formulae is recursive. A recursive linear order is 1-recursive if and only if it has the successivity relation $S(x, y)$ recursive. The following are examples of the type of results obtained.

A relation R is said to be *intrinsically recursive* on a recursive linear order $\underline{\underline{A}}$ if every isomorphism from $\underline{\underline{A}}$ to a recursive linear order carries R to a recursive relation. We show that a relation R is intrinsically recursive on a recursive linear order $\underline{\underline{A}}$ if and only if R is equivalent (in $\underline{\underline{A}}$) to a quantifier-free formula with a finite number of parameters.

A 1-recursive linear order $\underline{\underline{A}}$ is said to be *recursively categorical* if every 1-recursive linear order isomorphic to $\underline{\underline{A}}$ is recursively isomorphic to $\underline{\underline{A}}$. We characterize such 1-recursive linear orders as precisely those that can be partitioned, by a finite number of points, into intervals, each of which has order type in $\{\omega, \omega^*, \omega+\omega^*\} \cup \{k\cdot\eta : k < \omega\}$.

References

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