Grünbaum's book, but in Lecture 6 they and their variants, generalizations and applications receive a full treatment. One interesting topic here concerns whether the shape of a given face of a polytope can be specified (in some isomorphic polytope); examples are given to show that this is generally not possible. (This is in contrast to the case of dimension 3.) Diagrams also occur in Lecture 7, which covers *zonotopes*, or vector sums of line segments. These have close connexions with arrangements of hyperplanes and oriented matroids, a subject of independent interest.

If one cannot characterize face-lattices of polytopes, one might at least ask less detailed questions such as which sequences $(f_0, \ldots, f_{d-1})$ of numbers are *f-vectors*, with $f_j = f_j(P)$ the number of j-faces of some d-polytope P. Even this question cannot yet be answered if $d \geqq 4$, although it can when P is simple. This problem is only mentioned here, but the upper bound theorem (giving the maximum of each f_j when f_0 is fixed) is proved using the technique of shelling (established for polytopes after 1967), a favourable ordering of the facets of a polytope.

The final Lecture 9 deals with fibre polytopes, another recent development. These are concerned with subdivisions of one polytope arising from it as a projection of another. The lecture ends with a look towards the future.

In addition, each of the lectures concludes with (historical) notes and with problems and exercises; some of the last are unsolved. This is in keeping with the philosophy of the book as an introductory text which can be used by students to learn the subject by themselves. Finally, there is an extensive bibliography, enlivened by the provision of the authors' full names wherever known. My judgment is that the aim of the book, to give just such an introduction, is most successfully accomplished. But students would not be the only ones who could benefit from so sympathetic a treatment and I firmly recommend the volume to anyone who might wish to find out about a subject which is intrinsically fascinating and, in spite of having about the oldest pedigree in mathematics, is yet a lively area of reasearch and has increasingly many connexions with other branches.

P. McMULLEN

GLENDINNING, P. *Stability, instability and chaos* (Cambridge University Press, Cambridge 1994), xiii + 388 pp., hardcover: 0 521 41553 5, £45, paperback: 0 521 42566 2, £17.95.

This book adds to a fast growing library of undergraduate introductions to dynamical systems theory (where it joins titles by Perko, Drazin, Verhulst, to name only a few). It is clearly a reincarnation of a course lecture notes. These origins are reflected both in its (very many) strong points and in (the non-negligible number of) its shortcomings.

First, the good news. I enjoyed the light conversational style of this book (see Fig. 1.1 and the discussion of the Glendinning integral). I hope in this respect Dr. Glendinning's book sets a precedent. It is also very nicely typeset. I was also impressed with some of the material covered, in particular, the very coherent and readable discussion of resonances, a very modern exposition of invariant manifold theory, as well as computation of direction of Hopf bifurcation, discussions of Arnol'd tongues and of global bifurcation phenomena (chapter 12, based on the work by the author). Much of this material will find its way into the honours course on nonlinear ODEs which I am teaching.

I would say, however, that there is scope for improvement. There is no need for French spelling of Lyapunov's name; omission of Grobman from the theorem that usually bears his name as well requires an explanation. The style sometimes borders on the inadmissibly loose: for example, 'the solution can go all over the place before tending to the point' (p. 28) can easily be improved, as can the caption to Fig. 1.4. Fig. 5.8 is incomprehensible altogether. The motivation for defining stability for an arbitrary point in phase space (and not, say, for an invariant set) is not clear. The ordering of the material is also suspect. Centre manifolds belong by right in a discussion of invariant manifolds. Discrete dynamical systems are an interesting object on their own account and should not be hidden in a chapter on periodic orbits (Chapter 6). The section on canards (section 8.10) is too brief to say anything of use; furthermore, there are no references either to the work of nonstandard analysts on this subject [**1**], or to Eckhaus' famous rejoinder [**2**]. By the way, a much simpler example of canards is the modest first order non-autonomous equation

$$x' = x - x^3 + \lambda \cos(\varepsilon t),$$

with ε small and λ as a bifurcation parameter (this equation occurs in the theory of optical bistability) [**3**].

In this edition of the book there is much to praise; I am sure the next edition will merit nothing but praise.

M. GRINFELD

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2. W. Eckhaus, Relaxation oscillations including a standard chase on French ducks, in *Asymptotic Analysis II* (F. Verhulst ed., Lecture Notes in Mathematics **985**, Springer-Verlag 1983), 449–494.

3. P. Jung, G. Gray, R. Roy and P. Mandel, Scaling law for dynamical hysteresis, *Phys. Rev. Lett.* **65** (1991), 1873–1876.

Lunardi, A. *Analytic semigroups and optimal regularity in parabolic problems* (Progress in Nonlinear Differential Equations and their Applications Vol. 16, Birkhäuser, Basel, Boston, Berlin 1995) xvii + 424pp., 3 7643 5172 1, about £100.

One approach towards a general theory of (nonlinear, inhomogeneous, nonautonomous) parabolic differential equations is to start with the most special (linear, homogeneous, autonomous) case and then to proceed in several stages towards greater generality by using various approximation methods. The theory of one-parameter semigroups of operators arose as an abstract approach to (linear, homogeneous, autonomous) differential equations. Indeed, such semigroups correspond to solutions of the Cauchy problem

$$u'(t) = Au(t), \quad u(0) = u_0,$$

where A is an unbounded linear operator on a Banach space X. The solution is given by $u(t) = T(t)(u_0)$, where $\{T(t): t \geqq 0\}$ is the semigroup generated by A. Typical examples arise when A is an elliptic operator on an open subset Ω of $\mathbb{R}^n$ with suitable boundary conditions and X is a space of functions on Ω. In these examples the semigroup is analytic in the sense that it can be extended to a holomorphic semigroup of operators $T(z)$ defined in a sector given by $|\arg z| < \theta$. This allows one to exploit special properties of analytic semigroups such as the Spectral Mapping Theorem and estimates on $\|AT(t)\|$.

Having established that elliptic operators generate analytic semigroups, one can move to inhomogeneous equations by using the variation-of-constants formula and then to nonautonomous equations by perturbation techniques. These methods can then be extended to the simplest class of nonlinear cases, the semilinear equations of the form

$$u'(t) = Au(t) + f(t, u), \quad u(0) = u_0,$$

where typically A is a second-order elliptic operator and $f(t, u)$ may depend on the first-order derivatives of u. The fully nonlinear case can be put in the same form but now the nonlinear term $f(t, u)$ is of the same order as Au. This makes the analysis substantially more delicate and supplementary conditions have to be imposed even to ensure the existence of solutions.

The aim of this book is to follow this approach towards nonlinear parabolic equations. Where