SESSION III: THEORY

QUASI-STATIC EVOLUTION OF A FORCE-FREE MAGNETIC FIELD AND CONDITIONS FOR THE ONSET OF A STELLAR FLARE

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1 - INTRODUCTION

Magnetic fields in the solar corona are braught into an endless evolution by the never-ceasing motions of the subphotospheric plasma in which the feet of their lines are anchored. It is generally thought that this evolution is essentially quasi-static, the field passing through a sequence of force-free equilibrium states. Sporadically, however, the equilibrium is broken in a region of limited extent, and during a relatively short interval of time a catastrophic highly dynamic evolution takes place, giving rise to such well-known phenomena as flares or coronal transients. Understanding the factors which determine if a magnetohydrostatic coronal equilibrium is maintained or, on the contrary, destroyed, when boundary conditions change at the photospheric level, then appears as a central theoretical problem of solar physics. In this Communication, we report some recent results which shed some new light onto this old problem.

2 - EVOLUTION OF AN ARCADE FORCE-FREE FIELD EMBEDDED IN A CONDUCTING PLASMA

Let us consider in the half-space $\{z > 0\}$, assumed to contain a perfectly conducting plasma, a continuous time-sequence of x-invariant force-free fields $\underset{\sim}{B}_t(y,z) = \nabla A_t(y,z) \times \hat{x} + B_{tx}[A_t(y,z)]\hat{x}$ whose field lines have an arcade topology. This sequence describes a quasi-static evolution which is driven by a stationnary velocity field $\underset{\sim}{v}(y) = v(y)\hat{x}$ which is imposed on the boundary $\{z = 0\}$, and then the potential $A_t(y,z)$ is a solution of the initial-boundary value problem (see Aly, 1987)

$$- \Delta A_t = d\left(B_{tx}^2/2\right) / dA \tag{1}$$

$$X_t(A) = B_{tx}(A) \int_{C_{pt}(A)} \frac{ds_p}{|\nabla A_t|} = - B_{tx}(A) \frac{d\Sigma_t(A)}{dA} = t\zeta(A) \tag{2}$$

$$A_t(y,o) = g(y) \tag{3}$$

$$C_t[A_t] = \int_{\{z>0\}} |\nabla A_t|^2 \, dydz + t^2 \int_o^{\infty} \zeta^2 \left|\frac{dA_t}{d\Sigma}\right|^2 d\Sigma < \infty \tag{4}$$

$$\text{topology } \{C_{pt}\} \equiv \text{topology } \{C_{po}\} \text{ (arcade)} \tag{5}$$

Equation (1) expresses the force-free character of the field; (2) relates the shear $X_t(A)$ of a field line $C_t(A)$ labelled by a value A of the potential A_t - $X_t(A)$ is the difference between the x-positions of its left and right feet, respectively - to the velocity field on $\{z = 0\}$, which determines the function $\zeta(A)$, and to the time t; in this equation, $C_{pt}(A)$ represents the projection of $C_t(A)$ onto $\{x = 0\}$, while $\Sigma_t(A)$ stands for the area between $C_{pt}(A)$ and the y-axis; (3) is a boundary condition expressing that $A_t(y,o)$ is kept unchanged by the x-motions (g is a given function satisfying:

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$g(\pm\infty) = 0 \leqslant g(y) \leqslant A^m = g(o)$; $yg'(y) < 0$ for $y \neq 0$; and $y^2 g(y) = O(1)$); relation (4), which constraints the magnetic energy per unit of x-length to be finite, plays the role of an asymptotic condition for A_t; and (5) is a condition which expresses the frozen-in law. Clearly, at the initial time $t = 0$, the field coincides with the finite energy potential field A_0 associated with g.

We have yet been able to reach the following conclusions :

i) consider the associated variational problem, which consists to look at each time t for a function A_t^- which makes the energy $C_t[A]$, as defined by (4), an absolute minimum over the set of functions $\mathcal{H}$ belonging to an appropriate functional space and satisfying (3)-(5) in some sense; then this problem has always a solution, i.e.: $\forall t$, $\exists A_t^- \in \mathcal{H}$ such that $C_t[A_t^-] = \inf_{A \in \mathcal{H}} C_t[A]$. We shall assume here that A_t^- is sufficiently regular to be also a solution of the original problem. Of course, the field A_t^- is, by construction, absolutely non-linearly stable with respect to all 2D ideal perturbations;

ii) the energy $C_t^- = C_t[A_t^-]$ increases steadily from $C_0[A_0] = C_0^-$ up to infinity; for small (resp. large) values of t, $(C_t^- - C_0^-) \propto t^2$ (resp. $\propto \log t$) (Figure 1);

iii) when $t \to \infty$, A_t^- converges asymptotically towards a singular quasi-potential field A_∞^- which is completely (resp. partially) open if $A^1 = A^m$ (resp. $A^1 < A^m$), where A^1 is the smallest number such that $\zeta(A) = 0$ for $A^1 \leqslant A \leqslant A^m$ (see Figure 2a (resp. 2b)).

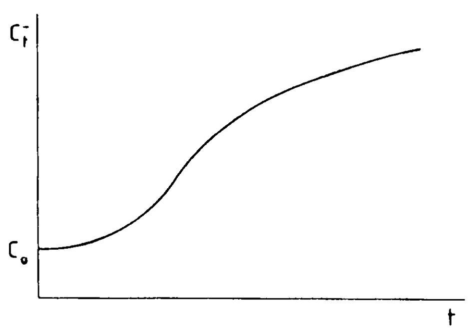

Figure 1 (see text)

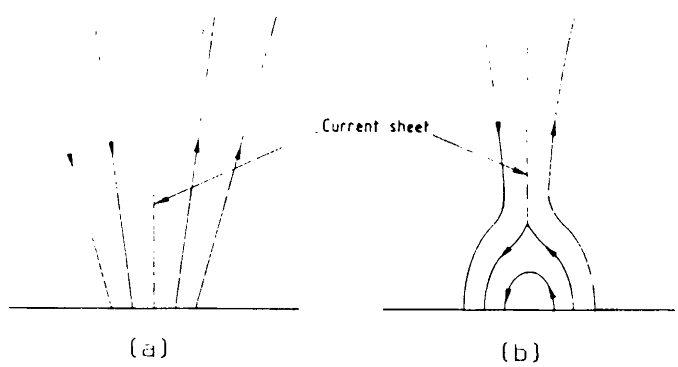

Figure 2 (see text)

3 - STABILITY OF THE ARCADE CONFIGURATIONS WITH RESPECT TO RECONNECTION

Let us now relax the assumption of perfect conductivity of the plasma and look for the possibility of new effects happening in a time scale much shorter than the irrelevant resistive time scale τ_r.

Shearing of the field creates some amount of "toroidal" magnetic flux (flux in the x-direction) which thus cannot be destroyed on a time-scale

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$\ll \tau_r$. However, it may be possible that a fast reconnection process acting in an arcade configuration rearranges this flux in a different way by cutting some of the lines into several pieces, as shown on Figure 3.

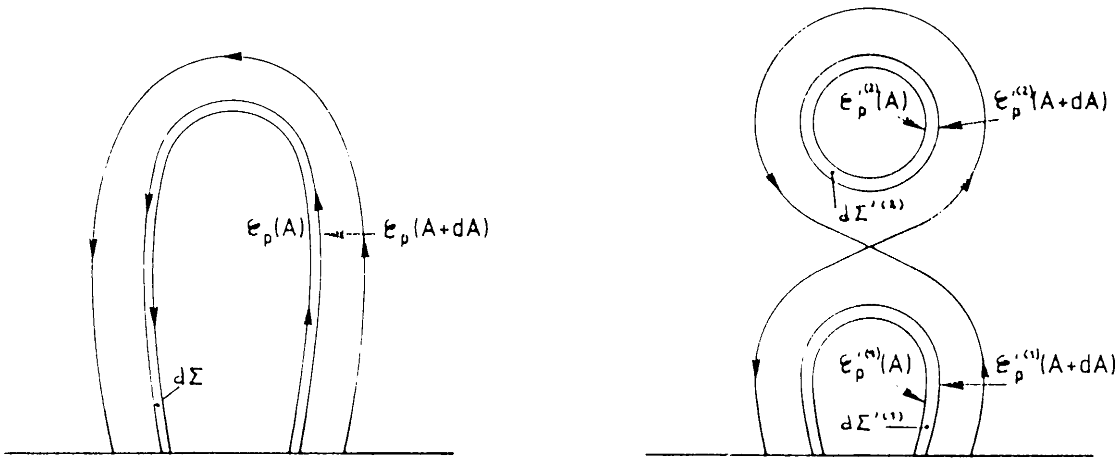

Figure 3: Transition by reconnection from an arcade to a more complex configuration.

In such a process, the topology of the lines changes, but the distribution of the magnetic fluxes are (quasi-)conserved. This means that if a new equilibrium field (A'_t, B'_{tx}) is obtained by reconnecting the arcade (A^-_t, B^-_{tx}), then: i) conservation of the poloidal fluxes: $A'_t(y,o) = g(y)$ and $0 \leqslant A'_t(y,z) \leqslant A^m$; ii) conservation of the toroidal fluxes:

$$X_t(A) = \left(-B^-_{tx}\frac{d\Sigma_t}{dA}\right)(A) = \sum_{i=1}^{p(A)}\left(-B'^{(i)}_{tx}\frac{d\Sigma'^{(i)}_t}{dA}\right)(A) = \sum_{i=1}^{p(A)} B'^{(i)}_{tx}(A)\int_{C'^{(i)}_{pt}(A)} ds'_p/|\nabla A'_t| \tag{6}$$

(the line $C_{pt}(A)$ being broken into $p(A)$ pieces).

Of course, reconnection may occur spontaneously at t only if there does exist among the configurations (A'_t, B'_{tx}) satisfying the requirements just stated above, one which has an energy smaller than C^-_t. Thus we are led to consider the following new minimization problem at each time t: "Minimize $C_t[A]$ over the set $\mathcal{H}'$ defined as $\mathcal{H}$, but without the topological constraint (5)" (note that we have taken here into account the fact that having $B'^{(i)}_{tx} \neq B'^{(j)}_{tx}$ for some $i \neq j$, increases the energy, and then taken $B'^{(i)}_{tx}(A) = B'_{tx}(A) = [-X_t(d\Sigma'_t/dA)^{-1}](A)$ for all i, $1 \leqslant i \leqslant p(A)$). This minimization problem has always a solution $A^{-\prime}_t$ ($C_t[A^{-\prime}_t] = C^{-\prime}_t = \inf_{\mathcal{H}'} C_t[A]$). Then we may have to face two possible situations: i) either $A^{-\prime}_t = A^-_t$ and $C^{-\prime}_t = C^-_t$: reconnection is not energetically favourable; ii) or $A^{-\prime}_t \neq A^-_t$ and $C^{-\prime}_t < C^-_t$: reconnection is energetically favourable.

Actually, one can show that there is a critical time $t_c[g,\zeta]$ such that the first (resp. the second) possibility arises if $0 \leqslant t < t_c$ (resp.

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$t_c < t < \infty$). The reason for this result may be easily understood. Indeed, for $t = 0$, $A_0 = A_0' = A_0$ by a well known property of potential fields and the minimizer of C_0 over $\mathcal{H}'$ then has an arcade topology; this property naturally also holds for small values of t. On the contrary, for large enough values of t, it is easy to see by using the asymptotic result of § 2 that the poloidal energy of A_t (Figure 4a) is decreased if we reconnect that field by making it potential in a small rectangle as shown on Figure 4b, while the toroidal energy may be made to change as little as we want (as $\lim B_{t} = 0$ for $t \to \infty$).

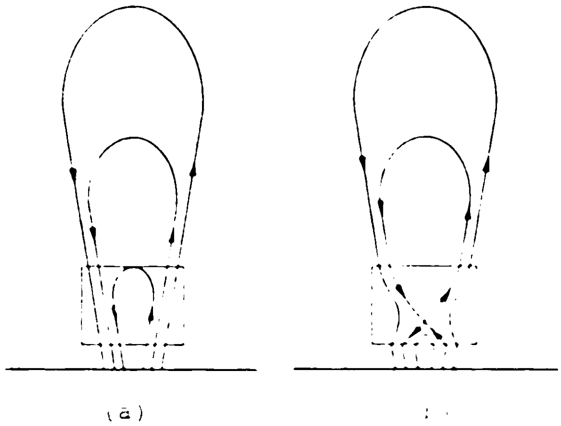

Figure 4

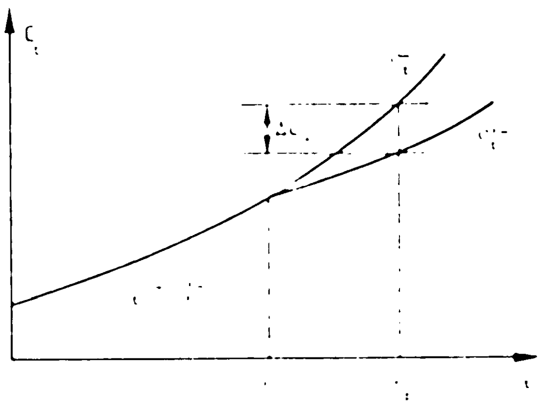

Figure 5 (see text)

Then, at $t > t_c$, there is an amount of energy ΔE which may be released by reconnection (Figure 5). However, it must be noted that there is no field with a non-arcade topology in a too small neighbourhood of A_t, which then appears to be stable against small enough amplitude perturbations in $\mathcal{H}'$. Therefore, the result above has to be interpreted as meaning that A_t becomes metastable with respect to reconnection in $\mathcal{H}'$ at t_c. There is an energetic barrier which has to be overpassed for an evolution of A_t towards A_t' to occur (see the symbolic Figure 6), and this necessitates the action of a finite perturbation. But metastability is just what is required to have a system evolving explosively and then to get a flare-like process (Sturrock, 1967). It is worth noticing, however, that the height of the barrier decreases with time, and then the transition becomes more and more easy.

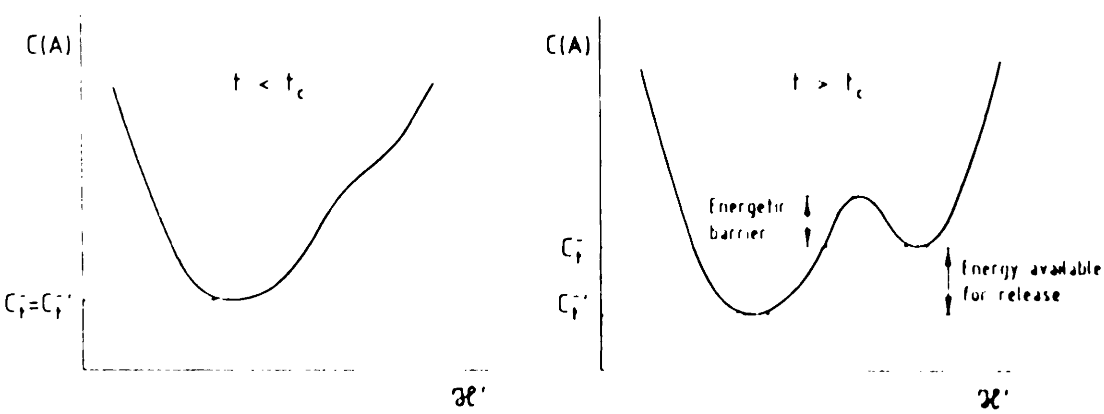

Figure 6: Energy of the various functions A in $\mathcal{H}'$ at a fixed time t.

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4 - APPLICATION TO TWO-RIBBONS FLARES

The theoretical analysis reported above suggests the following [illegible] rio for a two-ribbons flare: i) Phase 1: the sheared field evolves in a [illegible] way from some relaxed state, just expanding outwards; free energy gets [illegible], at time t_c, the field becomes metastable with respect to recon[illegible]; ii) Phase 2: one may speculate that at some $t_r > t_c$, a large amplitude perturbation creates a neutral point - and then a magnetic island - near the bott[illegible] of the stretched configuration; this island cannot be in equilibrium and erupts outwards, maybe creating a current sheet through which reconnect[illegible] proceeds further, releasing a part of the free energy stored in the [illegible]. At the end, one is left with an arcade wi[illegible] twisted tube standing above [illegible] the field is not potential [illegible] the fl[illegible] Remark: the final state does [illegible] need to coincide with the field (A_c, [illegible]) considered in § 3 to establish a necessary condition for reconnection to hold; indeed, the repartition of the toroidal fluxes between the islands according to (6) is determined by the non-equilibrium reconnection process itself; the subsequent evolution of the [illegible] is "adiabatic" and then one gets generally a final state in which [illegible] for [illegible]; this implies that the energy released at t_r is smaller than [illegible] shown in Figure 5 - but not very much, if t_r is large enough. There is also the possibility, still to be investigated, that the energy [illegible] the flare is not evacuated fast enough, what could lead to a final state quite different from that one considered here). It is worth noticing that the scenario proposed here allows to account for the most important observational features of a two-ribbons flare, as recently summarized by [illegible] and [illegible] ([illegible] also [illegible], 1983).

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