4. Interpretation of Type II Bursts
The theory illustrated in Sections 2 and 3 (and Cairns (1986a, b)) shows that a cutoff distribution arises naturally upstream of a Type II shock, irrespective of the particle distribution injected at the shock. Cutoff distributions have a positive slope \( \frac{dI}{dV} \) just above the cutoffs, and are therefore a source of free energy for generating Langmuir waves (Filbert and Kellogg 1979; Cairns 1986a, b), particularly when the positive slope occurs at much larger velocities than the background thermal velocity \( (V_T = 5 \times 10^7 \text{ m s}^{-1}) \) in the corona. However, cutoff distributions provide significant free energy for Langmuir wave growth only in a small fraction of the upstream region for three reasons. Firstly, the region with cutoff velocities significantly larger than the thermal velocity, i.e. \( V_c \geq 10^7 \text{ m s}^{-1} \) is small compared with the total volume of the foreshock (Figure 4). Secondly, the amount of free energy available for wave growth varies approximately as \( V_c^2 \), biasing the source region of the radiation to regions with \( V_c \) in excess of \( 10^7 \text{ m s}^{-1} \). Thirdly, the cutoffs do not reform when wave-particle scattering degrades them so that the level of Langmuir waves and resulting radiation should decrease with increasing distance from the shock. Thus, these cutoff distributions provide a naturally arising, spatially limited, and either continuous or transient source (for electrons produced continuously or impulsively at the shock, respectively) of free energy for Langmuir wave growth upstream of the Type II shock.

The continuous cutoff distribution produced by the shock continuously accelerating electrons into the foreshock provides a natural explanation for the Langmuir waves producing the Type II radiation. In particular, this model explains the long durations (typically 150 s) and narrow bandwidths of the Type II radiation. The Type II radiation is produced in both wings of the foreshock (ahead of the shock). Split-band Type II bursts are then interpreted in terms of shocks with more than one foreshock. Herringbone fine structures are interpreted in terms of plasma radiation produced by Langmuir waves generated by transient cutoff distributions.

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Asymmetric X-ray Pulsar Beam Emission

Whayne E. P. Padden and Michelle C. Storey,
Department of Theoretical Physics, University of Sydney

Abstract: Many of the X-ray pulsars for which X-ray light-curves have been presented in the literature exhibit asymmetric emission beams. We postulate the existence of an off-centre magnetic dipole field embedded in the rotating neutron star and show that such a field leads to varying rates of matter transfer between an accretion disc and the neutron star surface, over a rotation period. Assuming that the accretion rate onto the surface is simply related to the luminosity, we show that most of the observed asymmetries can be accounted for by this mechanism.

1. Introduction
Over 25 X-ray pulsars have so far been identified. White et al. (1983, hereafter W83) presented a description of 14 X-ray pulsars for which simultaneous multi-frequency observations are available and their paper represents a comprehensive collection of the available data. Many of the pulsars for which light curves are presented in W83 (see their Figure 1) have complicated pulse structures and several have asymmetric beam shapes, where the leading and trailing halves of the beam have different intensity.
In this paper we postulate a possible cause for such asymmetries in X-ray pulsar beam emission. We assume that a thin accretion disc forms around the neutron star and show that an off-centre dipolar magnetic field interacting with the accretion disc leads to time varying accretion onto the neutron star and subsequent asymmetries in the X-ray emission. Typical X-ray pulsar geometries are discussed below. In section 2 we discuss the effect that the neutron star magnetic field has on the accretion flow from the disc to the star. In section 3 we discuss the observational consequences of the proposed asymmetry in the magnetic field and in section 4 we compare our theoretical predictions with observations. Our conclusions are presented in section 5.

A number of observations have indicated that X-ray pulsars are highly magnetized neutron stars which are accreting material from a companion star. The mass of the compact star has been measured for six systems (Rappaport and Joss 1982), and the results are consistent with current theoretical estimates for a neutron star. Also the regularity of the pulsations (which range from 0.069s to 894s) indicates the clock mechanism is rotation of a neutron star. The existence of cyclotron features with energies \( \gtrsim 10 \text{ keV} \) in the spectra of several X-ray pulsars (Trümper et al. 1978, Voges et al. 1982), as well as measured period changes (Ghosh and Lamb 1979), provide strong evidence that they possess a magnetic field \( \gtrsim 10^{8} \text{T} \). Such a value can also be predicted on the basis of magnetic flux conservation at the time of the neutron star’s formation. Evidence that the X-ray pulsars are part of a binary system comes from (a) regular Doppler shifting of the pulse period and (b) in some cases occultation behind the companion star.

X-ray pulsars are powered by the accretion of matter from a companion star (e.g. Borsen 1980). Whether or not the accreting matter has sufficient angular momentum with respect to the neutron star to form an accretion disc, at some distance from the neutron star the radial inward diffusion of the matter is halted by the pressure of the magnetic field. Eventually, via poorly understood processes, the plasma enters the magnetosphere and becomes trapped by the magnetic field. Assuming that the field near the stellar surface is dipole-like the accreting plasma is then funnelled to the stellar surface near the magnetic poles.

The problem of how plasma is stopped at the stellar surface is also unresolved. For high luminosity X-ray pulsars i.e. \( L_x > 10^{30} \text{W} \), it appears likely that a radiative shock decelerates the flow, whereas for lower luminosity X-ray pulsars the deceleration may not be dominated by radiation pressure and the plasma flow may be decelerated by either a collisionless or collisional shock, or by nuclear collisions deep in the dense atmosphere formed above the surface. In any case, observed luminosities imply the kinetic energy of the infalling matter is efficiently converted to X-ray radiation, which is observed as a pulse of radiation once per period as the neutron star rotation brings the polar region into view.

In the case of high luminosity X-ray pulsars a column or cylindrical geometry for the dense atmosphere of accreted material is considered to be appropriate (Basko and Sunyaev 1976, Langer and Rappaport 1982, Wang and Frank 1981). Radiation is emitted predominantly from the sides of the column, probably producing a wide fan beam pattern. For lower luminosity X-ray pulsars a plane parallel or slab geometry is likely to be a better approximation to the shape of the atmosphere (Basko and Sunyaev 1975). In this geometry radiation is emitted primarily perpendicular to the top face of the slab. In this paper we restrict our attention to X-ray pulsars for which a slab geometry is appropriate.

Recently (Kirk et al. 1984, Kirk 1986) it has been shown that two-photon cyclotron emission dominates bremsstrahlung (free-free emission) as a source of soft photons from X-ray pulsar atmospheres. In this process an electron makes a transition from its first excited state to the ground state by emitting two photons. Kirk et al. (1984) showed the emission rate was greatest when one photon has frequency just below the cyclotron frequency \( \omega < \Omega \) and the other is a low frequency photon \( \omega < \Omega \) such that \( \omega + \omega' = 2 \Omega \). The theory for two-photon emission in X-ray pulsar environments has been outlined in Melrose and Kirk (1986) and Kirk and Melrose (1986).

The angular beam pattern resulting from two-photon cyclotron radiation and the associated Compton scattering process has been calculated by Kirk, Nagel and Storey (1986). Their calculations indicate the beam from the polar cap has a characteristic double-humped structure and they show that such a structure is observed in many X-ray pulsars. However, the asymmetric beam shapes observed in many X-ray pulsars could be obscuring such double humped structures.

2. Disc Accretion

Matter can be transferred to the neutron star from a binary companion by either a stellar wind or atmospheric Roche lobe overflow. In the case of wind accretion, plasma lost by the binary companion is gravitationally captured by the neutron star as it cools. The wind capture radius is generally larger than the radius of the neutron star’s magnetosphere. Depending on how much angular momentum the plasma possesses with respect to the neutron star, it may or may not be able to form an accretion disc. In the case of atmospheric Roche lobe overflow the plasma of the binary companion’s atmosphere flows over the gravitational saddle point and is immediately captured by the neutron star. Such plasma has a large angular momentum with respect to the neutron star and forms a Keplerian accretion disc. In such a disc the gravitational attraction of the neutron star acts as a centripetal force and the plasma slowly spirals inward as its angular momentum is transported outward by viscous shear stresses (e.g. Pringle 1981).

The inward diffusion is halted at some radius, \( r_i \), at which the magnetic pressure of the neutron star magnetic field becomes equal to the gas pressure of the disc, i.e.

\[
\frac{B^2}{2\mu_0} = P_{\text{disc}}
\]

Pringle and Rees (1972) found that this occurs at a radius

\[
r_i = 0.89 \left( \frac{v}{v_f} \right)^{2/7} \frac{L_x}{\pi^2}
\]

where \( v_f \) is the virial velocity.
where \( u_r \) is the inward radial drift velocity in the disc, \( h \) is the
disc semi-thickness, \( v_f \) the free-fall velocity. The factor \( ru_r/hv_f \)
has the approximate value 0.25 and only weakly depends on
the radial distance \( r \). \( l_0 \) is given by

\[
l_0 = 3.88 \times 10^{8} L_{30}^{2/7} \mu_{20}^{4/7} M_x^{1/2} R_4^{2/7} m
\]

Here \( L_{30} \) is the accretion luminosity in units of \( 10^{30} \) W, \( \mu_{20} \) is
the magnetic moment in units of \( 10^{20} \) Tm\(^3\), \( M_x \) is the mass
of the neutron star in solar masses and \( R_4 \) is the radius of the
neutron star in units of \( 10^4 \) m. Thus the inner radius of the disc
varies as \( B^{4/7} \).

For radii less than \( r_l \) the magnetic field pressure dominates
and matter is constrained to flow along the field lines down to
the magnetic poles of the neutron star rather than diffusing in
toward the star radially, as occurs for radii much greater than \( r_l \).

Of particular importance to our model are the time scales
for plasma in the accretion disc to enter the magnetosphere and
reach the neutron star surface. Ghosh and Lamb (1978, 1979)
have presented the most detailed model of the X-ray pulsar
accretion disc to date. They considered a situation in which the
stellar magnetic field is completely excluded from the disc and
found that because of (1) the development of the Kelvin-
Helmholtz instability, driven by a velocity discontinuity between
the disc and magnetic field, (2) turbulent diffusion of the stellar
field through the disc and (3) magnetic flux reconnection, the
magnetic field threads the disc over a sizeable region. The Kelvin-
Helmholtz instability dominates the other processes and its effect
is two-fold (Arons et al. 1984). First, it mixes the disc plasma
and stellar field together to give a magnetized medium which
can flow to the polar caps. Second, the magnetic stress on the
plasma as it is mixed pulls the plasma out of Keplerian
equilibrium, causing it to be in partial free-fall as it is mixed
onto the field lines. The time scale for the development of the
Kelvin-Helmholtz instability is of order \( 10^{-5} \) of the radial inward
drift time \( \tau_r \) of the disc plasma and occurs over a broad region
near the inner edge of the disc. Typically \( \tau \sim 10^3 \) s, so \( \tau_{KH} \sim 10^2 \) s.

Ghosh and Lamb also found the time scales for turbulent
diffusion and magnetic flux reconnection are of order 1 s and
0.1 s respectively.

3. Asymmetric Dipole Magnetic Field Model
In this section we discuss the effects of the preceding time scales
on the time-dependence of the accretion flow caused by an
asymmetric stellar magnetic dipole field.

Consider the case of an oblique rotator in which the magnetic
axis does not pass through the centre of the neutron star but
still lies in the meridional plane of the rotation axis, as shown
in Figure 1. As discussed above, the inner radius of the disc
is determined by a balance between the pressure of the stellar
magnetic field and gas pressure in the disc. For a magnetic dipole
centrally located in the star, the pressure balance at the inner
radius of the disc does not change as the star rotates. However,
for a field as in Figure 1, the pressure balance at any point on
the inner edge of the disc changes over a rotation period. Time
scales for the development of instabilities in this region have
been presented above. For many X-ray pulsars these time scales

Figure 1 — Non-aligned, non-axisymmetric dipole magnetic field interacting with an accretion disc. The magnetic axis lies in the meridional plane
of the rotation axis.
are short enough so that the decrease in magnetic field strength at the inner edge of the disc during the neutron star's rotation causes a significant amount of plasma to start diffusing inward again. Then as the magnetic field increases again, instabilities cause the plasma to enter the high magnetic field region. It then flows down onto the stellar surface. Thus, the accretion rate onto different areas of the polar cap varies, leading to asymmetries in the emitted pulse profile.

The electron and ion densities at the inner edge of the accretion disc (i.e. at the magnetospheric boundary) are believed to be $\sim 10^{19} - 10^{28} \text{ m}^{-3}$ (Ghosh and Lamb 1979).

If the position of the magnetic dipole axis is displaced by say 10% from the centre of the neutron star, the magnitude of the dipole magnetic field at the inner edge of the disc changes by $\sim 3 \times 10^3 \text{ G}$, assuming a typical magnetic field of $10^6 \text{ G}$ at the inner edge. As the inner radius of the disc is proportional to $B^{1/7}$, the change in magnetic field produces a change in the inner radius of the disc of $\Delta r / r \sim 1.5 \times 10^3 \text{ m}$.

This means there will now be an annular region of width $\sim \Delta r$ near the edge of the disc that contains disc plasma which becomes magnetically trapped when the rotation of the neutron star brings the increased magnetic field of the off-set dipole into contact with the plasma. This plasma is accreted onto the surface of the neutron star over a timescale of order half the neutron star rotation period. The increase in total mass transfer rate that results due to the accretion of this additional plasma can be calculated as follows.

The volume of the annular region of disc plasma which is accreted is given by

$$V = \pi \Delta r \Delta h h,$$

where $h$ is the scale height of the disc. Assuming a typical inner disc radius of $\sim 10^7 \text{ m}$, for a thin disc one usually takes $h \sim 10^{-2} \Delta r$ (e.g. Borner 1980), thus $h = 10^7 \text{ m}$. Hence, $V \sim 10^{15} \text{ m}^3$.

The number densities quoted imply mass densities $\rho \sim 2 \times 10^5 - 2 \times 10^6 \text{ g m}^{-3}$. Therefore, the mass contained in the annular region of width $\Delta r$ is $\sim 10^{10} - 10^{19} \text{ g}$. The matter will be transferred to the stellar surface in a time $\sim 1/2$ rotation period, so for periods of a few seconds a mass transfer rate $\dot{m} \sim 10^{10} - 10^{19} \text{ g s}^{-1}$ results. The total mass transfer rate onto the surface of an X-ray pulsar is expected to be $\dot{M} \sim 10^{15} - 10^{18} \text{ g s}^{-1}$ depending on its luminosity. Thus the analysis above shows that the small displacement in magnetic axis postulated, leads to significant change in the total mass transfer rate and thus significant changes in the X-ray pulsar luminosity.

In Figure 1 the rotation axis and line-of-sight are in the plane of the page. We see the centre of the main pulse when the magnetic axis is also in the plane of the page, as shown. The orientation of the magnetic axis at other phases of its orbit are shown in Figures 2a and 2b. We observe the main pulse from pole A and the interpulse from pole B. As discussed by Kirk et al. (1986) the main pulse is wider than the interpulse, but with the field configuration in Figures 1 and 2, the interpulse is more luminous as more plasma is trapped and channelled down onto pole B, than onto pole A.

The magnetic field on the right side of Figure 1 at the average disc radius is weakest when the pulsar is in the position shown. Matter from the disc is diffusing inward, effectively reducing the inner radius of the disc. On the left hand side the magnetic field at the average disc radius is at its maximum. Matter from the disc mixes with the magnetic field on the Kelvin-Helmholtz time scale, becomes magnetically trapped and then is funneled down onto the surface of the star. The transfer time for plasma to travel from the disc to the star is $\lesssim 0.1\text{s}$.

In Figure 2a, the pulsar has completed a quarter turn from its position in Figure 1. The field has increased on the right hand side and matter that was diffusing inward is now magnetically trapped and funnelled onto the stellar surface. On the left hand side the field has decreased and so the inner edge of the disc starts diffusing inward. In Figure 2b the right hand side is now experiencing maximum field and maximum disc plasma trapping, while on the left hand side the disc plasma has been depleted as the maximum field swept past and fresh plasma starts diffusing inward as this is now a region of weaker field.

In a discussion of the shape of the emitted pulse, we assume the intensity of emission at time $t$ is simply related to the amount of matter arriving at the stellar surface at that time. Emission we observe at time $t$ when the pulsar is in the position shown in Figure 1, is due to the matter that disconnected from the disc at time $t-t_1$, where $t_1$ is the time it took matter to travel from the disc to the stellar surface. The time $t_1$ depends on the length of the field lines on which disc matter must travel. It is apparent from Figures 1 and 2 that the leading part of the main pulse observed by us is emission due to plasma leaving the disc from a region that more recently experienced maximum field (and thus maximum disc plasma depletion) than the trailing part of the main pulse. The second half of the main pulse is thus brighter than the first half as more matter is being transferred onto the surface. A similar analysis shows that the interpulse is brighter during the leading part of the pulse than the trailing part. Kirk et al. (1986) discussed the conditions under which an interpulse is seen. Thus the possible observable pulse shapes for the field configurations of Figures 1 and 2 are as shown in Figure 3. An alternative magnetic field configuration for which the main pulse is brighter than the interpulse is shown in Figure 4. A similar analysis to that used above shows that the pulse asymmetry for the situation shown in Figure 4 leads to possible observable pulse shapes as indicated in Figure 5.

4. Comparison with Observations
The possible asymmetric pulse shapes that can be explained by the simple magnetic field asymmetry discussed above are shown in Figures 3 and 5. In summary, X-ray pulsars exhibiting only a single pulse can show either their leading or trailing edge brighter. Pulsars with an observable main pulse and interpulse that possess a brighter main pulse (the wider beam) should appear as in Figure 5b, whereas pulsars with a brighter interpulse should appear as in Figure 3a.

W83 present a unified description of 14 X-ray pulsars including multiple frequency light curves. We refer below to pulsars shown in Figure 1 of W83. Note that we do not include X-Per in our discussion as W83 notes that it appears to have
Figure 2—Different orientations of the magnetic axis during the neutron star's rotation.
Figure 3—Various possible pulse shapes observable assuming the magnetic field configuration illustrated in Figures 1 and 2.

a fundamentally different spectrum from all other higher luminosity pulsars and therefore may be emitting by a completely different mechanism. Also, the light curves are observed to be energy dependent. This implies that a variation in the mass accretion rate is not the only factor determining the pulse shape. Of the six remaining pulsars in Figure 1 with single profiles, four viz. Her X-1, 4U1258-62, Cen X-3 and 0A01653-40 show a sharp rise in emission, followed by a more gradual decline as in Figures 5a and 3b. One X-ray pulsar (GX1 + 4) exhibits this asymmetry for observations between 7-25 keV and 25-60 keV. The light curve for emission at frequencies between 2-7 keV does not have the same shape, but a more complicated structure with a deep notch (W83). Pulsar 4U1145-61 shows the opposite asymmetry as in Figure 3c.

Figure 4—An alternative configuration of the magnetic field and the accretion disc to that illustrated in Figure 1.
Figure 5 — Various possible pulse shapes observable assuming the magnetic field configuration illustrated in Figure 4.

Four of the pulsars in Figure 1 of W83 (4U1538-52, SMC X-1, 4U1223-62 and 4U0900-40) have light curves that contain both pulse and an interpulse. Pulsar SMC X-1 is an example of the asymmetry shown in Figure 5b. In 4U1538-52 there is suggestion of this asymmetry in the interpulse shape, but the asymmetry in the main pulse is only very slight. Pulsars 4U0900-40 (W83 Figure 7) and 4U1223-62 (W83 Figure 1) sometimes exhibit asymmetries that cannot be explained in our model.

5. Conclusion

In this paper we have proposed a new model to explain observed asymmetries in X-ray pulsar light curves. Our model consists of a thin accretion disc orbiting a rotating neutron star, which has an off-centre dipolar magnetic field. We have argued that such a geometry leads to a varying rate of accretion onto the magnetic polar caps, over the rotation period of the neutron star. By assuming the accretion rate onto the surface is simply related to the luminosity we then showed that a variety of asymmetric light curves result. Most of the X-ray pulsar light curves presented in W83 that exhibit asymmetries can be accounted for by our model. The only exception to this is the low frequency light curve for GX1+4 which has a more complicated structure and some observations of 4U0900-40 and 4U1223-62, which show different asymmetries. Thus, it is possible that most of the observed asymmetries in X-ray pulsar pulse shapes, are caused by asymmetries in the magnetic field of the sort discussed above and were it not for the reduction in emission on one side of the beam, many more X-ray pulsars would show the double-humped structure predicted by Kirk et al. (1986).

However, we note that the assumption of an off-centre magnetic dipole in the neutron star is ad hoc. Whether or not such a magnetic topology could exist in pulsars has not been explored in this paper and in fact theories of the generation and maintenance of neutron star magnetic fields are still in their infancy (Blandford et al. 1983). On the other hand, the fact that the qualitative model presented above can explain most of the observed asymmetries in X-ray pulsar beams, indicates that a more detailed investigation would be worthwhile.

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