FUNDAMENTAL PARAMETERS OF CEPHEIDS

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INTRODUCTION

In the two centuries since the discovery of δ Cephei and η Aquilae, the study of Cepheids, and of pulsating stars in general, has become a very important field in astronomy. The usefulness of Cepheids, both as distance indicators and as test objects for stellar astrophysics, has been amply demonstrated, and if one just looks at the flood of papers on Cepheids that appears each year, it is clear that the subject is not only old and respectable, but also still very much alive. This two-hundredth anniversary is therefore an appropriate occasion to evaluate what we have learnt about Cepheids, and where the remaining problems lie.

In this paper I will try to review some of our present knowledge of "fundamental parameters" of Cepheids. Which of the many parameters are "fundamental"? Since the period P_i of a given stellar pulsation mode, i, is primarily a function of mass M and radius R of the star, and since stellar evolution calculations usually give the luminosity L and effective temperature T_{eff} of a star as a function of its mass, age τ, and chemical composition (X,Y,Z), a useful list of fundamental parameters should contain at least: M, τ, (X,Y,Z), L, R, T_{eff}, P, i. Although the list has some redundancy, this is already a considerable number of parameters, and a proper discussion of these quantities would have to deal with almost every aspect of Cepheid behaviour. The scope of this paper will necessarily have to be much more limited. As a first limitation, I will discuss only the "classical" type of Cepheids, and say nothing about the W Virginis stars. Furthermore, I will concentrate on observational results, and on the constraints that can be derived from them on some of the above parameters, giving most attention to the calibration of L and T_{eff}.

REDDENING CORRECTIONS

The colour excess of a Cepheid can hardly be called a fundamental parameter, but it is usually such a difficult obstacle on the way to the intrinsic properties that I will start out with a few remarks on the reddening problem. For many years the intrinsic colours of Cepheids have been the subject of considerable debate. Uncertainties in the reddening corrections, in extreme cases as large as $0\overset{m}{.}2$ in E(B-V), have been mainly due to the following problems: 1) colour excesses for long-period Cepheids derived from a (spectral type)-$(B-V)_0$ relationship have been systematically larger than excesses based entirely on photometric calibrations; 2) up to

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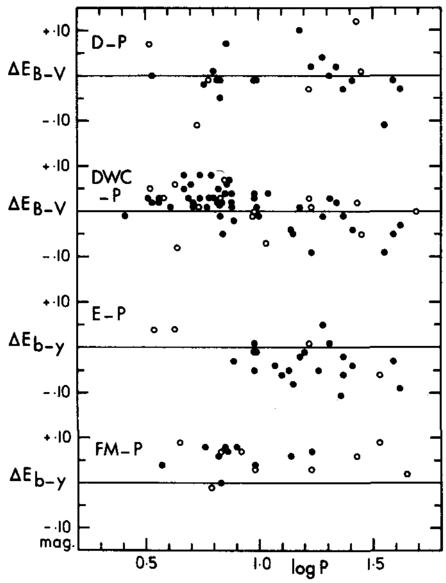
recently the large majority of the available Cepheid photometry has been in the UBV system, which is not very well suited for the determination of accurate Cepheid reddenings. For Cepheids in the Magellanic Clouds the situation has been complicated further by the effects of lower line blanketing.

During the last decade a large new collection of accurate photoelectric Cepheid photometry has been obtained in photometric systems that do allow good reddening determinations for F-G supergiants. Extensive surveys of galactic and Magellanic Cloud Cepheids have been made in the Kron-Cousins BVI system (Dean et al. 1978; Martin et al. 1979; Caldwell & Coulson 1984a), in the Walraven VBLUW system (Pel 1978), and in the Strömgren uvbyβ system (Feltz & McNamara 1980; Eggen 1983 a,b). A more limited set of data in the DDO system has been published by Dean (1981). The reddening results of this new generation of photometry show a much improved mutual agreement. It seems clear now that in the earlier studies Cepheid reddenings were generally overestimated, particularly in methods based on a spectral type - colour calibration, and at the longer periods. As for the Magellanic Cloud

Fig.1. Comparison of colour excesses for galactic Cepheids in five photometric systems. Since the largest common overlap is in the VBLUW data, all colour excess differences are given with respect to the VBLUW reddenings. $E(V-B)_{VBLUW}$ has been transformed to the other scales with the following "theoretical" transformations (computed from extinction law, passbands, and energy distributions): $E(b-y) = 1.75E(V-B)$,and

$$E(B-V)/E(V-B) = 2.41 -0.11(\log P -1) -0.18E(V-B).$$

Open symbols indicate stars with known or suspected companions or with other peculiarities.



Abbreviations used:

D: Dean (1981); DDO photometry, published excesses given in E(B-V).
DWC: Dean et al. (1978); Kron-Cousins BVI system.
E: Eggen (1983 a,b); Strömgren uvbyβ + RI.
FM: Feltz & McNamara (1980); Strömgren uvbyβ + khg.
P: Pel (1978); Walraven VBLUW system.

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Cepheids, better estimates for the abundance effects can now be made, and most new evidence indicates very small reddenings, at least in the outer regions of the Clouds (Caldwell & Coulson 1984 b; see also the review by Feast 1984).

This convergence in the newer reddening results is of course very good news, but let me point out that it is too early to call the reddening problem "solved". In Fig.1 I have compared reddenings for galactic Cepheids from BVI, DDO, Strömgren, and Walraven photometry. Evidently there still exist systematic zeropoint differences and period-dependent trends. These effects are much smaller than they used to be, but they are still disappointingly large compared to the photometric accuracy of well below $0^m.01$ of which all these systems are capable. The residuals can only partly be ascribed to shortcomings of the transformations that were used, and we can only conclude that systematic uncertainties of up to $0^m.05$ in E(B-V) are probably still present. Are errors at that level really disturbing? This can be judged from Table 1: the effects on radii and masses are not very serious, but those on T_{eff} and L are too large for a proper comparison of the empirical and theoretical positions of the Cepheid strip in the HR-diagram.

Table 1. Effect of $\Delta E(B-V) = +0^m.05$ for an average 10-day Cepheid.

parameter	relation used	resulting change	
temperature	$(B-V)_o - T_{eff}$	$\Delta T_{eff} = +130$ K	1)
gravity	typical photometric Balmerjump index	$\Delta \log g = +0.2$	2)
bol.corr.	$BC(T_{eff}, \log g)$	$\Delta BC = +0^m.01$	3)
luminosity	$A_V/E(B-V)$, 3), known distance	$\Delta L = +14$ %	4)
luminosity	$P-L-T_{eff}$ (theoretical), 1)	$\Delta L = +12$ %	5)
distance	P-L, with 4) for apparent L	$\Delta d = -7$ %	6)
distance	$P-L-T_{eff}$, 4) for app. L, 5) for true L	$\Delta d = +1$ %	7)
radius	$R \propto L^{\frac{1}{2}}.T_{eff}^{-2}$, 1), 4)	$\Delta R = +2$ %	8)
radius	Baade-Wesselink method	ΔR small, $\ll 1$ % ?	9)
mass	evolutionary M-L, 4)	$\Delta M_{evol} = +3.5$ %	10)
mass	pulsation relation $P-L-M-T_{eff}$, 1), 4)	$\Delta M_{puls} = +4.5$ %	11)

Fig.2. Temperature-colour relations for Cepheids and supergiants. $\Theta_{eff} \equiv 5040/T_{eff}$.

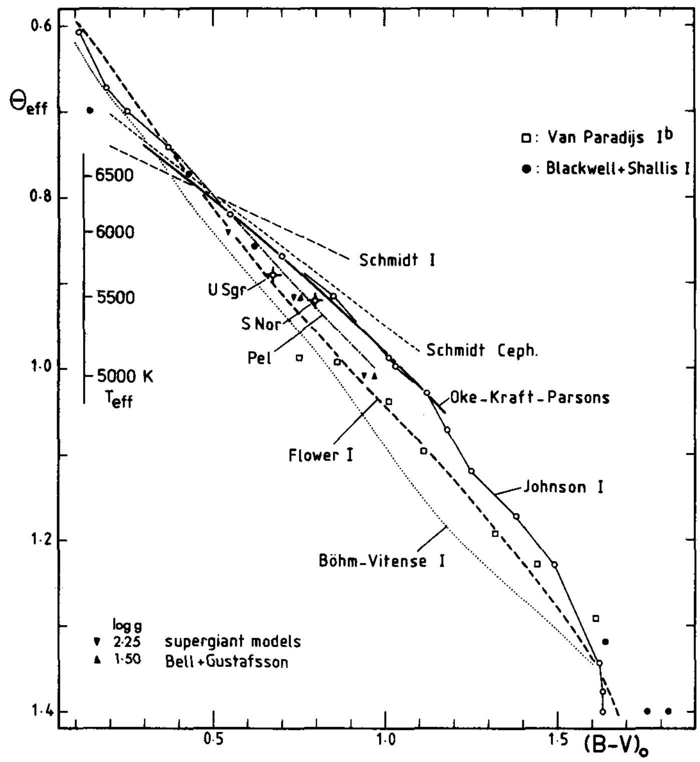
Supergiants:

Johnson I	: Johnson (1966).
Böhm-Vitense I	: Average for I^a and I^b from Böhm-Vitense (1972).
Schmidt I	: Schmidt (1972).
Van Paradijs I^b	: Van Paradijs (1973); 7 individual I^b supergiants.
Blackwell & Shallis I	: Individual supergiants from Blackwell & Shallis (1977). E(B-V) for δ CMa (at $(B-V)_0 = 0^m.62$) derived from VBLUW data. The other stars are within 200 pc, and small reddening corrections were made according to $E(B-V)=0^m.3\ kpc^{-1}$.
Flower I	: Flower (1977).
Bell & Gustafsson	: The hotter supergiant models from Bell & Gustafsson (1978), for Doppler broadening velocity 3.5 km s^{-1}.

Cepheids:

Oke-Kraft--Parsons	: Oke (1961); Kraft (1961); Parsons (1971, 1974). The relation by Rodgers (1970) is not shown, as it lies very close to the Oke-Kraft-Parsons line.
Schmidt Ceph.	: Schmidt (1972).
Pel	: Average relation derived from Pel (1978).

For the sake of clarity the Böhm-Vitense (1981) "best fit" relation is not shown; it lies halfway the Flower I and Böhm-Vitense I lines. Also not shown is the Bell & Parsons (1974) relation, which runs parallel to the Oke-Kraft-Parsons line, but $0^m.05$ bluer. For the "Wesselink temperatures" of U SGR and S NOR see text.



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That reddenings can be determined with an accuracy much better than $0^m.05$ is demonstrated by the following examples. From Strömgren photometry of NGC 6087, the cluster containing S NOR, Schmidt (1980 b) finds for the cluster members $E(b-y) = 0^m.123$, and Eggen (1980) $E(b-y) = 0^m.142$, whereas my own VBLUW photometry (Pel 1984) gives $E(V-B) = 0.076$, or $E(b-y) = 0^m.134$. The standard deviation for a single star is $0^m.01$ in each case. In M 25, with U SGR, the reddening is much larger, and inhomogeneous. For 18 cluster members within 6 arcmin from the Cepheid, Schmidt (1982) finds $E(b-y) = 0^m.358$, $\sigma = 0^m.031$. The VBLUW reddenings for 27 members within the same area correspond to $E(b-y) = 0^m.338$, $\sigma = 0^m.025$, showing that even in this difficult field a satisfactory agreement can be reached with different techniques.

EFFECTIVE TEMPERATURES

As can be expected from the Stefan-Boltzmann law, the effective temperature of a Cepheid is a very important quantity, and the estimates for some other, not directly observable parameters can be very sensitive to the accuracy of T_{eff}. An extreme example is the "pulsation mass", which depends very strongly on T_{eff} because of the pulsation relationship from which it is derived: $P \propto L^{0.83}.M^{-0.66}.T_{eff}^{-3.45}$ (Iben & Tuggle 1972). In this respect some numbers in Table 1 are misleading, because they suggest that T_{eff} has only minor effects on R and M. Without the compensating effect of the luminosity increase by A_V, however, $\Delta T_{eff} = +130$ K results in $\Delta R = -4.5$ % and $\Delta M_{puls} = -11.5$ % (Table 1 lines 8 and 11). Also in the HR-diagram, an error as small as 130 K is by no means negligible, as it corresponds to about one fifth of the width of the Cepheid strip at constant L.

By far the most common method of determining Cepheid temperatures is the use of a temperature-colour relation, where the colour is traditionally $(B-V)_o$. In order to estimate how well Cepheid temperatures are known, I have therefore collected in Fig.2 most of the $T_{eff}-(B-V)_o$ calibrations that exist in the literature for Cepheids and cool supergiants. Although my own photometric temperatures for Cepheids are actually based on a two-dimensional calibration, I have also represented those temperatures by a mean $T_{eff}-(B-V)_o$ relation, to allow a comparison.

Fig.2 should be interpreted with caution for several reasons. Firstly, not all of these relations are completely independent. The Flower calibration, for example, is based partly on the Van Paradijs data. Secondly, the problems with reddening also enter this diagram in some cases, via the reddening corrections for calibrating stars. Thirdly, it is well known that $(B-V)_o$ is not an ideal temperature index, because it is also sensitive to gravity and chemical composition. On the other hand, the uncertainties related to reddening are small compared to the very large differences in the temperature relations, and as long as we restrict ourselves to nearby classical Cepheids, differential line-blanketing can safely be neglected. Gravity effects within the Cepheid region do cause scatter in $(B-V)_o$, as can be seen from the Bell & Gustafsson colours in Fig.2 (see also Pel 1980), but also this scatter is small compared to the wide range in temperature scales in Fig.2. With some limitations, the

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comparison in this figure is therefore still instructive. But can it tell us what the most reliable temperature scale for Cepheids is?

The most refined synthetic spectra presently available for intermediate-type supergiants are those by Gustafsson & Bell (1979) and Kurucz (1979). The latter models are represented in Fig.2 by my own temperature relation for Cepheids, which is based on the Kurucz data. The overlap between these two sets of models is limited, but the agreement is reasonable, with differences that stay within 100 K over most of the common range. On the other hand, in her review of the effective temperature scale, Böhm-Vitense (1981) gives a best-fit relation for A-M supergiants which is mainly based on the calibrations by Böhm-Vitense (1972), Van Paradijs (1973), Bell & Parsons (1974), Luck (1977 a,b), and on a few stars with direct angular diameter measurements. The temperatures by Luck are in turn based on the Bell et al. (1976) model atmospheres. In the region of the Cepheids, Böhm-Vitense's "most probable" relation lies halfway between the Böhm-Vitense 1972 and the Flower relation, about 200-300 K cooler than my Cepheid temperatures.

How can we resolve this discrepancy? It is well known that the agreement between real and synthetic stellar spectra is still far from perfect. In particular, when synthetic (blue-visual) colours are normalized around A0 V, they systematically produce too low temperatures for F-G stars (cf. Relyea & Kurucz 1978; Gehren 1981). This problem is probably due to insufficient near-UV opacity in the models. I have tried to estimate the size of this effect in the Kurucz VBLUW colours, taking also into account improved photometric calibrations and results for new (unpublished) Kurucz models with improved treatment of convection. These estimated corrections, which are still very uncertain, indicate that my Cepheid temperatures may have to be decreased by about 100 K at the blue end of the colour-T_{eff} relation, and increased by a smaller amount at the red end. The gap in temperature between the Böhm-Vitense best-fit relation and my "best guess" would then become nearly constant over the whole Cepheid range, but it would still be about 250 K wide.

Unfortunately there are only two F-G supergiants for which interferometric diameter data allow a direct determination of T_{eff}: α CAR and δ CMA. Code et al. (1976) derive T_{eff}= 7460 K for α CAR and 6110 K for δ CMA, but with large uncertainties (σ = 450 K). Blackwell & Shallis (1977) find 7206 ± 173 K and 5877 ± 390 K for the same stars (see Fig.2), but also these numbers are too uncertain to improve the situation.

Since we are dealing with Cepheids, there is one other "direct" way available to estimate T_{eff}: via the Baade-Wesselink (BW) radius. The BW method in its original form, which assumes a unique correspondence between T_{eff} and colour, is entirely independent of temperature. This is not quite true for some modern versions of the BW method, but in all cases the BW radius depends very little on the adopted temperature scale. Moreover, temperatures derived from $L \propto R^2.T_{eff}^4$ are relatively insensitive to errors in R and L, and the errors in R_{BW} and L should be uncorrelated when L is derived from cluster distances.

In Fig.2 I have indicated "Baade-Wesselink" temperatures for the two cluster-Cepheids with the best cluster data, S NOR and U SGR. The BW radii of these stars were taken from the recent discussion of Cepheid radii by Fernie (1984); they are based on many BW determinations (14 for U SGR, 9 for S NOR). The luminosities were derived from the cluster photometry (cf. next section). The resulting temperatures are 5470 K for S NOR and 5655 K for U SGR. I am of course pleased to see that these temperatures fit well with my own temperature calibration, but unfortunately this agreement may be entirely accidental. The errors in the BW temperatures are still large (σ = 180 K for S NOR, 210 K for U SGR), mainly due to the uncertainties in L (10-15 %). BW radii may also be systematically too small (Fernie 1984). An upward correction of the BW radii by 10 % would lower the BW temperatures by 275 K. We are therefore left with the conclusion that the slope of the $T_{eff}-(B-V)_0$ relation is probably fairly well established, but that the zeropoint remains uncertain within a range of about 250 K.

The preceding discussion may leave a rather disappointing impression about the status of temperature determinations for Cepheids. With 250 K uncertainty in the temperature scale, and reddening errors adding another 130 K, Cepheid temperatures could be wrong by 380 K. This would correspond to more than half of the width of the Cepheid strip, and it would make mass estimates or comparisons with theoretical blue edges almost impossible. Some optimism is also justified, however. It has been shown that a very good fit to theoretical blue edges can be obtained with Cepheid temperatures derived from multicolour photometry and synthetic spectra (e.g. Pel & Lub 1978). This can not be used to support a particular calibration of reddenings and temperatures, of course, but it indicates at least that a systematic error in T_{eff} as large as 380 K is unlikely. It should also be remembered that relative temperature determinations are much more accurate than the absolute scale. Within a given photometric calibration , an accuracy of $0\overset{m}{.}03$ in E(B-V) is well attainable (see Fig.1), which means that the relative positions of Cepheids inside the strip can be determined to within ± 12 % of the strip width.

LUMINOSITIES

It is very difficult to discuss the luminosities of Cepheids without entering into a discussion of the Cepheid distance scale. Since the role of Cepheids as distance indicators will be the subject of a separate review at this colloquium, I will concentrate here on only a few aspects of the luminosity calibration. This I will do mainly from the physical viewpoint, i.e. the calibration of bolometric luminosities (L) rather than of absolute magnitudes (M_V).

The transformation of M_V into L requires only one step, the bolometric correction (BC), but for most stars this is by no means an easy one. Fortunately Cepheids have nearly solar temperatures, and consequently their bolometric corrections are on average small. On the other hand, the temperature variations of large-amplitude Cepheids are so large - 1000 K or more - that the differences between visual and bolometric lightcurves can become quite significant. Since the "equilibrium luminosity" L of a Cepheid is the time average of the bolometric lightcurve, it is

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good to check whether bolometric corrections are indeed only a minor source of uncertainty in the calibration of L.

In order to estimate how well the BC scale for Cepheids is determined, I have compared in Fig.3 the empirical BC calibration of Flower (1977) with theoretical BC values based on the Kurucz (1979) model atmospheres. Flower's BC relations in this temperature range are mainly based on the data of Johnson (1966) and Code et al. (1976). The BC values for the Kurucz models were taken from Lub & Pel (1977); the supergiant relations correspond to a microturbulence of 4 km s^{-1}, and the BC scale was normalized by adopting $BC_{\odot} = -0\overset{m}{.}07$. The agreement between both BC calibrations is quite satisfactory. The Flower supergiants relation and the "mean locus" occupied by Cepheids in the Kurucz calibration differ by about $0\overset{m}{.}02$, but the important thing is that this difference is nearly constant with T_{eff}. We can therefore conclude that the shape of the BC curve for Cepheids is quite accurately known, and that the uncertainty in the zero-point is probably $\lesssim 0\overset{m}{.}02$, corresponding to an error in L of about 2 %. This is not entirely negligible, but it is only a small factor in the total uncertainty of L, as we will see.

For the calibration of Cepheid parameters, and particularly for the luminosity scale, the Cepheids in open clusters and associations remain of crucial importance. In recent years there has been a strong renewed effort to increase the number of these calibrating Cepheids, and to improve the quality of the cluster data. I refer especially to the photometric programs by Turner (1977 1978 1980 1983) and Schmidt (1983 and op. cit.) The results of this new work are in many respects promising. Although for very long it seemed impossible to increase the number of 13 "classical" calibrators of Sandage & Tammann (1969), Fernie & McGonegal (1983) now list 27 Cepheids for which cluster/association membership is likely. These stars cover the whole range of Cepheid periods, and they define a P-L relationship with the remarkably small r.m.s. scatter of $0\overset{m}{.}16$ in M_V. It is not certain, however, that this small scatter really reflects the present level of accuracy in the Cepheid luminosity scale.

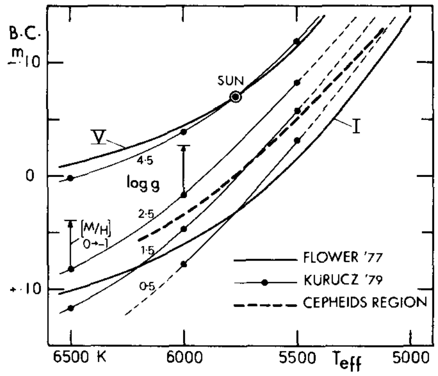

Fig. 3
Empirical (Flower 1977) and theoretical (Kurucz 1979) BC calibrations for Cepheids. The arrows [M/H] 0 → -1 indicate the effect of a factor 10 decrease in metallicity.

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Two persistent problems cause considerable uncertainty in the distances of galactic open clusters. Firstly, there are the well-known complications related to the Hyades zeropoint. The recent increase of the Hyades distance appears well established now (Hanson 1980), but it remains uncertain how the distances to other clusters should be corrected for the effect of a high metallicity of the Hyades, and even the composition of the Hyades is still a matter of dispute (Cayrel de Strobel 1980; Flower 1980). Secondly, there exists a discrepancy between the cluster distances derived from UBV photometry and those from the new uvbyβ data of Schmidt, in the sense that Schmidt's distances are systematically smaller by 20-40 % (Schmidt 1980a; Caldwell 1983). This discrepancy is alleviated partly by the recent recalibration of the Hβ index by Balona & Shobbrook (1984), but even these last corrections leave systematic differences corresponding to about 20 % in distance. It is obvious that an error of this size would ruin the Cepheid distance scale, and that it would also be very serious for comparisons with Cepheid theory: a 20 % distance error means 40 % in L, 20 % in R (at given T_{eff}), 10 % in M_{evol}, and 50 % in M_{puls}.

In an attempt to shed some new light on this problem, I will now discuss recent results from VBLUW photometry of NGC 6087 and M 25, the parent clusters of S NOR and U SGR. These clusters are probably the two most reliable points in the calibration of Cepheid luminosities. The ages of NGC 6087 and M 25 are so close to that of the Pleiades ($\tau \sim 7.5 \times 10^7$ yr), and the main-sequences of the three clusters are so similar, that it should be possible to derive very accurate differential distance moduli with respect to the Pleiades. Since the Pleiades distance can be determined from a direct fit to nearby parallax stars (Van Leeuwen 1983), this offers the very attractive opportunity of distance determinations for U SGR and S NOR that are entirely independent of the Hyades. The uncertainty due to possible composition differences between the clusters cannot be bypassed in this way, of course. On the other hand, with respect to metallicity the Pleiades are probably much more typical for young open clusters than the Hyades (cf. Nissen 1980).

Let us see now what comes out of such a Hyades-independent calibration. The available VBLUW data consists of the extensive Pleiades photometry by Van Leeuwen (1983), and my own VBLUW photometry for about 140 stars in each of the clusters NGC 6087 and M 25 (Pel 1984). It is not possible to discuss any details of the photometric analysis here, but a few important points should be mentioned. The question of cluster membership causes no problems for the brighter Pleiades, but this is not so for the other two clusters. Similarly to the uvbyβ photometry, the VBLUW system provides 4 criteria to decide about cluster membership: position in the HR-diagram, colour excess, and spectral classifications based on two reddening-independent two-colour diagrams. The latter diagrams are calibrated in terms of T_{eff} and log g, and they allow two independent 2-dimensional classifications for each star. Accurate individual reddening corrections can then be made, and a de-reddened magnitude-T_{eff} diagram can be constructed.

Fig.4 shows the HR-diagrams of the three clusters, for the upper main-sequences down to about A0. The T_{eff} scale used here is based on the synthetic spectra by Kurucz (Kurucz 1979; Lub & Pel 1977), but I want to stress that this is unimportant for the distance determination. A diagram with the reddening-independent temperature index [B-U] along the abscissa looks extremely similar to Fig.4, and its leads to the same results. A comparison of Fig.4 with theoretical isochrones (e.g. Hejlesen 1980) confirms that the three clusters differ very little in age, $\Delta\tau \lesssim 25$ %. This comparison shows also that differential evolution effects in the late B and A stars are extremely small. In the main-sequence fits I have therefore given the highest weight to the lower parts of the HR-diagrams in Fig.4. Small corrections for the age differences were applied, based on the theoretical isochrones. The final results for the distances of M 25 and NGC 6087 are given in Table 2, together with results from other studies.

Can we learn anything from Table 2 with respect to the discrepancy between UBV and uvbyβ distances? After adjusting Schmidt's distances with the corrections by Balona & Shobbrook, there appears to be excellent agreement between the VBLUW and uvbyβ results for M 25, but not for NGC 6087. I have the strong suspicion that the disagreement for NGC 6087 is related to the fact that the uvbyβ distance is based on only 11 of the brightest cluster stars, all near the main-sequence turnoff, and clearly evolved. It could very well be that the Hβ calibration for such stars is less reliable. The uvbyβ distance for M 25 should be much more accurate, since it is based on 27 cluster stars which cover the main-sequence nearly down to $\log T_{eff} = 4.0$.

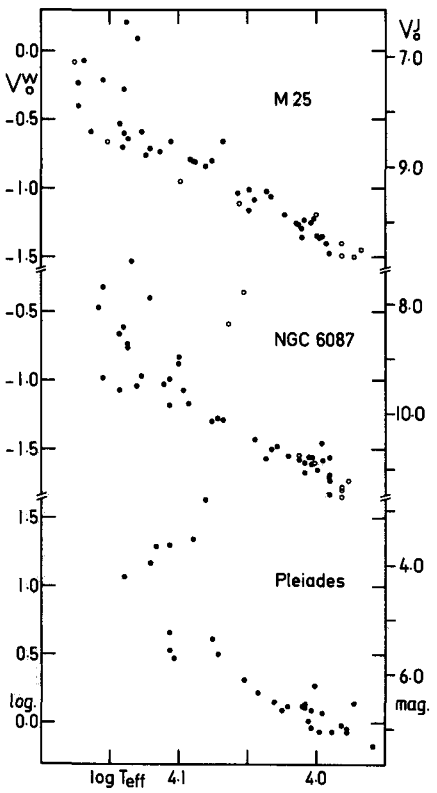

Fig.4.
Reddening-free HR-diagrams for M 25, NGC 6087, and Pleiades, as determined from VBLUW photometry. The Walraven V_o scale is in log units, the (transformed) UBV V_o scale in magnitudes. Temperatures have been derived from reddening-independent colour-indices. Open circles denote stars with less accurate reddening values, or cases where there is some doubt about cluster membership. Binaries have not been removed from the Pleiades diagram.

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In this respect it is probably significant that Schmidt's preliminary result for M 25 (Schmidt 1980a), which was still based on only 12 cluster members, gave a modulus of only $8\overset{m}{.}44$. It is likely that this first set of 12 stars covered mainly the bright, evolved cluster stars.

After all the work on the Hyades modulus and the cluster distance scale over the last fifteen years, it is ironic that the new VBLUW distances for S NOR and U SGR are so close to the 1969 Sandage & Tammann values. This is no argument against the new Hyades distance, but it indicates strongly that the increase in the Hyades distance is indeed to a large fraction offset by composition effects (cf. Martin et al. 1979; Caldwell 1983). Since more reliable theoretical data on composition effects in cluster main-sequences are now available (VandenBerg & Bridges 1984), we can show that the Pleiades distance used in the VBLUW method does not conflict with the revised Hyades modulus. Adopting a Hyades modulus of $3\overset{m}{.}30 \pm 0\overset{m}{.}06$ from Hanson (1980), and a modulus difference Pleiades-Hyades $\Delta(m-M)_0 = 2\overset{m}{.}52 \pm 0\overset{m}{.}05$ (mean value from Turner 1979 and Jones 1981), the uncorrected Pleiades modulus becomes $5\overset{m}{.}82$. If the Hyades are more metal-rich by $\Delta[Fe/H] = +0.15$, the Pleiades modulus should be corrected by $-0\overset{m}{.}22$ ($\pm 0\overset{m}{.}03$?) according to the VandenBerg & Bridges data. This gives a final Pleiades modulus of $5\overset{m}{.}60 \pm 0\overset{m}{.}08$. Van Leeuwen's fit to parallax stars gave $5\overset{m}{.}57 \pm 0\overset{m}{.}08$.

Returning now to the aim of this whole exercise – the calibration of Cepheid luminosities – I will modestly adopt the VBLUW moduli as probably the most reliable distances for S NOR and U SGR. From the VBLUW photometry of the Cepheids (Pel 1978) we finally get $\log(L/L_\odot) = 3.536$ for S NOR, and 3.436 for U SGR (see Table 3). Taking into account the uncertainties in BC and reddening of the Cepheids, the r.m.s. error in both luminosities is $\sigma(\log L) = 0.044$, i.e. 11 % in L. This represents the best accuracy that is

Table 2. Distance moduli for M 25 and NGC 6087.

	M 25		NGC 6087	
VBLUW photometry, fit to Pleiades		σ		σ
$\Delta(m-M)_0$ relative to Pleiades	$3\overset{m}{.}41$	$0\overset{m}{.}06$	$4\overset{m}{.}29$	$0\overset{m}{.}06$
differential evolution corrections	−0.03	0.01	−0.02	0.01
$(m-M)_0$ Pleiades (Van Leeuwen 1983)	5.57	0.08	5.57	0.08
total distance modulus $(m-M)_0$	8.95	0.10	9.84	0.10
Distances from other methods				
Sandage & Tammann (1969); UBV,old Hyades distance $(m-M)_0 = 3\overset{m}{.}03$	8.98		9.76	
Caldwell (1983); UBV,Hyades at $(m-M)_0 = 3\overset{m}{.}28$, correction for high metallicity Hyades	9.14		9.81	
Fernie & McGonegal (1983); UBV, Hyades at $3\overset{m}{.}29$, no correction for metallicity	9.21		9.95	
Schmidt (1980 b, 1982); uvbyβ, no explicit fit to Hyades	8.76		9.60	
Schmidt, as corrected by Balona & Shobbrook (1984)	8.91		9.64	

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attainable for a calibrating Cepheid. It should be kept in mind, however, that this 11 % standard error does not yet account for the intrinsic scatter in main-sequences due to variations in composition, rotation, binary frequency. The contributions from these extra error sources are very uncertain, but they could very well increase σ_L to about 15 %.

MASSES AND RADII

The excellent review of Cepheid masses given by A.N.Cox a few years ago (Cox 1980) is still fully relevant to the present situation, so I will not attempt to repeat that analysis here. In his review Cox came to the important conclusion that the "primary" Cepheid mass problem - the fact that $M_{evol} > M_{puls}$ for all Cepheids with known L - had been solved by the increased Hyades distance and by lower Cepheid temperatures due to improved reddening and temperature calibrations. The factor that contributed most in removing the discrepancy was the larger Hyades modulus. We can expect therefore that the reduction in cluster distances that I have just discussed will cause mass problems again.

Table 3 lists the masses and radii for U SGR and S NOR that follow from the temperatures and luminosities that I adopted as "best values". The different mass estimates (following Cox's nomenclature) were computed for (Y=0.28,Z=0.02) from the same relations used by Cox: the evolutionary M-L relation for Cepheids from Becker et al. (1977), and the Faulkner (1977) fitting formulae for the P-M-L-T_{eff} relation. The results in Table 3 are not very surprising. The "theoretical" masses M_{th} are hardly affected by the new calibration, but the reduction in L moves M_{evol} and M_{puls} apart. This brings back a mass discrepancy that is not very large, but nevertheless significant.

Table 3. Fundamental parameters of U SGR and S NOR. Numbers in parentheses are corresponding values from Cox (1980). Masses and radii are in solar units.

parameter	U SGR		S NOR	
period	$6^d.745$		$9^d.755$	
$(m-M)_o$	$8^m.95$		$9^m.84$	
T_{eff} K	5754	(5734)	5425	(5502)
$\log(L/L_\odot)$	3.436	(3.59)	3.536	(3.65)
$R(L,T_{eff})$	52.5	(63.8)	66.4	(74.2)
M_{evol}	6.44	(7.09)	6.85	(7.36)
M_{th}	6.74	(6.72)	7.22	(7.37)
M_{puls}	5.15	(8.79)	5.45	(7.31)
M_{evol}/M_{puls}	1.25	(0.81)	1.26	(1.01)
M_{th}/M_{puls}	1.31	(0.76)	1.32	(1.01)
R_{BW} Baade-Wesselink radius (Fernie 1984)	54.3	(55.3)	65.2	(53.4)
$M_{BW} = M_{puls}$ from R_{BW}	5.61	(6.63)	5.25	(3.59)

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If we assume that the discrepancy is caused by the observational data, and not by the theoretical relations, the problem can be cured with the well-known remedy: a decrease in T_{eff}, an increase in L, or a combination of both (Iben & Tuggle 1975; Cox 1980). In view of the uncertainty limits in the T_{eff} and L scales, and of the blue edge position in the HR-diagram, the least unattractive compromise would be ΔT_{eff}= −155 K and ΔL= +10 %. This would bring perfect agreement (within 1 %) between M_{evol}, M_{puls}, and M_{th} for both Cepheids (note that the new mass ratios in Table 3 are nearly identical for both stars), and it would not spoil the good fit of observed and theoretical blue edges too much. We should not accept this solution too easily, however. It clearly would not solve all problems, as $R(L,T_{eff})$ would increase by 10 %, and M_{BW} would remain too low. We should also keep in mind that the data in Table 3 can hardly be called representative for the whole Cepheid strip, as they are based on only two Cepheids with not too different periods.

The determinations of M and R for Cepheids are so closely related, that the problems with Cepheid masses could also be described as radius-discrepancies. In order to see what the situation is at the long- and short-period ends of the Cepheid strip, I will now switch from M to R, and discuss some results from a recent excellent review of Cepheid radii by Fernie (1984). Fernie discusses the following four types of radii:
1) Baade-Wesselink radii, R_{BW}, for which average values are taken from many sources.
2) $R(L,T_{eff})$ for cluster/association Cepheids, R_{CL}, based on the data by Fernie & McGonegal (1983).
3) "beat/bump radii", R_{BB}, based on R values from mixed-mode period ratios and phases of bumps in velocity curves (Cogan 1978).
3) "theoretical radii", R_{TH}, determined by solving the Becker et al.(1977) M-L relation, and the Iben & Tuggle (1975) P-M-L-T_{eff} relation at the blue and red edges of the Cepheid strip.
For these different radii he finds the following mean P-R relations:

$$\log R_{BW} = 1.244\ (\pm 0.023) + 0.587\ (\pm 0.022) \log P$$
$$\log R_{CL} = 1.042\ (\pm 0.015) + 0.824\ (\pm 0.010) \log P$$
$$\log R_{BB} = 0.833\ (\pm 0.018) + 0.956\ (\pm 0.022) \log P$$
$$\log R_{TH} = 1.179\ (\pm 0.006) + 0.692\ (\pm 0.006) \log P$$

Fernie describes this as "a sorry situation", even if one omits the R_{BB} radii on the grounds that the correct interpretation of the beat and bump phenomena is still unclear. For periods between 5 and 10 days the R_{BW}, R_{CL} and R_{TH} radii agree reasonably well, but outside this range there are serious discrepancies, of up to 50 % at the longest periods.

These results show that the good agreement between $R(L,T_{eff})$ and R_{BW} in Table 3 is misleading. It is also clear now that a simple zeropoint shift in L and T_{eff} can not solve all mass and radius problems over the whole period range. Adopting the distance scale of Table 3, instead of the uncorrected Hyades modulus of $3^{m}_{.}29$ used by Fernie, would make all R_{CL} 12 % smaller. This would bring no real improvement; it would merely shift the point of best agreement between R_{TH}, R_{CL} and R_{BW} towards longer periods.

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The R values that stand out most strongly are R_{BB} at the short periods, and R_{BW} at the long periods. The solution for the very small beat and bump radii (and masses) almost certainly has to come from the theory, but it is still difficult to decide whether surface He-enrichment (cf. Cox 1980) is the real answer to this problem. As for the BW radii, I should point out that Fernie's mean R_{BW} relation is an average for several rather different versions of the BW method, and that in general the more refined versions give steeper relations. It is particularly noteworthy that the BW method of Sollazzo et al. (1981) gives radii that agree very well with R_{TH}. Their method does not depend at all on the assumption of a unique colour-temperature correspondence, but it uses the detailed surface brightness variation derived from multicolour photometry and model atmospheres. Also the "Barnes-Evans" version of the BW method (Barnes et al. 1977), which is based on a (V-R) vs. surface brightness calibration, gives a much steeper R-P relation; in a recent paper Gieren et al. (1984) find $\log R \propto 0.79 \log P$ from this method. It seems therefore that the R_{BW} radii can be improved to agree much better with R_{CL} and R_{TH}. The $R_{TH} - R_{CL}$ discrepancy is probably more difficult to solve, however, as is also shown in Fernie's discussion. Changes in the theoretical relations are not sufficient (opacities, chemical composition), or go the wrong way (mass loss), whereas the empirical calibrations of L and T_{eff} allow some shift in the zeropoint of the R_{CL} relation, but hardly in its slope.

It should be noted that in a recent paper by Burki (1984) it is also concluded that good agreement can be reached for R_{BW} and R_{TH}, but in a very different way. Burki shows that R_{BW} and R_{TH} agree satisfactorily for short-period Cepheids if one corrects R_{BW} for the effects of duplicity. The long-period R_{BW} values remain smaller than R_{TH}, but Burki argues that these small R_{BW} are in fact the correct radii, and that R_{TH} should be reduced by taking into account mass loss. Although this reconciles R_{BW} and R_{TH}, I am afraid that it leaves a large discrepancy of both R_{BW} and R_{TH} with respect to the R_{CL} values (not considered by Burki) at long periods.

The conclusion from all this can only be that full agreement between empirical and theoretical data for Cepheids has not yet been reached, and that the solution of all mass and radius problems will require improvements on both the observational and the theoretical side. The above discussion also emphasizes once again how important the Cepheids in clusters and associations are in this respect. The basic parameters of these stars should be determined with the highest accuracy possible. In the list of 27 cluster/association Cepheids of Fernie & McGonegal (op.cit.) there are still several cases for which more reliable data are needed, particularly among the long-period Cepheids in associations. The cluster distance scale can hopefully be improved significantly within a few years with the Hipparcos satellite, which should give the distances to Hyades, Pleiades, and a few other nearby clusters to better than 1 %. For a small number of nearby Cepheids direct parallax measurements of useful accuracy should even be possible. Of the various discrepancies mentioned, those related to the double-mode Cepheids are probably the most disturbing. This is primarily a challenge to the theoreticians, but observers could make an important contribution if double-mode Cepheids in the Magellanic Clouds could be found.

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At the end of this review, I realize that I have been talking mainly about uncertainties and discrepancies. This may have give the wrong impression that in the field of Cepheids we are just discovering problem after problem. If you would conclude that there has been no real progress, it is good to remember that after the discovery of the first Cepheids it took 130 years before it became clear that Cepheids pulsate, and 170 years before it was known why. It also took 170 years before Cepheid luminosities were known to better than a factor four. It is only fair that it takes a bit longer to solve the more subtle problems of these fascinating stars.

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REFERENCES

Balona, L.A. & Shobbrook, R.R. (1984). Mon. Not. R. astr. Soc. In press.
Barnes, T.G., Dominy, J.F., Evans, D.S., Kelton, P.W., Parsons, S.B. & Stover, R.J. (1977). Mon. Not. R. astr. Soc. 178, 661.
Becker, S.A., Iben, I. & Tuggle, R.S. (1977). Astrophys. J. 218, 633.
Bell, R.A. & Parsons, S.B. (1974). Mon. Not. R. astr. Soc. 169, 71.
Bell, R.A., Eriksson, K., Gustafsson, B. & Nordlund, A. (1976). Astron. Astrophys. Suppl. 23, 27.
Bell, R.A. & Gustafsson, B. (1978). Astron. Astrophys. Suppl. 34, 229.
Blackwell, D.E. & Shallis, M.J. (1977). Mon. Not. R. astr. Soc. 180, 177.
Böhm-Vitense, E. (1972). Astron. Astrophys. 17, 335.
Böhm-Vitense, E. (1981). Ann. Rev. Astron. Astrophys. 19, 295.
Burki, G. (1984). Astron. Astrophys. 133, 185.
Caldwell, J.A.R. (1983). Observatory 103, 244.
Caldwell, J.A.R. & Coulson, I.M. (1984a). SAAO Circ. In press.
Caldwell, J.A.R. & Coulson, I.M. (1984b). Mon. Not. R. astr. Soc. In press.
Cayrel de Strobel, G. (1980). In Star Clusters, IAU Symp. 85, ed. J.E. Hesser, p. 91, Reidel, Dordrecht.
Code, A.D., Davis, J., Bless, R.C. & Hanbury Brown, R. (1976). Astrophys. J. 203, 417.
Cogan, B.C. (1978). Astrophys. J. 221, 635.
Cox, A.N. (1980). Ann. Rev. Astron. Astrophys. 18, 15.
Dean, J.F., Warren, P.R. & Cousins, A.W.J. (1978). Mon. Not. R. astr. Soc. 183, 569.
Dean, J.F. (1981). Mon. Not. R. astr. Soc. 197, 779.
Eggen, O.J. (1980). Astrophys. J. 238, 919.
Eggen, O.J. (1983a). Astron. J. 88, 361.
Eggen, O.J. (1983b). Astron. J. 88, 1187.
Faulkner, D.J. (1977). Astrophys. J. 218, 209.
Feast, M.W. (1984). In Structure and Evolution of the Magellanic Clouds, IAU Symp. 108, eds. S. van den Bergh & K.S. de Boer, p. 157, Reidel, Dordrecht.
Feltz, K.A. & McNamara, D.H. (1980). Publ. astr. Soc. Pacif. 92, 609.
Fernie, J.D. & McGonegal, R. (1983). Astrophys. J. 275, 732.
Fernie, J.D. (1984). Astrophys. J. In press.
Flower, P.J. (1977). Astron. Astrophys. 54, 31.

Flower, P.J. (1980). In Star Clusters, IAU Symp. 85, ed. J.E. Hesser, p. 119, Reidel, Dordrecht.
Gehren, T. (1981). Astron. Astrophys. 100, 97.
Gieren, W., Coulson, I.M. & Caldwell, J.A.R. (1984). Preprint; cf. also this conference.
Gustafsson, B. & Bell, R.A. (1979). Astron. Astrophys. 74, 313.
Hanson, R.B. (1980). In Star Clusters, IAU Symp. 85, ed. J.E. Hesser, p.71, Reidel, Dordrecht.
Hejlesen, P.M. (1980). Astron. Astrophys. Suppl. 39, 347.
Iben, I. & Tuggle, R.S. (1972). Astrophys. J. 178, 441.
Iben, I. & Tuggle, R.S. (1975). Astrophys. J. 197, 39.
Johnson, H. (1966). Ann. Rev. Astron. Astrophys. 4, 193.
Jones, B.F. (1981). Astron. J. 86, 290.
Kraft, R.P. (1961). Astrophys. J. 134, 616.
Kurucz. R.L. (1979). Astrophys. J. Suppl. 40, 1.
Lub, J. & Pel, J.W. (1977). Astron. Astrophys. 54, 137.
Luck, R.E. (1977a). Astrophys. J. 212, 743.
Luck, R.E. (1977b). Astrophys. J. 218, 752.
Martin, W.L., Warren, P.R. & Feast, M.W. (1979). Mon. Not. R. astr. Soc. 188, 139.
Nissen, P.E. (1980). In Star Clusters, IAU Symp. 85, ed. J.E. Hesser, p.51, Reidel, Dordrecht.
Oke, J.B. (1961). Astrophys. J. 134, 214.
Parsons, S.B. (1971). Mon. Not. R. astr. Soc. 152, 121.
Parsons, S.B. (1974). In Stellar Instability and Evolution, IAU Symp. 59, eds. P. Ledoux, A. Noels, A.W. Rodgers, p.56, Reidel, Dordrecht.
Pel, J.W. & Lub, J. (1978). In The HR-Diagram, IAU Symp. 80, eds. A.G. Davis Philip & D.S. Hayes, p. 229, Reidel, Dordrecht.
Pel, J.W. (1978). Astron. Astrophys. 62, 75.
Pel, J.W. (1980). In Current Problems in Stellar Pulsation Instabilities, eds. D.Fischel, J.R.Lesh & W.M.Sparks, p.1, NASA Tech.Mem.80625.
Pel, J.W. (1984). In preparation.
Relyea, L.J. & Kurucz, R.L. (1978). Astrophys. J. Suppl. 37, 45.
Rodgers, A.W. (1970). Mon. Not. R. astr. Soc. 151, 133.
Sandage, A. & Tammann, G.A. (1969). Astrophys. J. 157, 683.
Schmidt, E.G. (1972). Astrophys. J. 174, 605.
Schmidt, E.G. (1980a). Space. Sci. Rev. 27, 449.
Schmidt, E.G. (1980b). Astron. J. 85, 158.
Schmidt, E.G. (1982). Astron. J. 87, 650.
Schmidt, E.G. (1983). Astron. J. 88, 104.
Sollazzo, C., Russo, G., Onnembo, A. & Caccin, B. (1981). Astron. Astrophys. 99, 66.
Turner, D.G. (1977). Astron. J. 82, 163.
Turner, D.G. (1978). J. R. astr. Soc. Canada, 72, 248.
Turner, D.G. (1979). Publ. astr. Soc. Pacif. 91, 642.
Turner, D.G. (1980). Astrophys. J. 235, 146.
Turner, D.G. (1983). J. R. astr. Soc. Canada. 77, 31.
VandenBerg, D.A. & Bridges, T.J. (1984). Astrophys. J. 278, 679.
Van Leeuwen, F. (1983). Thesis, Leiden University.
Van Paradijs, J.A. (1973). Astron. Astrophys. 23, 369.