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MATERIAL PROPERTY AND BOUNDARY CONDITION EFFECTS ON STRESSES IN AVALANCHE SNOW-PACKS

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Abstract. A linear elastic finite element computer program was applied to determine the stress distributions in multi-layered snow-packs typical of those found at Berthoud Pass, Colorado. The effect on stress distribution of wide variations in elastic material properties was examined. Also, an attempt was made to model the shear failure of a weak sub-layer in the snow-pack by relaxing the condition that the bottom snow layer be firmly attached to the ground.

Résumé. *Effets des propriétés mecaniques du materiau et des conditions aux limites sur les contraintes dans un manteau neigeux susceptible de produire des avalanches.* Un programme pour calculateur à élément fini linéaire et élastique a été appliqué à la détermination de la distribution des contraintes dans un manteau neigeux à plusieurs couches, typiques de ceux rencontrés à Berthoud Pass, Colorado. Les effets sur la distribution des efforts des grosses variations dans les propriétés élastiques du matériau, ont fait l'objet d'examen. Ainsi, un essai a été fait de simuler la rupture au cisaillement d'un niveau fragile sous-jacent dans le manteau neigeux en abandonnant la condition que le niveau de base de la neige soit fermement fixé au sol.

Zusammenfassung. *Auswirkungen von Materialeigenschaften und Randbedingungen auf die Spannungen in Lawinen-Schneedecken.* Mit Hilfe eines Rechenprogrammes für linear-elastische, finite Elemente wurden die Spannungsverteilungen in mehrschichtigen Schneedecken, wie sie typisch am Berthoud Pass, Colorado, gefunden werden, bestimmt. Die Auswirkung auf die Spannungsverteilung wurde für einen grossen Variationsbereich der elastischen Materialeigenschaften untersucht. Darüber hinaus wurde versucht, durch Abschwächen der Forderung, dass die unterste Schneeschicht fest mit dem Boden verhaftet sein soll, das Scherversagen einer schwachen Unterschicht in der Schneedecke darzustellen.

Introduction

This paper discusses the progress made in a joint research effort between the Rocky Mountain Forest and Range Experiment Station and Colorado State University to develop a mathematical model for the prediction of stresses and displacements in an avalanche snow-pack. In this work, the finite element method is being used to provide a flexible stress analysis tool which can easily accommodate the geometric complexities of a realistic snow-pack. At present the finite element model only accounts for elastic effects and therefore ignores the realistic effects of creep and plasticity. In spite of this limitation a considerable amount of insight to the stress distribution may be gained by such analysis, and to this end the following work is presented.

A set of numerical experiments were conducted on the finite element model. The purpose of these experiments was to predict the variations in stress distribution which result from varying both the material properties of the snow and the boundary conditions along the bottom of the snow-pack. This work is an extension of previous work by Smith (1972) in which the type of boundary condition along the bottom of the snow-pack was very restrictive, and in which the study of material property effects was incomplete.

In earlier studies the bottom layer of the snow-pack was assumed to be firmly attached to the ground. To simulate a weak sub-layer the modulus of elasticity of the bottom layer was taken to be an order of magnitude smaller than for the rest of the snow-pack. An attempt has been made in the present work to refine the modelling of the weak sub-layer condition by application of shear and normal stress perturbations to the bottom layer, an idea investigated by Perla and LaChapelle (1970). This is an attempt to model a condition in which a shear failure has occurred along the bottom of the snow-pack.

There has been considerable interest in the orientation of the principal axes of stress, particularly in the region of the fracture line. This paper also presents information regarding the influence of varying material properties and boundary conditions on the principal stress directions.

Problem statement and analysis technique

The analytical model was applied to a multi-layered snow-pack (Fig. 1) found on Berthoud Pass, Colorado, U.S.A. This snow-pack was measured and observed during the winter of 1967–68 by members of the Rocky Mountain Forest and Range Experiment Station. A natural avalanche did take place on this location in December of 1967 with the observed fracture line noted in Figure 1.

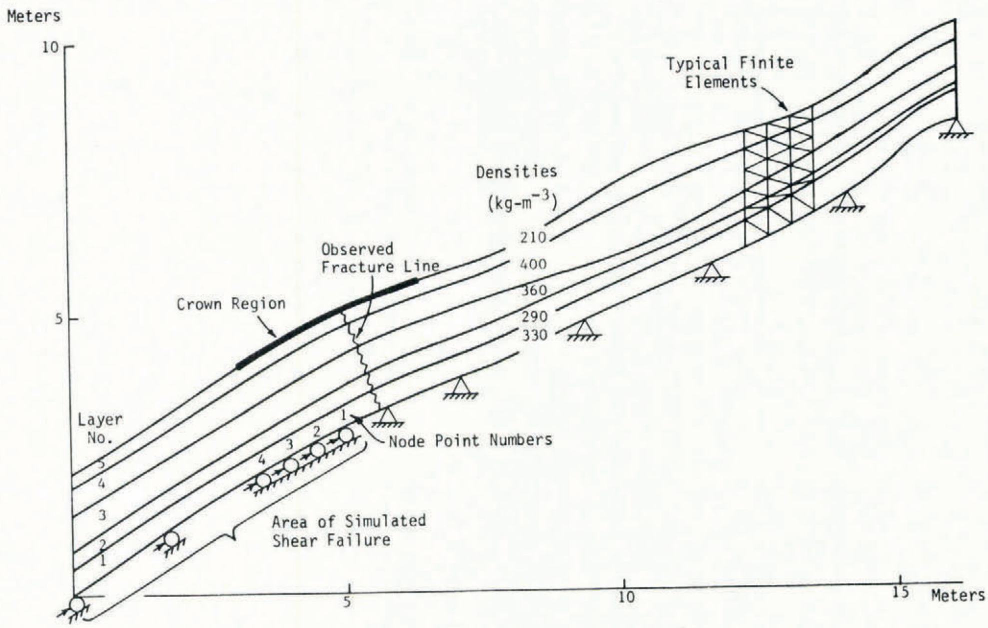

Fig. 1. Schematic of a realistic snow-pack.

The linear elastic finite element computer program, utilizing triangular constant strain elements, was applied with the following assumptions:

1. Plane strain conditions exist in the snow-pack.
2. The top surface is stress free.
3. The snow in each layer is isotropic.
4. A constant value of Young's modulus and Poisson's ratio exists throughout the snow-pack.

A realistic snow-pack such as the one shown in Figure 1 is much longer than the figure indicates. In order to minimize possible errors due to truncating the snow-pack, the approximate stress distribution due to Mellor (1968) was applied to the left- and right-hand ends.

For part of the studies, a shear failure along the bottom layer of snow was assumed to exist over the area shown in Figure 1, and was simulated by the following procedure. In the finite element method, as applied here, the snow-pack is divided into triangular sub-regions with nodes at the vertices of each triangle. Once the numerical solution to a particular problem is complete, the stress and strain throughout each element as well as the net force and displacement at each node is known. To model the shear failure a computer run was first made in which the nodes along the bottom of the snow-pack were assumed to be rigidly attached to the slope. This is called the "base-line" condition. The forces acting on the bottom nodes were then isolated for special study. The tangential components of these forces were reduced, and a new computer run was made to simulate a shear perturbation which might occur along the bottom of a snow-pack due to the presence of a shear failure in a weak sub-layer. For the present analysis, the forces were reduced in the following way. Node point 1, Figure 1, had 100% of the base-line force applied to it. Node point 2 had 90%, node point 3 had 80%, while node point 4 and the remaining bottom nodes had 70% of the "base-line" force applied.

This pattern of reduced forces is only one of many possible combinations which might be tried. The attempt here has been to determine whether this approach to modelling the weak sub-layer has any validity at all in terms of the type of stress distribution produced compared to that expected based on field experience.

Results

Computer runs were made to predict the stress distribution within the snow-pack for several combinations of values of the material properties (Young's modulus E and Poisson's ratio ν). Analyses were performed for the "base-line" boundary conditions and for conditions of a simulated shear failure. Table I summarizes the cases which were run.

Table I. Combinations of material properties used in computer analyses

Young's modulus	*Base-line*			*Shear failure in a weak sub-layer*		
	Poisson's ratio			*Poisson's ratio*		
N m^{-2}	0.0	0.25	0.45	0.0	0.25	0.45
2×10^7	X	X	X	X	X	X
2×10^8		X				
2×10^9		X				

Effects of material properties

Varying Young's modulus, E, had no effect on the stress distribution as long as the value of E was held constant throughout the snow-pack. This is to be expected since the thickness of the five-layered snow-pack model was fairly constant everywhere. However, recent studies have indicated that varying Young's modulus within a variable-thickness layer of snow does affect stress distribution (Curtis, unpublished).

The effect of Poisson's ratio on stresses in the snow-pack is shown in Figures 2 through 4 where attention has been drawn to the crown region shown in Figure 1. In these figures, the top graph presents the variation of the principal and maximum shear stresses with position along the top layer. The center diagram shows line segments which are aligned with the direction of maximum principal stress. The lower graph presents the variation of the principal and maximum shear stresses with position along the bottom layer.

Two trends should be noted in these figures. First, values of principal stresses in the bottom layer become more compressive with increasing Poisson's ratio. The second trend to note is that there is a re-orientation of the principal elements in the critical region of the snow-pack. An examination of the principal element directions in the lower layer of snow shows that as Poisson's ratio increases, the principal directions rotate in a clockwise manner. (Haefeli (1963) has discussed rotation of principal directions due to settling of snow in the neutral zone. This is not the situation being examined in this paper.) The rotation is so great that for a Poisson's ratio of 0.45 the principal elements all lie at an angle of nearly 45° with respect to the slope of the layer below the observed fracture line. Remembering that the plane of maximum shear is oriented at an angle of 45° to the direction of maximum principal stress, it may be concluded that if a failure were to occur in the bottom layer of snow, it could occur as a shear failure. This result is interesting in light of a snow-pack failure theory in which it is presumed that avalanche release initiates as a shear failure in a lower layer below the fracture line producing a tensile failure at the fracture line (Quervain, 1966, p. 16).

To carry this point further one should compare the stress distribution resulting from the "base-line" runs with experimentally derived failure envelopes. Let us examine the maximum shear values found in lower layers below the observed fracture line for the $\nu = 0.45$ case.

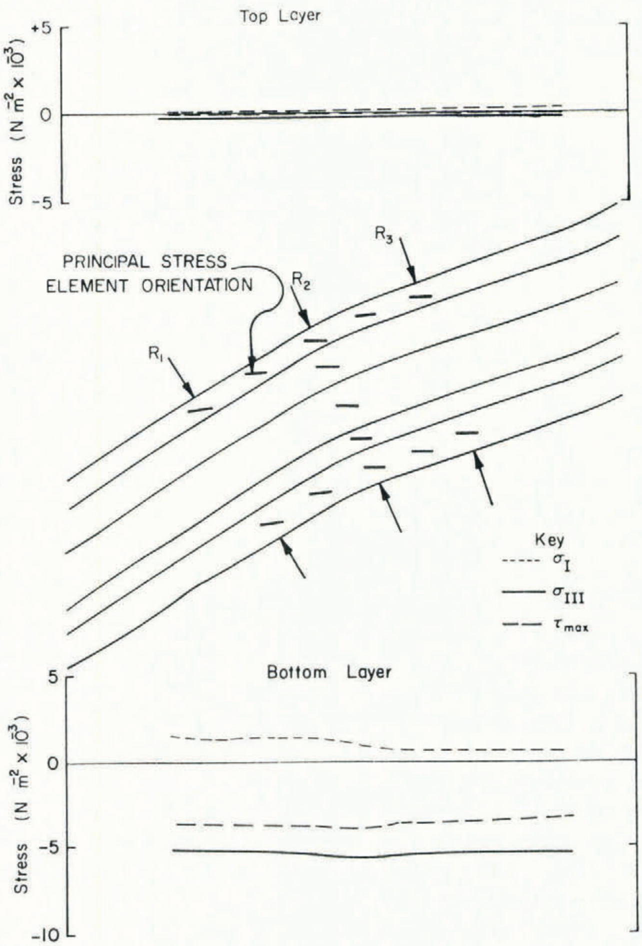

Fig. 2. Principal stress magnitudes and directions in the crown region, base-line, ν = 0.0c.

Figure 5 shows that maximum shear stress values for layers 1 and 2 do lie at the lower boundary of the shear failure envelope as constructed by Mellor (1966), while shear values for the other layers do not. Thus it is conceivable that shear failure could take place in either of these two layers. Field observations after the avalanche had released revealed that a shear failure did take place between layers 1 and 2.

One important difference between the shear stresses in these two layers is that the directions of maximum shear are different. Maximum shear in the bottom layer is oriented parallel to

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the hill's slope below the fracture line. This is not the case for layer 2. Therefore, one would expect that if a shear failure was to occur in the bottom layer, it would be in a direction parallel to the ground surface, while in layer 2 it would not.

Figure 6 presents the "base-line" run for Poisson's ratio of 0.25 plotted in a different way to illustrate the variation of stress through the snow-pack in a direction perpendicular to the snow surface. A step-wise linear variation in the stresses is noted. This is due to the fact that the stresses are heavily controlled by the snow density. Since the density of each layer is different, the slope of the stress variation in that layer is expected to be different.

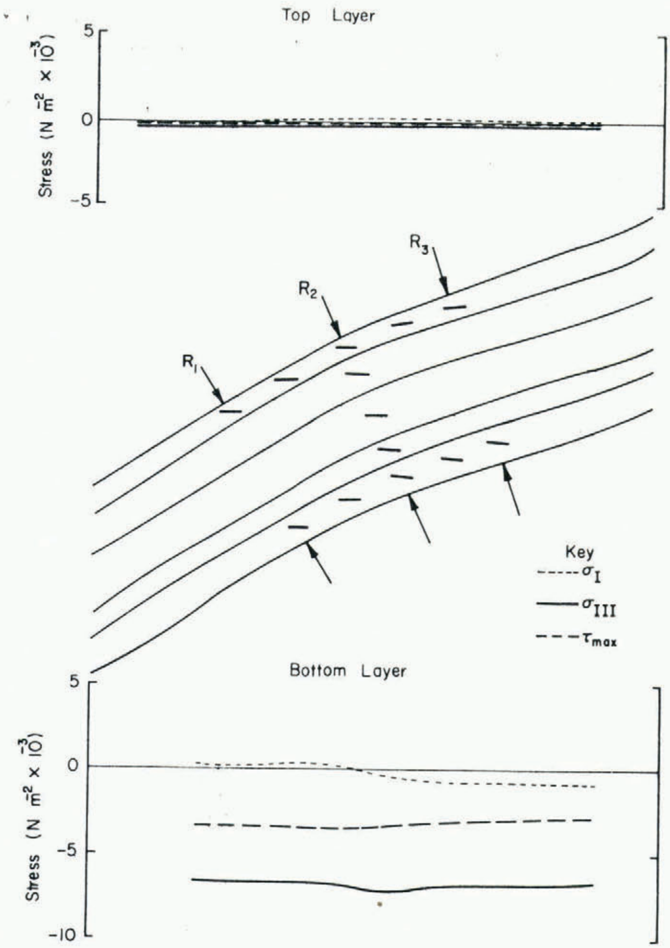

Fig. 3. Principal stress magnitudes and directions in the crown region, base line, ν = 0.25.

Effects of a shear failure in a weak sub-layer

Figures 7 and 8 graphically display the results of reducing tangential loads at the base of the snow-pack to model the shear failure. There are two primary effects to be noted. The first and most obvious effect produced by the weakening of the sub-layer is to create a stress concentration just below the observed fracture line. Comparison of Figures 3 and 7 for a typical value of Poisson's ratio clearly demonstrates how the stresses change in the snow-pack as a result of weakening the bottom layer. Along the top layer, the state of stress changes from one of near zero stress to one in which the maximum principal stress is fairly large and tensile. One also finds that the principal element in the reduced load case is directed parallel to the top surface. Both observations tend to support the theory of avalanche release that points to a shear failure in the snow-pack followed by a tensile failure near the surface as the release mechanism.

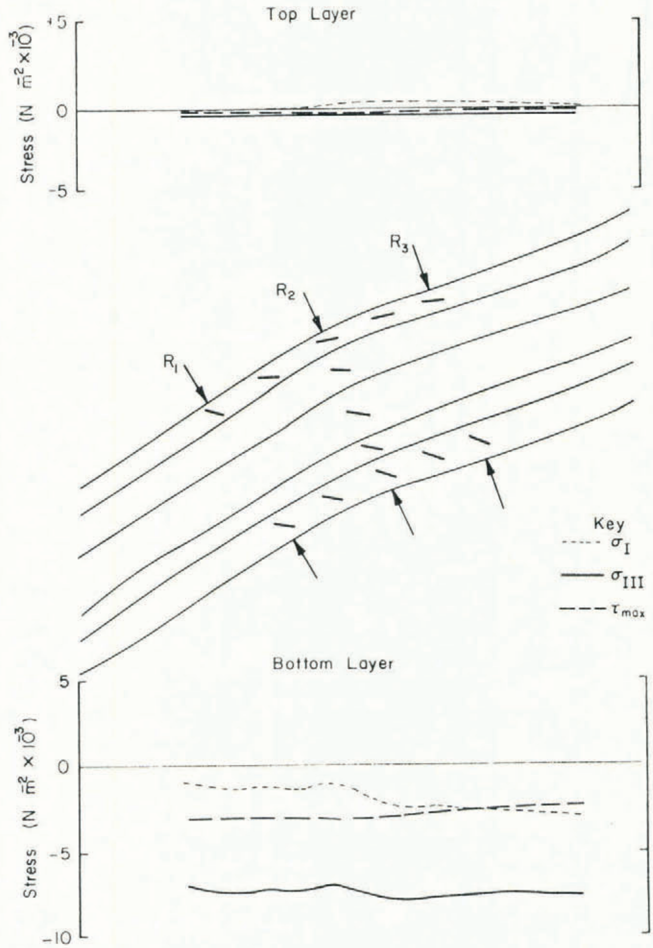

Fig. 4. Principal stress magnitudes and directions in the crown region, base-line, ν 0.45.

Changing Poisson's ratio affected the reduced load stress distribution in nearly the same way that it did for base-line runs. Without referring to additional figures, the analysis showed that increasing Poisson's ratio had little effect on principal stress magnitudes for the shear failure calculations, but that there was a re-orientation of the principal elements. Increasing Poisson's ratio caused a small clockwise rotation of the principal elements in the bottom layer, but had no effect on principal stress directions in the top layer.

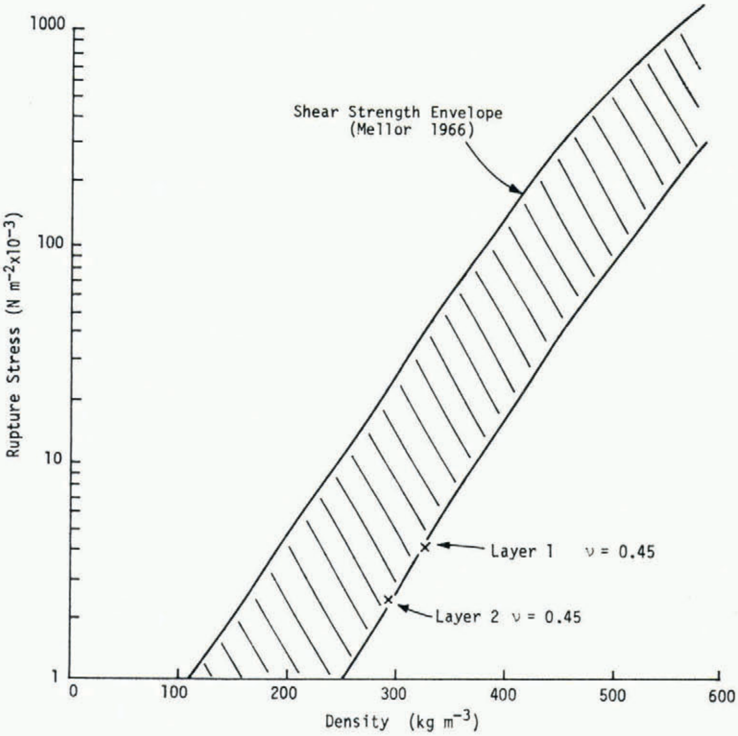

Fig. 5. Shear strength of snow.

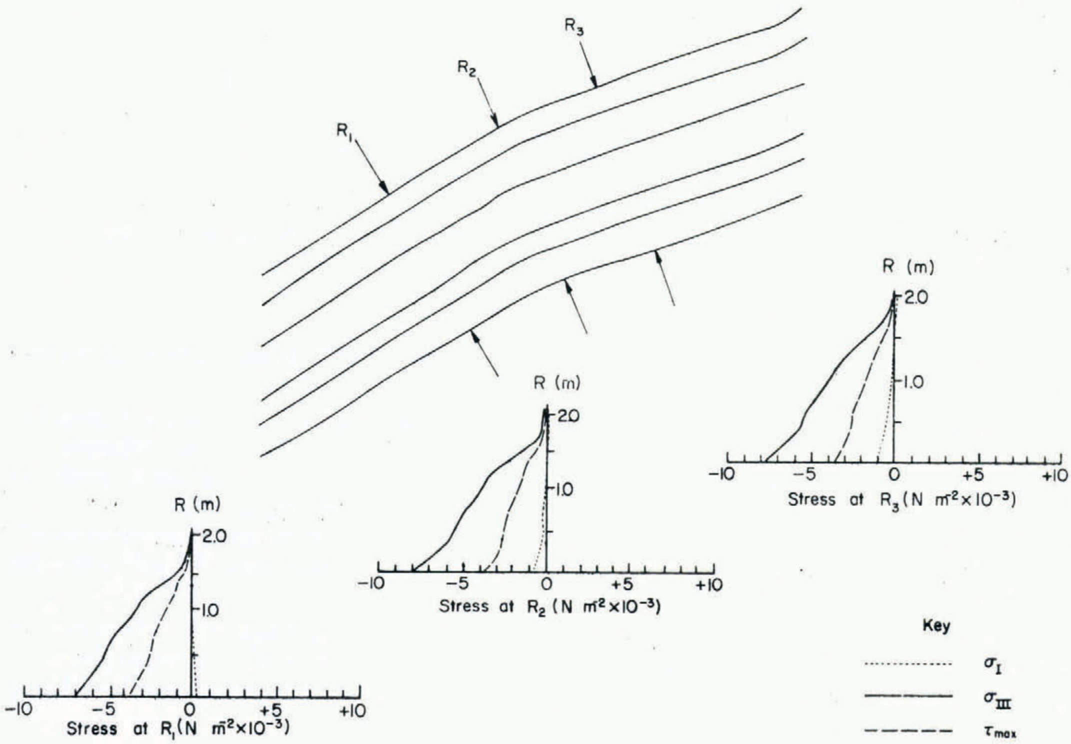

Fig. 6. Principal stresses in the crown region, base-line, ν = 0.25.

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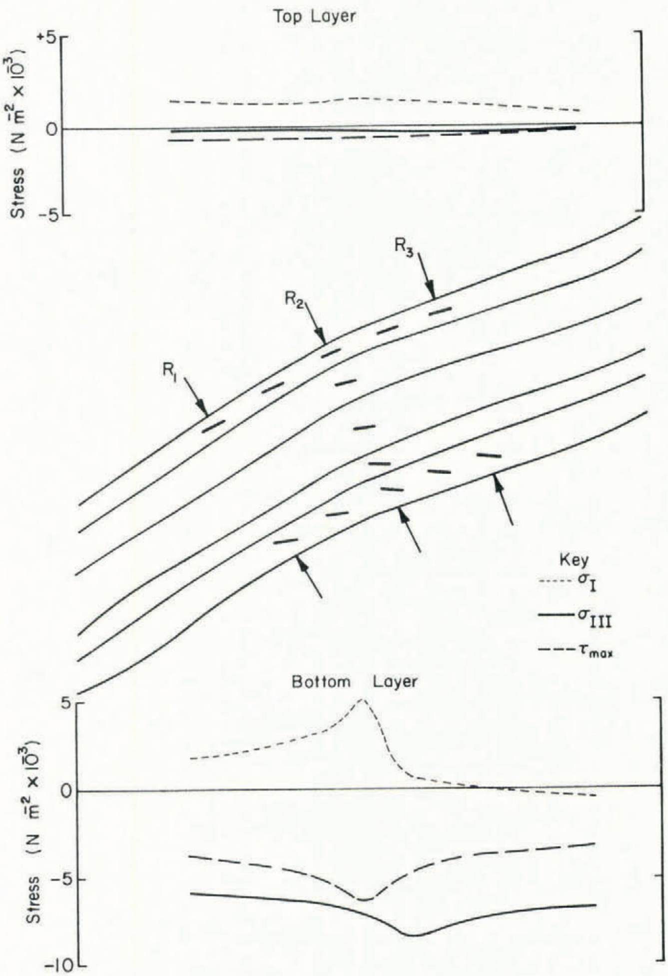

Fig. 7. Principal stress magnitudes and directions in the crown region, simulated shear failure, $\nu = 0.25$.

Summary and conclusions

Conclusions that can be drawn with respect to the effect of varying material properties are:

1. Varying a Young's modulus which is uniform throughout the snow-pack does not influence the stress distribution in a snow-pack having layers of uniform thickness.
2. Increasing Poisson's ratio makes the principal stress values more compressive, while for the most part rotating the principal elements in a clockwise direction.
3. Layers of variable density snow influence the variation of stresses as a function of depth. Density will play an even greater part when Young's modulus and Poisson's ratio are made functions of this parameter.

The above analysis was not an attempt to verify any particular avalanche failure theory; however, two conclusions can be drawn that tend to substantiate field observations of this and other avalanche snow-packs. First, the base-line runs revealed that a shear failure could have

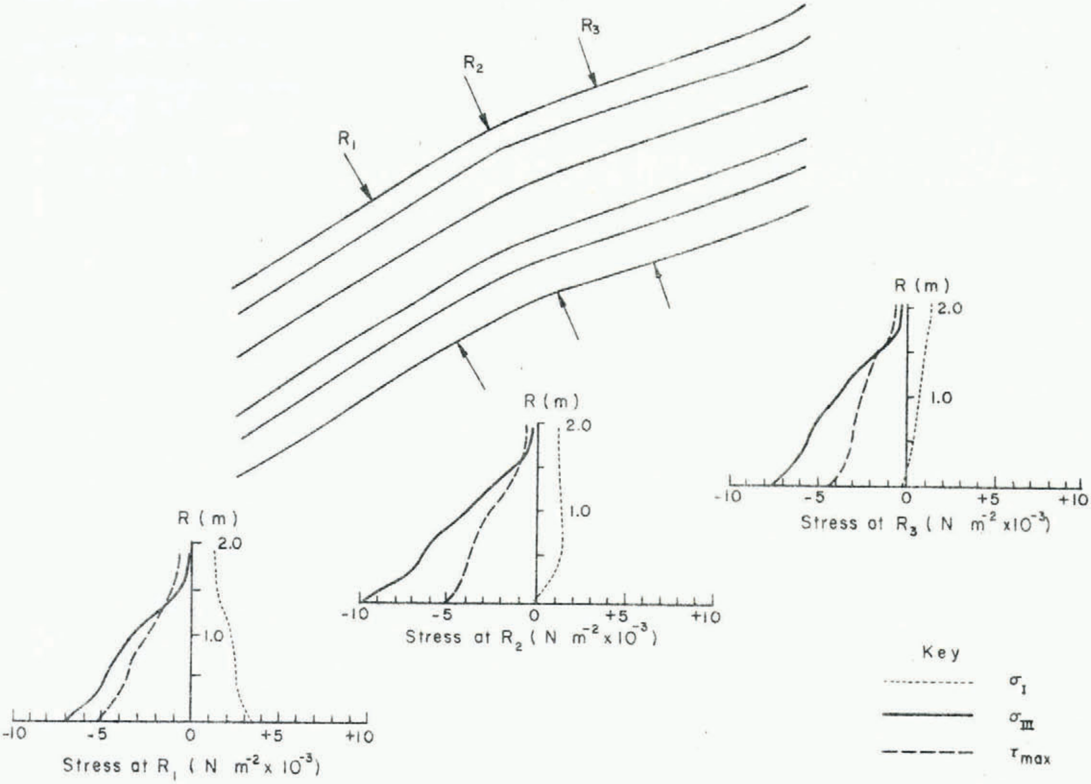

Fig. 8. Principal stresses in the crown region, simulated shear failure, $\nu = 0.25$.

taken place in the bottom layer of snow below the observed fracture line. Post-release investigations of the actual snow-pack revealed that the shear plane was between the first and second layers. Another important result of the analysis is that through shear failure simulation, large tensile stresses parallel to the snow surface can be generated within the top layer of snow. This condition is consistent with the field observation that the crown fracture surface was perpendicular to the slope.

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REFERENCES

Curtis, J. O. Unpublished. Linear elastic analysis of dry slab avalanche release mechanisms. [M.S. thesis, Colorado State University, 1973.]

Haefeli, R. 1963. Stress transformations, tensile strengths and rupture processes of the snow cover. (*In* Kingery, W. D., *ed. Ice and snow; properties, processes, and applications: proceedings of a conference held at the Massachusetts Institute of Technology, February 12–16, 1962.* Cambridge, Mass., M.I.T. Press, p. 560–75.)

Mellor, M. 1966. Snow mechanics. *Applied Mechanics Review*, Vol. 19, No. 5, p. 379–89.

Mellor, M. 1968. Avalanches. U.S. Cold Regions Research and Engineering Laboratory. *Cold regions science and engineering*. Hanover, N.H., Pt. III, Sect. A3d.

Perla, R. I., *and* LaChapelle, E. R. 1970. A theory of snow slab failure. *Journal of Geophysical Research*, Vol. 75, No. 36, p. 7619–27.

Quervain, M. R. de. 1966. Problems of avalanche research. *Union de Géodésie et Géophysique Internationale. Association Internationale d'Hydrologie Scientifique. Commission pour la Neige et la Glace. Division Neige Saisonnière et Avalanches. Symposium international sur les aspects scientifiques des avalanches de neige, 5–10 avril 1965, Davos, Suisse*, p. 15–22.

Smith, F. W. 1972. Elastic stresses in layered snow packs. *Journal of Glaciology*, Vol. 11, No. 63, p. 407–14.