# LABORATORY WORK ON THE SHAPES OF ASTEROIDS 

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Photometric lightcurves of about 50 asteroids have been obtained over the past 20 yr , yet very little is known about the shape of these objects. Perhaps 100 lightcurves (including photographic ones) of 433 Eros have been obtained with amplitudes up to 1.5 mag. Some authors ${ }^{1}$ have attempted to calculate the dimensions of Eros assuming it to be a three-axis ellipsoid. The most recent determination ( $35 \mathrm{~km}, 16 \mathrm{~km}, 7 \mathrm{~km}$ ) was given by Roach and Stoddard in 1938. In the case of 624 Hektor, amplitudes up to 1.1 mag were observed on lightcurves in which the primary and secondary maxima differed by less than 0.04 mag. Van Houten (1963) noted that for lightcurve amplitudes greater than 0.2 mag , the two maxima were about the same level and differed by 0.04 mag on the average. This is an indication of the small effect of reflectivity differences between the opposite sides. Assuming then that the light variation of Hektor is due almost entirely to shape, Dunlap and Gehrels (1969) used a cylindrical model with rounded ends to calculate a length of 110 km and a diameter of 40 km . The nearly constant ( $\pm 0.02 \mathrm{mag}$ ) absolute magnitude of the maxima ruled out a third axis being significantly different from the second. More recent lightcurves of 1620 Geographos have been obtained with amplitudes up to 2.0 mag (Dunlap and Gehrels, 1971). If all of this variation is caused by shape, Geographos might be nearly six times longer than wide! However, the 0.1 mag difference between maxima (and an even larger difference in the minima) suggests a possible reflectivity effect that appears to reduce the length to-width ratio to about 4 . It was decided to make a laboratory investigation of the lightcurves of models to clarify our understanding of light variations caused by shape and perhaps enable us to find a particular shape that would reproduce the observations of Geographos. The work is still in progress, but we already have obtained some interesting results.

## PRODUCTION OF MODEL LIGHTCURVES

Figure 1 illustrates some of the first of 12 models that have been observed. Each model was made with a Styrofoam center covered with a thin layer of Plasticene and finally dusted with powdered rock. The model was turned about

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Figure 1.-A sample of some early models.
its shortest body axis by a stepping motor, and integrations were usually made every $3^{\circ}$ (or $5^{\circ}$ ) over $240^{\circ}$ (or $360^{\circ}$ ) of rotation using a photometer as used at the telescopes (Coyne and Gehrels, 1967).

Figure 2 defines the geometry of the observations. The model's rotation axis can be oriented in space around two perpendicular directions. One is the line of sight, a rotation about which causes a change in asterocentric obliquity. ${ }^{2}$ The


Figure 2.-The geometry of the laboratory observations. (Note that obliquity as defined here is not the same as the obliquity defined in the glossary.)

[^1]other direction is perpendicular to the line of sight at the model's center, about which a rotation causes a change in aspect (the angle between the rotation axis and the line of sight). The light source can be moved horizontally to change the phase. For each of the models, up to 27 lightcurves were produced by varying the aspect $\left(90^{\circ}, 60^{\circ}, 35^{\circ}\right)$, the obliquity $\left(90^{\circ}, 50^{\circ}, 15^{\circ}\right)$ and the phase $\left(20^{\circ}\right.$, $40^{\circ}, 60^{\circ}$ ). The average probable error estimate of all angle measurements is $\pm 1^{\circ}$.

Figure 3 illustrates all the lightcurves obtained from a smooth-surfaced, long, cylindrical model with rounded ends. One end and part of one side were artificially darkened with graphite powder to produce the apparent reflectivity differences between primary and secondary features seen in the Geographos lightcurves (Dunlap and Gehrels, 1971). Ignoring these differences, the lightcurves illustrate in general the effects of changing the aspect, phase, and obliquity. Several characteristics of the lightcurves can be identified that are used later in making comparisons of models:
(1) Amplitude: the height of the curve from minimum to maximum (The estimated probable error of the amplitudes is $\pm 0.01$ mag.)
(2) Shape of minima: sharp, flat, and/or asymmetric
(3) Width of minima at half amplitude


Figure 3.-Model lightcurves in a 3 by 3 by 3 matrix. The model was a cylinder with hemispherical ends having one end darkened and an overall length-to-width ratio of about 4. The lightcurve at $90^{\circ}$ obliquity, $90^{\circ}$ aspect, and $20^{\circ}$ phase (top left) has an amplitude of 2.12 mag.
(4) Time shifts of maxima or minima relative to the observation at $90^{\circ}$ aspect, $90^{\circ}$ obliquity, $20^{\circ}$ phase
(5) Lightcurve inversions: maxima become minima and vice versa (time shift is $90^{\circ}$ )
(6) Primary and secondary maxima and minima

Looking horizontally from left to right in figure 3, one sees the changes produced by decreasing the aspect; most noticeable is the decrease in amplitude and the time shifts (leading to two lightcurve inversions and two partial inversions at the top right of the figure). The inversions are understood roughly as occurring when the illuminated part of the "true" maxima has a smaller area (as seen by the detector) than the illuminated part of the "true" minima. Looking vertically, one sees sometimes a noticeable change in amplitude with obliquity and sometimes changes in asymmetry. Looking diagonally (in groups of three), one sees the changes due to phase-usually small changes in amplitude with some asymmetries and time shifts.

## AMPLITUDE-ASPECT RELATIONS

Figure 4 is the set of nine amplitude-aspect curves for the lightcurves from figure 3 (using secondary amplitudes to avoid reflectivity effects). The turnup in the curves at $90^{\circ}$ obliquity and $40^{\circ}$ and $60^{\circ}$ phase is associated with


Figure 4.-Amplitude-aspect curves obtained from the secondary amplitudes of figure 3.
lightcurve inversions. (See fig. 3). Curves for the other models are similar but not exactly the same as these. It is clear, however, that there is no unique amplitude-aspect function for this or any of the models studied. Therefore, it is not possible, in general, to determine a rotation axis precisely by using a single amplitude-aspect function. Of course, approximations can be made; and they may be better if the phase angles are always small. However, the amplitudeaspect function is model dependent in an as-yet-unknown way.

## COMPARISONS OF MODELS

Table I is a brief summary of the results of five comparisons of models. To see how differences in the shape affect the observed light variation, each model was compared with one having a different shape; finally, the lightcurves made at the same orientations were examined for differences in the characteristics described earlier. The changes in the light variation usually depend not only on the shape of the model, but also on aspect, obliquity, and phase. We cannot, for example, look at a single asteroid lightcurve and deduce the shape of the asteroid. Therefore, before comparisons can be made with actual observations, the orientation of the rotation axis in space must be known precisely $\left(\sim \pm 1^{\circ}\right)$. Probably the weakest point in our present method for obtaining the rotation axis (see Taylor ${ }^{3}$ ) is in accounting for differential time shifts in the maxima (or minima) that depend on aspect, obliquity, and phase. It may be possible to utilize the time shifts from the models to improve our determination of rotation axes. We notice also in table I that the presence of a third body axis is clearly evident in the change in brightness of the maxima as the aspect changes.

## COMPARISON WITH TELESCOPIC OBSERVATIONS

Figure 5 shows the August 31, 1969, lightcurve of Geographos and also the average of the two model lightcurves from figure 3 that are closest to the calculated orientation of Geographos if its pole is at $\lambda_{0}=113^{\circ}, \beta_{0}=84^{\circ}$.

In the laboratory we were modeling direct rotation, but Geographos' rotation is retrograde. Making the necessary corrections to the lightcurve and moving the dark side to again follow the dark end, the model will reproduce the asymmetries in the minima; but the difference in the widths of the minima is still unexplained. We are currently developing a computer method of using the model data along with observed amplitudes to determine a pole, but results are not yet available. Other asteroids with large amplitudes (433 Eros, 624 Hektor, and possibly 15 Eunomia, 39 Laetitia, and 44 Nysa) might also be used for comparisons.

## CONCLUSION

The extreme smoothness ( $\leqslant 0.004 \mathrm{mag}$ ) of all the model lightcurves is not usually seen in asteroid lightcurves, although asteroids with large light

[^2]TABLE I.-Comparisons of Models

| Model compared with reference ${ }^{\text {a }}$ | Observed changes in light variation relative to the reference model ${ }^{\mathbf{b}}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Amplitude | Shape of minima | Time shifts | Other |
| 1. Same shape; crosssectional area ratio 2.3 times larger | Larger by up to a factor of 2 | Narrower, sharper, more asymmetry | Some larger shifts in maxima | Lightcurve inversion is possible at smaller phase angle. |
| 2. Two tangent spheres of equal radii; 5 percent larger area ratio | Usually smaller | Narrower, sharper | Some smaller shifts in maxima, more in minima |  |
| 3. Both ends pointed (conical); $\sim 5$ percent larger area ratio | Somewhat larger; amplitude-aspect curves have less or opposite curvature | Complicated changes with less asymmetry | No significant change | Occasionally the minima are wedge-shaped. |
| 4. One end pointed (conical); some brighter spots; 5 percent smaller area ratio | Primary and secondary amplitudes larger, except at $90^{\circ}$ aspect | Wider minima with one resembling the reference and the other those of model 3 (above) | Some smaller shifts in maxima and minima | Primary and secondary minima are frequently interchanged. |
| 5. Elongation along a second body axis 0.7 times the major axis; 3 percent larger area ratio at $90^{\circ}$ aspect | Smaller, except at $90^{\circ}$ aspect | Somewhat wider | Usually larger shifts in maxima and minima | Increase in brightness of the maxima as aspect decreases (up to 0.32 mag at $35^{\circ}$ aspect). |

${ }^{\mathrm{a}}$ The reference model is a cylinder with hemispherical ends and a cross-sectional area ratio $A_{\max } / A_{\min }$ of 1.9 .
${ }^{\mathrm{b}}$ These changes usually depend on the aspect, phase, and obliquity and therefore do not characterize all the lightcurves.


Figure 5.-Comparison of model to telescopic observations: - Geographos, August 31, 1969; o model whose lightcurves are in figure 3.
variations appear to have somewhat smoother lightcurves. Two models with rougher surfaces (about 5 and 20 percent deviations from an average dimension) were also observed. The roughest model produced the only deviation: a smooth tertiary hump seen in certain orientations near the minimum of light. A few lightcurves were also made using a very irregularly shaped and somewhat porous rock, and they show several small features with some indication of deviations from smoothness. More work needs to be done in modeling the surface texture, which is apparently the major source of small features in the lightcurves.

There is no unique amplitude-aspect function for any of the models studied, and this method should not be used for precise determinations of rotation axes. The differences in the amplitude-aspect function as well as in other characteristics of the lightcurves are partly due to the shape, but the orientation of the rotation axis must be precisely known-by photometric astrometry (see Taylor ${ }^{4}$ )-before the shape of an asteroid may be confidently determined.

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## DISCUSSION

BANDERMANN: Is there any obvious reason why the $|\Delta m|$ of successive minima in the lightcurves is usually larger than the $|\Delta m|$ of successive maxima?

DUNLAP: Perhaps small differences in surface reflectivity are relatively more important at low than at high levels of brightness.

ALFVEN: I think laboratory work of this kind is very important. It is so healthy to see in the laboratory what is correct in theory and how many different solutions we can have. The theoretical models that are used always imply a number of assumptions that may not be applicable in nature.


[^0]:    ${ }^{1}$ See p. 133.

[^1]:    ${ }^{2}$ Throughout this paper, obliquity refers to the dihedral angle between the plane determined by the line of sight and the axis of rotation, and by the plane perpendicular to the scattering plane and containing the line of sight. (See fig. 2.)

[^2]:    ${ }^{3}$ See p. 121.

[^3]:    ${ }^{4}$ See p. 128.

