PARALLEL MACHINE SCHEDULING WITH JOB DELIVERY COORDINATION

J. M. DONG\(^1\), X. S. WANG\(^2\), L. L. WANG\(^3\) and J. L. HU\(^4\)

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Abstract

We analyse a parallel (identical) machine scheduling problem with job delivery to a single customer. For this problem, each job needs to be processed on \(m\) parallel machines non-pre-emptively and then transported to a customer by one vehicle with a limited physical capacity. The optimization goal is to minimize the makespan, the time at which all the jobs are processed and delivered and the vehicle returns to the machine. We present an approximation algorithm with a tight worst-case performance ratio of \(7/3 - 1/m\) for the general case, \(m \geq 3\).

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1. Introduction

Coordinated production–transportation scheduling problems have received much attention from both industry and academic researchers. In this paper, we consider the scheduling problem in a parallel machine environment, in which jobs finished on the parallel machines need to be delivered to a customer by a single vehicle with a limited load capacity. And, the vehicle has to return to the machine for another shipment after it delivers a shipment to the customer. The goal is to minimize the makespan, that is, the time at which all the jobs are processed and delivered to the customer and the vehicle returns to the machine.

Our target scheduling problem is formally described as follows. We are given \(m\) parallel (identical) machines and a set of jobs \(\mathcal{J} = \{J_1, J_2, \ldots, J_n\}\) to be processed on the machines. Each job \(J_j\) has a non-pre-emptive processing time \(p_j\) on the machine...
and a physical size $s_j$. The finished jobs are to be delivered to a customer by a single vehicle and they can be transported by the vehicle in batches. The size $s_j$ of the job $J_j$ represents its fractional space requirement on the vehicle and a batch is valid only if the total size of the jobs therein does not exceed 1. The vehicle takes a constant time $T$ for a round trip between the machine and the customer. The objective is to minimize the makespan $C_{\text{max}}$. This problem can be denoted as $\text{(Pm} \rightarrow \text{D}, k = 1 | v = 1, c = 1 | C_{\text{max}})$, where “Pm $\rightarrow$ D, $k = 1$” states that the jobs are first processed on $m$ parallel machines and then delivered to one customer; “$v = 1, c = 1$” means that only a vehicle of capacity 1 is employed to deliver the jobs; the last term states the objective being the makespan $C_{\text{max}}$. As the problem is NP-hard, we focus on polynomial time approximation algorithms. We use the worst-case performance ratio to estimate such algorithms. In general, for an instance $I$ of a minimization NP-hard problem, let $C^H(I)$ and $C^*(I)$ denote, respectively, the objective value generated by an approximation algorithm $H$ and the optimal value. The worst-case ratio of $H$ is defined by the largest ratio $C^H(I)/C^*(I)$ for any $I$. If an algorithm has a worst-case ratio of $\rho$, then we call it the $\rho$-approximation.

When $m = 1$, that is, when jobs are first processed on a single machine, Chang and Lee [1] showed that the problem is NP-hard and presented a $5/3$-approximation algorithm. He et al. [4] presented an improved $53/35$-approximation algorithm, while Zhong et al. [8] presented an improved $(3/2 + \epsilon)$-approximation algorithm. Lu and Yuan [5] designed a best possible $3/2$-approximation algorithm. When $m = 2$, that is, when jobs are first processed on two parallel (identical) machines, Chang and Lee [1] also presented an approximation algorithm with worst-case performance ratio 2; Zhong et al. [8] presented an improved $5/3$-approximation algorithm. Later, Su et al. [7] presented an improved $8/5$-approximation algorithm.

In this paper, we study the target problem $\text{(Pm} \rightarrow \text{D}, k = 1 | v = 1, c = 1 | C_{\text{max}})$. We present an approximation algorithm for the general case when $m \geq 3$. The algorithm has a tight worst-case performance ratio of $7/3 - 1/m$. Note that the target problem in the general case when $m \geq 3$ has not been studied formally in the literature, and it cannot be easily solved by applying the similar methods of dealing with the special cases, $m \leq 2$. Thus, this $(7/3 - 1/m)$-approximation algorithm is the first of such results on this type of approximation algorithm.

Since the bin-packing problem is an important component problem inside the target problem during the job delivery stage, to reduce the number of shipments, we will employ the classical algorithm first-fit decreasing (FFD) for the bin-packing problem. In the algorithm FFD, the jobs are sorted into a nonincreasing order of the size, and in this order a job is placed into the lowest indexed batch to which it fits, or otherwise a new batch is created for the job. We use $b'$ to denote the minimum number of batches required in the bin-packing instance associated with the target problem; and let $b^\text{FFD}$ denote the number of batches in the bin-packing solution generated by the algorithm FFD. Let $b^*$ denote the number of batches in an optimal schedule to the target problem. The algorithm FFD has a tight absolute worst-case performance guarantee of $b^\text{FFD} \leq 3b'/2$ [6] and a tight asymptotic worst-case performance guarantee of
Clearly, $b' \leq b^\ast$. We conclude that
\[
b_{FFD} \leq \min\{\frac{3}{2}b^\ast, \frac{11}{9}b^\ast + \frac{6}{9}\}.
\] (1.1)

2. Approximation algorithm and proof of the worst-case performance ratio

We show in this section that the target problem ($Pm \rightarrow D, k = 1 | v = 1, c = 1 | C_{\text{max}}$), in the general case of $m \geq 3$, admits a $(7/3 - 1/m)$-approximation algorithm, which may be described as follows.

Algorithm 1 $H_1$

Step 1. Use FFD to pack the jobs of $J$ by their sizes into batches.
1.1. Let $k_1$ be the total number of resulting batches, denoted as $B_1, B_2, \ldots, B_{k_1}$.
1.2. Calculate the sum of processing times for jobs in $B_i$ and denote it by $P_i$ for $i = 1, 2, \ldots, k_1$.
1.3. Re-index the batches such that $P_1 \leq P_2 \leq \cdots \leq P_{k_1}$.

Step 2. Sort the jobs within each batch into a nonincreasing order of the processing time.
2.1. Starting with $B_1$, assign each job in $B_i$ to a machine with the least amount of work so far (list scheduling rule [3]) for $i = 1, 2, \ldots, k_1$.

Step 3. Deliver a finished batch as early as possible. If multiple batches have finished when the vehicle becomes available, deliver the batch with the smallest index.

We denote this approximation algorithm as $H_1$. The achieved schedule by the algorithm is denoted as $\pi_1 = \langle B_1, B_2, \ldots, B_{k_1} \rangle$, which is a sequence of job batches. Note that only after all of the jobs of a batch are finished can the batch be said to be finished and ready for transportation to its customer. The makespan of the schedule $\pi_1$ is denoted as $C_{\pi_1}^\ast$. Let $\rho_i$ denote the latest completion time on the machine of the jobs assigned to $B_i$. Thus, by $H_1$,
\[
C_{\pi_1}^\ast = \max_{1 \leq i \leq k_1} \{\rho_i + (k_1 - i + 1)T\}. \tag{2.1}
\]

Next we estimate the makespan of an optimal schedule for the target problem, denoted as $C_{\text{max}}^\ast$. Let $k^\ast$ denote the number of batches in the optimal schedule. For simplicity, the sum of the processing times of all jobs in $J$ is denoted as $P$, and let $p_{\text{max}}$ denote the largest processing time of the jobs among $J$. Without loss of generality, we assume that jobs are processed consecutively on each machine. Secondly, since we have only one transporter, and the processing of all jobs on the $m$ parallel machines needs at least $\max\{P/m, p_{\text{max}}\}$ times, we have the lower bound
\[
C_{\text{max}}^\ast \geq \max\left\{\frac{P}{m} + T, k^\ast T, p_{\text{max}} + T\right\}. \tag{2.2}
\]
Lemma 2.1. For the schedule \( \pi_1 \),
\[
\rho_i \leq \frac{iP}{mk_1} + \frac{m-1}{m} \min\{p_{\max}, P_i\}
\]
for every \( i = 1, 2, \ldots, k_1 \).

Proof. Assume that the last job in batch \( B_i \) is \( J_i \). Since batches are sequenced in nondecreasing order of \( P_j \) for all \( j \), we get \( \sum_{j=1}^i P_j \leq iP/k_1 \). Since the list scheduling rule assigns the current job to the first available machine at the earliest possible time, and the jobs in each batch are assigned to machines by this rule in the algorithm,
\[
\rho_i \leq \frac{\sum_{j=1}^i P_j - P_i}{m} + P_i \leq \frac{iP}{mk_1} + \frac{m-1}{m} \min\{p_{\max}, P_i\},
\]
where the last inequality holds due to \( P_i \leq \min\{p_{\max}, P_i\} \). This completes the proof. \( \Box \)

Consequently, using Lemma 2.1 and equations (2.1)–(2.2), we conclude that
\[
\frac{C'_{\max}}{C^*_{\max}} = \frac{\max_{1 \leq i \leq k_1}\{\rho_i + (k_1 - i + 1)T\}}{C^*_{\max}}
\]
\[
\leq \frac{\max_{1 \leq i \leq k_1}\{(m-1)p_{\max}/m + iP/mk_1 + (k_1 - i + 1)T\}}{C^*_{\max}}
\]
\[
\leq \frac{m-1}{m} + \frac{\max_{1 \leq i \leq k_1}\{iP/mk_1 + iT/k_1 + (k_1 - i + 1 - (m-1)/m - i/k_1)T\}}{C^*_{\max}}
\]
\[
\leq \frac{m-1}{m} + \max_{1 \leq i \leq k_1}\left\{ \frac{i}{k_1} + \frac{k_1 - i + 1 - (m-1)/m - i/k_1}{k^*} \right\}.
\]
Therefore,
\[
\frac{C'_{\max}}{C^*_{\max}} \leq \frac{m-1}{m} + \max_{1 \leq i \leq k_1}\left\{ \frac{i}{k_1} + \frac{k_1 + 1/m - i/k_1}{k^*} \right\}.
\tag{2.3}
\]

Lemma 2.2. If \( k_1 \leq k^* \), then \( C'_{\max}/C^*_{\max} \leq 7/3 - 1/m \).

Proof. If \( k_1 < k^* \), then
\[
\max_{1 \leq i \leq k_1}\left\{ \frac{i}{k_1} + \frac{k_1 + 1/m - i/k_1}{k^*} \right\} = \frac{k^2 + (k^* - k_1 - 1)k_1 + k_1/m}{k_1 k^*} < 1,
\]
since \( k^* - k_1 - 1 \geq 0 \) and \( m \geq 3 \). Consequently, equation (2.3) reduces to \( C'_{\max}/C^*_{\max} \leq 2 - 1/m \).

If \( k_1 = k^* \), then
\[
\max_{1 \leq i \leq k_1}\left\{ \frac{i}{k_1} + \frac{k_1 + 1/m - i/k_1}{k^*} \right\} = \max_{1 \leq i \leq k_1}\left\{ \frac{k_1 - i/k_1 + 1/m}{k_1} \right\} \leq 1 - \frac{1}{k_2} + \frac{1}{k_1} \leq \frac{4}{3},
\]
since \( m \geq 3 \). Consequently, equation (2.3) becomes \( C'_{\max}/C^*_{\max} \leq 7/3 - 1/m \).

Thus, we have \( C'_{\max}/C^*_{\max} \leq 7/3 - 1/m \), which proves the lemma. \( \Box \)

Lemma 2.3. If \( k_1 > k^* \) and \( k^* \leq 5 \), then \( C'_{\max}/C^*_{\max} \leq 7/3 - 1/m \).
PROOF. If \( k_1 > k^* \), we have \( k^* \geq 2 \) by equation (1.1). We consider the following two cases.

**Case 1.** Let \( k^* = 2 \). From equation (1.1), we have \( k_1 = 3 \); thus, equation (2.3) becomes

\[
\frac{C_{\pi_1}}{C^*_{\pi_1}} \leq \frac{m - 1}{m} + \max_{1 \leq i \leq 3} \left\{ \frac{i}{3} + \frac{3 - 4i/3 + 1/m}{2} \right\} \leq \frac{m - 1}{m} + \frac{4}{3} = \frac{7}{3} - \frac{1}{m},
\]

where the second inequality holds due to \( m \geq 3 \).

**Case 2.** Let \( 3 \leq k^* \leq 5 \). From equation (1.1), we have \( k_1 = k^* + 1 \). When \( (k^*, k_1) = (3, 4) \), equation (2.3) becomes

\[
\frac{C_{\pi_1}}{C^*_{\pi_1}} \leq \frac{m - 1}{m} + \max_{1 \leq i \leq 4} \left\{ \frac{i}{4} + \frac{4 - 5i/4 + 1/m}{3} \right\} \leq \frac{41}{18} - \frac{1}{m},
\]

where the last inequality holds due to \( m \geq 3 \). Similarly, when \( (k^*, k_1) = (4, 5) \) or \( (k^*, k_1) = (5, 6) \), it follows that

\[
\frac{C_{\pi_1}}{C^*_{\pi_1}} \leq 67/30 - \frac{1}{m} \quad \text{or} \quad \frac{C_{\pi_1}}{C^*_{\pi_1}} \leq 11/5 - \frac{1}{m}.
\]

Thus, we always have \( C_{\pi_1}/C^*_{\pi_1} \leq 7/3 - 1/m \), which completes the proof. \( \square \)

**Lemma 2.4.** If \( k_1 > k^* \) and \( 6 \leq k^* \leq 9 \), then \( C_{\pi_1}/C^*_{\pi_1} \leq 7/3 - 1/m \).

**Proof.** If \( 6 \leq k^* \leq 9 \), from equation (1.1), we have \( k_1 \leq k^* + 2 \). When \( k_1 = k^* + 1 \), equation (2.3) becomes

\[
\frac{C_{\pi_1}}{C^*_{\pi_1}} \leq \frac{m - 1}{m} + \max_{1 \leq i \leq k_1} \left\{ \frac{k_1 - i/k_1 + 1/m}{k^*} \right\} \leq \frac{m - 1}{m} + \frac{k_1 - 1/k_1 + 1/m}{k^*} \leq \frac{20}{9} - \frac{1}{m},
\]

where the first inequality holds due to \( k_1 > k^* \) and the last inequality holds due to \( k_1 = k^* + 1, k^* \geq 6 \) and \( m \geq 3 \). When \( k_1 = k^* + 2 \), we consider the following two cases.

**Case 1.** \((m \geq 4)\) Since \( k_1 = k^* + 2, 6 \leq k^* \leq 9 \) and \( m \geq 4 \), equation (2.3) becomes

\[
\frac{C_{\pi_1}}{C^*_{\pi_1}} \leq \frac{m - 1}{m} + \max_{1 \leq i \leq k_1} \left\{ \frac{k_1^2 + (k^* - k_1 - 1)i + k_1/m}{k_1 k^*} \right\}
\]

\[
= \frac{m - 1}{m} + \frac{(k^* + 2)^2 - 3 + (k^* + 2)/m}{(k^* + 2)k^*} \leq \frac{m - 1}{m} + \frac{(k^* + 2)^2}{(k^* + 2)k^*} \leq \frac{7}{3} - \frac{1}{m}.
\]

**Case 2.** \((m = 3)\) From equation (2.1), we further distinguish the following three sub-cases according to the value of \( C^*_{\pi_1} \).
Case 2.1. \( C_{\text{max}}^{\pi_1} = \rho_{k_1} + T \). It follows from Lemma 2.1 and equation (2.2) that
\[
\frac{C_{\text{max}}^{\pi_1}}{C_{\text{max}}^*} \leq \frac{p/m + (2/3)p_{\text{max}} + T}{C_{\text{max}}^*} \leq \frac{5}{3}.
\]

Case 2.2. \( C_{\text{max}}^{\pi_1} = \rho_{k_1-1} + 2T \). From Lemma 2.1,
\[
\rho_{k_1-1} \leq \sum_{j=1}^{k_1-1} p_j + \frac{2}{3} \min\{p_{\text{max}}, p_{k_1-1}\}.
\]
Consequently, using \( P_{k_1-1} \leq P_{k_1} \) and equation (2.2),
\[
\frac{C_{\text{max}}^{\pi_1}}{C_{\text{max}}^*} \leq \frac{\left( \sum_{j=1}^{k_1-1} p_j / 3 + (2/3) \min\{p_{\text{max}}, p_{k_1-1}\} + 2T \right)}{C_{\text{max}}^*}
\]
\[
\leq \frac{\left( \sum_{j=1}^{k_1-1} p_j + p_{k_1-1} + p_{\text{max}} \right) / 3 + 2T}{C_{\text{max}}^*}
\]
\[
\leq \frac{\left( \sum_{j=1}^{k_1} p_j + p_{k_1} + p_{\text{max}} \right) / 3 + 2T}{C_{\text{max}}^*}
\]
\[
= \frac{(P + p_{\text{max}}) / 3 + 2T}{C_{\text{max}}^*}
\]
\[
\leq 2.
\]

Case 2.3. \( C_{\text{max}}^{\pi_1} = \max_{1 \leq i \leq k_1-2} (\rho_i + (k_1 - i + 1)T) \). From Lemma 2.1, \( \rho_i \leq (\sum_{j=1}^i p_j) / 3 + 2P_i / 3 \). Also, since \( P_1 \leq P_2 \leq \cdots \leq P_{k_1} \),
\[
\sum_{j=1}^{i} p_j + 2P_i \leq \sum_{j=1}^{i+2} p_j \leq (i + 2) \frac{P}{k_1}
\]
when \( i \leq k_1 - 2 \). Consequently, using equation (2.2), \( k_1 = k^* + 2 \) and \( 6 \leq k^* \leq 9 \) yields
\[
\frac{C_{\text{max}}^{\pi_1}}{C_{\text{max}}^*} \leq \frac{\max_{1 \leq i \leq k_1-2} \left( \left( \sum_{j=1}^{i} p_j / 3 + 2P_i / 3 + (k_1 - i + 1)T \right) \right)}{C_{\text{max}}^*}
\]
\[
\leq \max_{1 \leq i \leq k_1-2} \left\{ \frac{(i + 2)P/3k_1 + (k_1 - i + 1)T}{C_{\text{max}}^*} \right\}
\]
\[
\leq \max_{1 \leq i \leq k_1-2} \left\{ \frac{(i + 2)P/3k_1 + (i + 2)T/k_1}{(P/3) + T} + \frac{(k_1 - i + 1)T - (i + 2)T/k_1}{k^*T} \right\}
\]
\[
= \max_{1 \leq i \leq k_1-2} \left\{ \frac{i + 2}{k_1} + \frac{k_1 - i + 1 - (i + 2)/k_1}{k^*} \right\}
\]
\[
= \frac{(k^* + 2)^2 + 3k^* - 3}{k^*(k^* + 2)}
\]
\[
< 2.
\]
In summary, we always have \( C_{\text{max}}^{\pi_1}/C_{\text{max}}^* \leq 7/3 - 1/m \), which proves the lemma. \( \square \)
Table 1. An instance $I$ of the problem $(Pm \rightarrow D, k = 1 \mid v = 1, c = 1 \mid C_{\text{max}})$ with $m = 3$ and $T = 1$.

<table>
<thead>
<tr>
<th>Job index $j$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Processing time $p_j$</td>
<td>$\epsilon$</td>
<td>$\epsilon$</td>
<td>$\epsilon$</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Size $s_j$</td>
<td>31/60</td>
<td>29/120</td>
<td>29/120</td>
<td>1/3</td>
<td>1/3</td>
<td>9/40</td>
</tr>
</tbody>
</table>

**Lemma 2.5.** If $k_1 > k^*$ and $k^* \geq 10$, then

\[
\frac{C_{\pi_1}^*}{C_{\text{max}}^*} \leq \frac{209}{90} - \frac{1}{m}.
\]

**Proof.** Inequality (2.3) changes to

\[
\frac{C_{\text{max}}^*}{C_{\text{max}}^*} \leq \frac{m - 1}{m} + \max_{1 \leq i \leq k_1} \left\{ \frac{k^* - i/k^* + 1/m}{k^*} \right\} \leq \frac{m - 1}{m} + \frac{11k^*/9 + 6/9 + 1/3}{k^*} \leq \frac{209}{90} - \frac{1}{m}.
\]

Here the first inequality holds due to $k_1 > k^*$, the second inequality holds due to equation (1.1) and $m \geq 3$ and the last inequality holds due to $k^* \geq 10$. This proves the lemma.

**Theorem 2.6.** For the problem $(Pm \rightarrow D, k = 1 \mid v = 1, c = 1 \mid C_{\text{max}})$, in the general case of $m \geq 3$, the algorithm $H_1$ is a $(7/3 - 1/m)$-approximation.

**Proof.** We prove the theorem by considering all feasible combinations of $k^*$ and $k_1$ satisfying equation (1.1). All four of the different combinations of $k^*$ and $k_1$ have been separately dealt with in Lemmas 2.2–2.5. The maximum ratio between the makespan achieved by the algorithm $H_1$ and the optimal makespan $C_{\text{max}}^*$ among the four lemmas is $7/3 - 1/m$, which completes the proof.

In addition, we present an instance $I$ of the problem $(Pm \rightarrow D, k = 1 \mid v = 1, c = 1 \mid C_{\text{max}})$ in Table 1 to show that the performance ratio $7/3 - 1/m$ of the algorithm $H_1$ is tight. In the instance $I$, there are six jobs, three machines and one vehicle with a capacity of 1. A round trip between the machine and the customer is 1 (that is, $T = 1$).

The six jobs in the instance $I$ can be formed into two batches; $B^*_1 = \{J_1, J_2, J_3\}$ and $B^*_2 = \{J_4, J_5, J_6\}$. Since the processing times of the batches are $P^*_1 = 3\epsilon$ and $P^*_2 = 3$, by setting the job batch processing order as $\langle B^*_1, B^*_2 \rangle$, we achieve a makespan $2 + 2\epsilon$ when $\epsilon > 0$ is sufficiently small. In fact, observe that this is actually an optimal schedule, that is, $C_{\text{max}}^* = 2 + \epsilon$.

On the other hand, the six jobs in the instance $I$ can be formed into three batches $B_1 = \{J_6\}$, $B_2 = \{J_1, J_4\}$ and $B_3 = \{J_5, J_2, J_3\}$ by algorithm $H_1$ and $\pi_1 = \langle B_1, B_2, B_3 \rangle$. Since $P_1 = 1$, $P_2 = 1 + \epsilon$ and $P_3 = 1 + 2\epsilon$, we have $\rho_1 = 1$, $\rho_2 = 1$ and $\rho_3 = 1 + \epsilon$. Consequently, by equation (2.1), we have $C_{\text{max}}^* = 4$. It follows that the performance ratio of the algorithm $H_1$ on the instance $I$ is $4/(2 + \epsilon) \rightarrow 2$ when $\epsilon \rightarrow 0$.  

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3. Conclusions

In this paper, we studied a parallel machine scheduling problem with the finished job delivery to a single customer, \((Pm \rightarrow D, k = 1 | v = 1, c = 1 | C_{\text{max}})\). We presented an algorithm with a tight bound of \(7/3 - 1/m\) for the general case when \(m \geq 3\). It would be interesting to see whether the problem admits a better approximation algorithm, for example, by using a better bin-packing algorithm or finding some relationship between the job size and its processing times on the parallel machines. In addition, it is worth extending the results to the problem of \(m\) uniform machines, which can be denoted as \((Qm \rightarrow D, k = 1 | v = 1, c = 1 | C_{\text{max}})\), and has not been studied even when \(m = 2\).

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