

BENEDETTO, J. J. *Harmonic analysis and applications* (CRC Press, Boca Raton–New York–London–Tokyo, 1966), xix+336pp., 0 8493 7879 6, \$64.95.

When Klein looked at the Fourier analysis of his day he felt that it had lost its way: “. . . whereas the true motive of all Fourier’s creations was the usefulness and applicability of his methods to the great practical problems posed by nature, an abstract, purely function theoretical interest in this ever improving mathematical tool later won the upper hand. If I may be permitted a simile: The mathematics of our day appears to me like a large weapon shop in peace time. The store window is filled with showpieces whose ingenious, artful, and pleasing design enchants the connoisseur. The real origin and purpose of these things, to attack and defeat the enemy, has retreated so far into the background of consciousness as to be forgotten.”

Fortunately the 20th century provided Fourier analysis with a host of powerful new tools ranging from Lebesgue measure and distribution theory to Fejér’s theorem and the remarkable family of inequalities grouped together as “Heisenberg type inequalities”, together with fascinating new practical problems from crystallography to mapping the sea bottom. The electronic computer and the discovery (or rediscovery) of appropriate methods of numerical analysis provided a further stimulus to the practical side of the subject and, indirectly but just as powerfully, to the theoretical side.

Although the first part of the century saw major contributions to the subject from such British analysts as Young, Hardy, Littlewood and Titchmarsh, Klein’s pessimistic image persists in Britain, where Fourier analysis is believed to be practised either in a crude and non-rigorous way by physicists seeking solutions to rather old-fashioned physical problems or in an over-sophisticated way by classical analysts seeking solutions to ever more delicate and irrelevant problems. However, France and the United States have recently seen the successful marriage of classical analysis and practical communication theory. The most notable of the resulting offspring is the theory of wavelets, but other thriving children romp through the pages of this book.

Benedetto’s book is a celebration of that marriage. It is written with pride and enthusiasm by someone for whom the Hilbert transform and bandpass filters are two sides of the same coin. Everyone who intends to give a course on Fourier analysis should read this book. Old topics are approached in new ways and new topics shown to be teachable. However, they may come to feel that this is one book which could have been made longer with advantage. If I ever have to teach the method of stationary phase I shall certainly start by considering the introduction given here, but I regret the absence of the fascinating further development which the author could no doubt have given us.

One of the strengths of this book is the way that it shows how apparently unconnected topics are linked by the methods and ideas of Fourier analysis. In a few places the desire to show that everything is connected to everything else combines with the drive for brevity with unhappy results, for example: “*Surgery invariants* can also be determined by evaluating Gauss sums of this type. The purpose of surgery in *differential topology* is to characterise simply connected manifolds in higher dimensions . . .”; we may be better informed after reading this, but we are certainly no wiser.

On the other hand, we are both better informed and wiser after reading the treatment of the Fast Fourier Transform which follows. Always up to date, sometimes irritating, more often fascinating, this book is never dull.

John Benedetto set out to write both a textbook and an essay. He has written a marvellous essay which can be recommended without reservation, but the resulting textbook is only suitable for abler and better prepared students than can be found in most British universities. No one who intends to give a proper course on Fourier analysis can afford not to consult it and no mathematics library should be without it, but as a course text I would still stick with Helson’s *Harmonic Analysis* (Addison-Wesley, 1983) for those who, like me, admire “ingenious, artful, and pleasing design” and Dym and McKean’s *Fourier Series and Integrals* (Academic Press, 1972) for those whose main object is “to attack and defeat the enemy”.

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