A NECESSARY CONDITION TO THE SIMULTANEOUS CORRECTION OF CELESTIAL REFERENCE FRAMES AND MINOR PLANETS ORBITAL ELEMENTS

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Abstract. In order to obtain corrections to orbital elements of minor planets and to determine parameters of rotation of a reference frame, a geometrical model with restrictions in the declination, longitude of the ascending node and argument of the perihelion to minimize a residual function involving all named parameters was presented (Marco *et al.*, 1996, 1997). On the other hand, such rotational model of correction should not modify the semiaxis, eccentricity nor the mean anomaly, so new restrictions on these parameters seem to be necessary. These conditions should reflect the invariability in the size of the orbit and they should be included in the model to complete a consistent set of restrictions.

1. Introduction

Let $\overrightarrow{X}(\alpha, \delta)$ be an unitary vector position in equatorial coordinates for a body in elliptic motion around the Sun. We suppose the existence of errors on its position which are modeled by means of a rotational model given by

$$\Delta\overrightarrow{X} = R\overrightarrow{X} \text{ with } R = \begin{bmatrix} 1 & -\Delta\xi & -\Delta\eta \\ \Delta\xi & 1 & -\Delta\varepsilon \\ \Delta\eta & \Delta\varepsilon & 1 \end{bmatrix} \tag{1}$$

From (1) the incremental errors induced by the rotational model and relating the spherical coordinates at whatever time t are given by

$$\begin{aligned} \Delta\alpha &= \Delta\xi + \Delta\eta \sin\alpha \tan\delta - \Delta\varepsilon \cos\alpha \tan\delta \\ \Delta\delta &= \Delta\eta \cos\alpha + \Delta\varepsilon \sin\alpha \end{aligned} \tag{2}$$

2. Rotation Model Applied to the Orbital Plane at t_0

We denote the different unitary vectors (at t_0) defining the orbit orientation by $\overrightarrow{p_1^0}$ (in the perihelion direction), $\overrightarrow{p_3^0}$ (normal to the orbit) and $\overrightarrow{p_2^0}$ the necessary one to built a direct triad. The application of the rotational model to the triad is carried out by means of the expressions

$$R_1(\varepsilon)RR_1(-\varepsilon)\overrightarrow{p_i^0} = \overrightarrow{p_i^0} + \overrightarrow{\Delta p_i^0} \qquad \text{for i=1,2,3} \tag{3}$$

J. Henrard and S. Ferraz-Mello (eds.), Impact of Modern Dynamics in Astronomy, 425–426.

where R is the rotation matrix from (1) and $R_1(\pm\varepsilon)$ the rotation matrix relating equatorial and ecliptic coordinates. The relation (4) follows from (3)

$$\overrightarrow{\Delta p_i^0} = \overrightarrow{p_i^0} \times R_1(\varepsilon)\overrightarrow{Y} \text{ being } \overrightarrow{Y} = [\Delta\varepsilon, -\Delta\eta, \Delta\xi]^t \tag{4}$$

If we express these $\overrightarrow{p_i^0}$ vectors in ecliptic heliocentric coordinates, due to the fact that they are functions of the incremental values $\Delta\Omega_0$ (argument of ascending node), $\Delta\omega_0$ (argument of perihelion) and Δi_0 (inclination of the orbit), we can obtain from (3)

$$\begin{bmatrix} \Delta\varepsilon \\ -\Delta\eta \\ \Delta\xi \end{bmatrix} = \begin{bmatrix} 0 & \sin i_0 \sin\Omega_0 & \cos\Omega_0 \\ -\sin\varepsilon & \cos i_0 \sin\varepsilon + \sin i_0 \cos\Omega_0 \cos\varepsilon & -\sin\Omega_0 \cos\varepsilon \\ \cos\varepsilon & \cos i_0 \cos\varepsilon - \sin i_0 \cos\Omega_0 \sin\varepsilon & \sin\Omega_0 \sin\varepsilon \end{bmatrix} \begin{bmatrix} \Delta\Omega_0 \\ \Delta\omega_0 \\ \Delta i_0 \end{bmatrix} \tag{5}$$

3. Necessary Compatibility Conditions for the Rotational Model

Let $\overrightarrow{W} = P\overrightarrow{v}$ be the ecliptic coordinates of the unitary vector position $\overrightarrow{v} = [\cos u, \sin u, 0]^t$ in the orbital plane at t, where u represents the true anomaly and let $\overrightarrow{X} = R_1(-\varepsilon)\overrightarrow{W}$ be its equatorial representation. Then, we have the incremental vector $\overrightarrow{\Delta W} = R_1(\varepsilon)\overrightarrow{\Delta X}$,

$$\overrightarrow{\Delta W} = \left[\overrightarrow{\Delta p_i} v_i + \overrightarrow{p_i}\Delta v_i\right] \tag{6}$$

Also, from (4) this infinitesimal vector is expressed as follows

$$\overrightarrow{\Delta W} = R_1(\varepsilon)\left[\overrightarrow{X} \times \overrightarrow{Y}\right] = \overrightarrow{W} \times R_1(\varepsilon)\overrightarrow{Y} = P\overrightarrow{v} \times R_1(\varepsilon)\overrightarrow{Y} \tag{7}$$

Through the expression of $P\overrightarrow{v}$ in vectorial form and due to the fact that $R_1(\varepsilon)\overrightarrow{Y}$ is an infinitesimal vector we obtain

$$\overrightarrow{\Delta W} = \sum_{i=1}^{2} \overrightarrow{p_i} v_i \times R_1(\varepsilon)\overrightarrow{Y} \simeq \sum_{i=1}^{2} \overrightarrow{p_i^0} v_i \times R_1(\varepsilon)\overrightarrow{Y} \tag{8}$$

applying (5) and (6) , we obtain

$$\overrightarrow{\Delta W} = \sum_{i=1}^{2} \left\{ \left[\frac{\overrightarrow{\partial p_i}}{\partial\Omega}\Delta\Omega + \frac{\overrightarrow{\partial p_i}}{\partial\omega}\Delta\omega + \frac{\overrightarrow{\partial p_i}}{\partial i}\Delta i \right] v_i + \overrightarrow{p_i}\Delta v_i \right\} \tag{9}$$

which give us together with (8) the complete set of restrictions.

References

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