

INCREASING POWER OF M-TEST THROUGH PRE-TESTING

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The idea of improving the properties of estimators by pre-testing the uncertain nonsample prior information (NSPI) is adopted in the testing regime to achieve better power of the ultimate test. In this thesis, studies on increasing power of the ultimate test through pre-testing the uncertain NSPI are carried out for four types of regression models, namely the simple regression model, the multivariate simple regression model, the parallelism model and the multiple linear regression model.

In this thesis, procedures are developed for:

- testing the intercept of a simple regression model, when the NSPI on the slope is (i) unknown, (ii) certain or (iii) uncertain, or equivalently, when the slope is (i) completely unspecified, (ii) specified to a fixed value, or (iii) suspected to be a fixed value;
- testing the intercept vector of a multivariate simple regression model when the NSPI on the slope vector is (i) unknown, (ii) certain or (iii) uncertain, or equivalently, when the slope vector is (i) completely unspecified, (ii) specified to a fixed value, or (iii) suspected to be a fixed value;
- testing the intercepts of $p (> 1)$ simple regression models when the NSPI on the slopes is (i) unknown, (ii) certain or (iii) uncertain, or equivalently, when the slopes are (i) completely unspecified, (ii) equal at a fixed value or (iii) suspected to be equal at a fixed value;
- testing a set of parameters of the multiple linear regression when the NSPI on the other set of parameters is (i) unknown, (ii) certain or (iii) uncertain, or equivalently, when the other set of parameters is (i) completely unspecified, (ii) zero or (iii) suspected to be zero.

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Under the three scenarios, the (i) unrestricted test (UT), (ii) restricted test (RT) and (iii) pre-test test (PTT) functions are used to formulate the M-tests. The M-tests are derived using the score function in the M-estimation methodology. The sensitivity of the M-test to aberrant observations depends on the choice of the score function.

For each regression model, the following steps are carried out: (i) the test statistics for the UT, RT and PTT are proposed; (ii) the asymptotic distributions of the test statistics under the local alternative are derived; (iii) the asymptotic power functions of the tests are derived; (iv) the performance (size and power) of the UT, RT and PTT is compared analytically; (v) the performance of the UT, RT and PTT is compared computationally using illustrative data of a two-sample case or data simulated using the Monte Carlo method.

Under a sequence of local alternative hypotheses when the sample size is large, the sampling distributions for the UT, RT and PT (pre-test) of the simple regression model follow a normal distribution. However, that of the PTT is a bivariate normal distribution. For the multivariate simple regression model, parallelism model and multiple linear regression model, the sampling distributions of the UT, RT and PT follow a univariate noncentral chi-square distribution under the alternative hypothesis when the sample size is large. However, that of the PTT is a bivariate noncentral chi-square distribution. For all regression models, there is a correlation between the UT and PT but there is no such correlation between the RT and PT. To evaluate the power function of the PTT, a package in R is used to compute the probability integral of the bivariate normal while a new R code is used to compute the probability integral of the bivariate noncentral chi-square distribution.

The robustness properties of the M-test are studied computationally on the simulated data for the simple regression model and the multivariate simple regression model. The power of the M-test using the Huber score function is better than in the least-squares based test because the former is not significantly affected by slight departures from the model assumptions while the latter depends heavily on the normality assumptions. For all regression models, the PTT demonstrates a reasonable domination over the other two tests asymptotically when the suspected NSPI value is not too far away from that under the null hypothesis.

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