however act as a valuable supplement to a more rigorous text, both in first year university service courses, and at A-level. The price however is rather high, particularly so for the hardback edition.

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Second review

This book covers the simpler aspects of calculus required for entry to many scientific courses in Scottish and English universities and colleges. The author, himself a senior lecturer at Paisley College of Technology, clearly sees a need for a text which revises those topics which should be, but which are not necessarily, at the finger tips of his new students and which revises them in a fashion which can be readily understood by all of them.

Because there are no constraints of a specific syllabus—Mr. Johnson only considers those topics which, presumably, he considers useful—there is an air of freshness in the book which is so often absent in texts written for the mass A-level market. This is heightened by imaginative examples of which there are enough when one remembers that this is really a revision text. The examples seem to be nice and easy; any which require much thought have useful hints which is, of course, what the customers want!

The text is carefully written and thoughtfully produced which makes it an easy text to follow. This is mainly because the author is, on the whole, quoting results which he then shows his readers how to use. Again, this is precisely what his clientele requires. I do not suppose that many of them would be that interested in why the results are true. While the author is very methodical and spends many useful pages explaining about limits before going on to differentiation, for example, there are a few things which I find strange, pompous or wrong.

I wonder why he uses $\Delta x$ instead of the more usual $\delta x$ for an increment in $x$? Why introduce the differential, $dy = f'(x) \, dx$, where $y = f(x)$, and then say, “Note that we can divide the differential $dy$ by the differential $dx$ to obtain $dy \div dx = f'(x) \ldots$ Thus the quotient of the differentials $dy/dx$ is in fact the same as the symbol $dy/dx$ used in Chapter 1.”?

The reason I object to this sort of thing is that the readers, in the main, will not really understand what they are doing and it encourages them to think fallaciously, even though they will invariably obtain the correct answers. $dy/dx$ is to be thought of as a transformation applied to the element $y$ of a given set; differentiation is something which is done to $y$ to produce its derivative. Does not Mr. Johnson understand that, if he is not careful, he will have his students thinking $\int f'(x) \, dx = \int f'(x)/du \, du$ because, in this “expression”, $dx$ can be replaced by $dx/du \, du$? Reading on in the chapter on integration that is precisely what is taught! Very poor; all students of mathematics should be given the truth even if it does make demands upon their intellect.

The other main things I must object to are (i) stating that integration is the reverse of differentiation, as a definition, as opposed to its being the limit of a sum and (ii) starting with $a^x$ as the basis for all work on exponential and logarithmic functions. How do you prove that $a^x$ is continuous, Mr. Johnson?

I was amused by the fact that derivation of the formula for radius of curvature “has been given by Thomas and Finney (1979).” May I suggest that all authors put footnotes in their texts for the general education of their innocent readers as to the period of the ideas being studied? Of course, Mr. Johnson merely wanted to suggest an up-to-the-minute text where a rigorous proof of the said formula could be found. But, seriously, how many students at this level are aware that integration, for example, is Greek in its origins?

I would not wish to diminish the care which Mr. Johnson takes over his teaching; he clearly faces the same kind of problem as A-level teachers. In particular, the chapter entitled...
Stationary Points and Points of Inflexion is full of advice which can only have been gained by many long years at the chalk face. Again, I would quibble with some of the mathematics. Finding the closest approach of two ships of different courses is always better done using vectors and not by the more obvious but tedious method using Pythagoras' Theorem. It was good to see that Simpson's Rule is determined for a cubic rather than the more common quadratic but, as usual, it was not clear why Simpson's Rule approximates to the required integral in the same sense that the Trapezium Rule does.

The power of the book lies in showing students how to use the techniques of elementary calculus right up to second-order differential equations. In this respect it is very clear indeed and is worth a look.

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An argument for putting mathematical analysis in the undergraduate curriculum is that it gives the student an opportunity to learn why calculus works. With some courses and texts the student probably wishes (s)he never asked and leaves feeling the theory of calculus is the creation of pedants. There is a fine line between accuracy and exactitude. David Stirling treads it carefully.

His book opens with two short chapters which guard the reader away from sloppy logic. The basic tools are gathered in Chapter 3: the algebra and order axioms for real numbers (the completeness axiom is left to Chapter 6), the principle of induction, proof by contradiction, and some tricks with inequalities.

The fun starts in Chapter 4: Limits. The definition of limit for a real sequence is followed by almost a page of comments. The reader is firmly and quickly immersed in the sea of analytical symbology. Too quickly I feel, for an introductory text. The student of analysis must jump two hurdles not one. First, the abstract concept—the intuitive idea if you like—of the definition or result must be understood. Diagrams and examples help here. Second, (s)he must learn the notation with which the concept is succinctly expressed. To help the student, perhaps it is better to linger a little with colloquial language. "For every positive number e, no matter how small, there is a sufficiently large positive integer N such that . . . ." looks a lot more friendly, is more informative and no less precise than "∀ e > 0 ∃ N ∈ N s.t. . . .". The hieroglyphs are great for notes and the chalkboard but make difficult reading first up. The student will appreciate their value a little later and learn to use them correctly soon enough. A nice feature in this chapter is the way in which the proofs of the limit theorems for real sequences are presented. The proofs are conventional but alongside we get a column of "helpful grafitti" which explains the intuitive ideas for constructing the proofs. It removes some of the mystery of analysis. I was disappointed to find no such running commentary in the rest of the book.

Chapter 5 gives the basic results for real series and tests for convergence. A couple of quibbles: the proof of Lemma 5.4 needs Theorem 4.3 as well as 4.4; for an introductory text Cauchy's condensation test could well be omitted and the convergence of \( \sum \frac{1}{n^p} \) resolved directly.

We learn about the difference between the real and rational number systems in Chapter 6. The author starts with Dedekind's Axiom of Continuity rather than the equivalent Completeness Axiom claiming (in his preface) that the former is more plausible. Functions, continuous, bounded, monotonic, and invertible are discussed in Chapter 7. Again, a softer colloquial definition of limit would be preferred, at least initially.

Chapter 8, Differentiation, includes Rolle's theorem and its corollaries, the mean value theorem, L'Hopital's Rule, Taylor's theorem. It culminates with some interesting applications, for example, using Taylor's theorem to find (for \( a \geq 1 \)) the leading terms of \( 1 + 2^x + 3^x + \ldots + n^x = n^{x+1}/(x+1) + n^x/2 + \ldots \). Chapter 9 tells us all we need to know.