THE PATTERN OF CONVECTION IN THE SUN

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Abstract. The structure of solar magnetic fields is dominated by the effects of convection, which should be incorporated in any model of the solar cycle. Although mixing length theory is adequate for calculating the structure of main sequence stars, a better description of convection is needed for any detailed dynamo model. Recent work on nonlinear convection at low Prandtl numbers is reviewed. There has been some progress towards a theory of compressible convection, though there is still no firm theoretical evidence for cells with scales less than the depth of the convecting layer. However, it remains likely that the pattern of solar convection is dominated by granules, supergranules and giant cells. The effects of rotation on these cells are briefly considered.

1. Introduction

Over the past decade our understanding of the Sun (and particularly of small scale photospheric features) has been transformed by a wealth of detailed observations. Theoreticians lag behind observers but it has become clear in the last few years that progress requires detailed calculations rather than qualitative, order-of-magnitude arguments. Although few of these calculations have yet been carried out it is at least possible to outline a sequence of increasingly complicated problems that must be solved if we are to explain what has been observed. We have attained the stage that geographers had reached by 1500. The age of fantasy is over: now we can map out areas of ignorance and have to determine programmes of systematic exploration.

Solar magnetic fields are intimately related to the pattern of convection in the outer layers of the Sun. A full description of the structure of convection is necessary for any theory of the solar cycle. In particular, we need to supply a detailed velocity field that can be fed into more sophisticated kinematic dynamo models. Astrophysical convection has recently been reviewed by Spiegel (1971b, 1972). Here I shall first outline our limited understanding of the problem and then indicate what progress is being made towards a proper theory. Laboratory convection is still poorly understood and in stars further difficulties are posed by compressibility – the density changes by a factor of 10^5 over the Sun's convective zone – and rotation; since Dr Gilman and Dr Durney have already discussed the latter in some detail I shall concentrate on compressibility and its effect on the characteristic scale of convective motion. Finally, I shall atempt to provide what seems to me the best available description of the pattern of solar convection and its interactions with magnetic fields.

2. Mixing Length Theory

Mixing length theory, as developed by Biermann 40 years ago and elaborated since, still provides the only quantitative method of relating the temperature gradient to the heat flux in a stellar convective zone. Consider a plane horizontal layer, referred to

cartesian coordinates with the z-axis pointing downwards in the direction of the gravitational acceleration **g** and a temperature gradient

$$\frac{\mathrm{d}T}{\mathrm{d}z} = \beta_0 > \beta_{\mathrm{ad}}, \tag{1}$$

where β_{ad} is the adiabatic gradient. Then the degree of superadiabaticity is measured by the dimensionless Rayleigh number

$$R = \frac{g\alpha\beta l^4}{\kappa\nu}, \tag{2}$$

where l is a characteristic length scale for convection, $\beta = \beta_0 - \beta_{\mathrm{ad}}$, α is the coefficient of thermal expansion ($1/T$ for a perfect gas) and κ, ν are the thermal and viscous diffusivities. When $\kappa \gg \nu$ it is convenient to introduce a modified Rayleigh number

$$S = \frac{g\alpha\beta l^4}{\kappa^2} = pR, \tag{3}$$

where the Prandtl number

$$p = \nu/\kappa. \tag{4}$$

The efficiency of convection is given by the Nusselt number

$$N = \frac{(\text{Total heat flux}) - c_p\rho\kappa\beta_{ad}}{c_p\rho\kappa\beta}, \tag{5}$$

which is a dimensionless measure of the superadiabatic heat flux, where ρ is the density and c_p the specific heat at constant pressure. The aim of a convection theory is to predict N as a function of R and p.

To derive mixing length theory in its simplest form, let us consider vigorous convection, with eddies of a characteristic length scale l, so that $N \approx F/c_p\rho\kappa\beta$, where F is the convective heat flux. If w is the upward vertical velocity and θ the temperature excess of a rising fluid element then

$$F \sim c_p\rho w\theta. \tag{6}$$

For rapid convection, heat losses through (radiative) diffusion can be ignored and so the potential temperature variation

$$\theta \sim \beta l. \tag{7}$$

The velocity can be estimated by balancing the rate of working of the buoyancy force against the rate of dissipation of energy through the nonlinear inertial term in the equation of motion:

$$\rho g\alpha\theta \sim \rho w^2/l. \tag{8}$$

Then, from (7) and (8), the reduced free fall velocity

$$w \sim (g\alpha\beta)^{1/2}l \tag{9}$$

and the flux

$$F \sim c_p\rho(g\alpha\beta^3 l^4)^{1/2} . \tag{10}$$

Finally, from (6),

$$N \sim (pR)^{1/2} = S^{1/2} . \tag{11}$$

This result holds for any local Boussinesq theory of convection, whether it is expressed in terms of bubbles, cells or eddies. To calculate the heat flux it is only necessary to calibrate various constants, all of which can be absorbed into the mixing length l.

In practice, l is set equal to a multiple of the local pressure (or density) scale height and the arbitrary constant is calibrated by ensuring that a solar model, with a given metal abundance, evolves from the zero age main sequence to the Sun's present radius and luminosity in its known lifetime of 4.7×10^9 yr. This procedure can be followed for Biermann's simple theory (described above) or for the more elaborate local theories of Öpik and Böhm-Vitense, which allow for lateral radiative losses. When this is done, all local theories give the same depth, around 150 000 km, for the solar convection zone and also provide consistent models of lower main-sequence stars (Gough and Weiss, 1976). (Mullan's (1971) application of Öpik's theory employs arbitrary constants with values that reduce the efficiency of convection but are incompatible with evolution of the Sun.)

It is not difficult to devise descriptions of convection that are adequate for calculating the structure of stars like the Sun. (Radiative losses are significant only in a very shallow photospheric zone; of course, these losses are important at levels where the granulation is observed (Travis and Matsushima, 1973; Spruit, 1974) but all available theories of convection are too crude to be valid in this region.) It is only necessary to calculate the jump in entropy across a narrow region, about 1000 km deep, immediately below the photosphere, where the temperature gradient is strongly superadiabatic. Below this region, the stratification is virtually adiabatic throughout the convective zone. Unfortunately, this simple description no longer holds for red giants (Schwarzschild, 1975; Gough and Weiss, 1976), nor can it be used to predict the detailed structure of convection in the Sun.

Mixing length theory depends on the assumption that there is a characteristic local length scale l, related to some local scale height. Provided the viscosity is small, the theory then predicts that the Nusselt number $N \propto S^{1/2}$ and is independent of ν. Both this result and the underlying assumption need to be verified. However, it is difficult to compare the theory with laboratory experiments, which are dominated by the effects of thermal boundary layers. Moreover, few experiments have so far been carried out at low Prandtl numbers ($p = \frac{1}{40}$ for mercury but, owing to radiative diffusion, $p \approx 10^{-9}$ in the Sun) and significant density variations across the layer

cannot be reproduced in an experiment. Hence any improvement must rely on theory and, for such a nonlinear problem, on numerical experiments. Various idealized problems have been studied over the last few years and the following sections describe the progress that has so far been made.

3. Low Prandtl Number Convection

It is generally accepted that in a fluid with a low Prandtl number kinetic energy is dissipated through some turbulent process and that the heat transport should not depend explicitly on the viscosity. Thus N should depend not on the Rayleigh number but on the product $S = pR$. If heat transport in the boundary layers is laminar and limited by the thermal diffusivity then $N \propto S^{1/3}$; if the heat flux F is independent of κ also then $N \propto S^{1/2}$ as in Equation (11) (Spiegel, 1971a, b). However, there is as yet no firm theoretical basis for this belief.

Numerical experiments on Boussinesq convection between free boundaries in two-dimensional rolls showed that at high Reynolds numbers N is proportional to $R^{0.36}$ and independent of Prandtl number as $p \to 0$ (Veronis, 1966; Moore and Weiss, 1973). It might appear that three-dimensional geometry, which introduces an asymmetry between hot fluid rising at the centre of a cell and cold fluid sinking at its periphery, would cause a reduction in N as p decreases for a fixed Rayleigh number. Indeed, Gough *et al.* (1975) studied a simple model of nonlinear convection in which the horizontal ground plan of the cells was specified (the modal approximation) and found that $N \sim (S \ln S)^{1/5}$ for $p \le 1$, $S \gg 1$. However, computations of steady laminar convection in an axisymmetric cylindrical cell showed no such effect: N depended on R only and was independent of p as $p \to 0$, as for two-dimensional rolls, so the heat transport did depend on the viscosity (Jones *et al.*, 1976).

What happens in these numerical experiments is that fluid in the cell turns over many times, gradually picking up speed as it does so. Although viscosity is slight, the frictional force increases with the speed until it is eventually able to balance the buoyancy force and an equilibrium is reached. This flywheel mode of convection has velocities much greater than the free-fall velocity in (9) and can therefore carry far more energy than a cell that only turns over once. Are such flywheels likely to be realized or will their growth be limited by some instability?

One possibility is that cells become unstable to non-axisymmetric perturbations and split up into segments (Jones, 1975). Indeed, inspection of any photograph of the photospheric granulation shows a number of exploding granules (Musman, 1972) that resemble the unstable vortex rings described by Widnall and Sullivan (1973) and Widnall (1975). However, it is also likely that the flywheels suffer from some collective instability that limits the growth of the velocity. Such an instability would allow adjacent vortex rings to merge and disappear, like opposing magnetic fields at a current sheet. Energy would then be dissipated neither by laminar viscous friction nor through an inertial range of eddies (as in homogeneous turbulence) but locally in spasmodic bursts.

To describe such a process requires a fully three-dimensional calculation. For the moment, we must assume that more sophisticated - and considerably more

expensive – time-dependent computations would then demonstrate that the lifetime of a cell is of the order of the turnover-time $\tau \sim l/w$ (as seen in the solar granulation), that the velocity would be of the same order as that in (9) and that the heat flux might follow a law of the form predicted by mixing length theory in Equation (11).

4. Effects of Compressibility

The density gradient in the solar convective zone introduces an asymmetry between upward and downward motion. Rising elements of fluid expand and dominate convection; sinking elements contract and can disappear into the interstices between the rising columns (Schwarzschild, 1961). In a Boussinesq fluid, cells with fluid rising or sinking at their centres are equally probable but this degeneracy is removed by non-Boussinesq effects. In air (where the predominant effect is the increase of viscosity with temperature) motion is downward at the centres of convection cells. In the Sun the density gradient is important, favouring upward motion in the core. This asymmetry is of course observed, and is essential for the α-effect in solar dynamos (Steenbeck *et al.*, 1966).

Mathematically, the dominance of rising and expanding gas is expressed through the continuity equation

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{u}) \tag{12}$$

where $\mathbf{u}$ is the velocity. Provided that convective motion is slow compared with the sound speed c_s (so that the Mach number $M = |\mathbf{u}|/c_s \ll 1$) we may filter out acoustic waves by adopting the anelastic approximation (Gough, 1969), which is a generalization of the familiar Boussinesq approximation. Equation (12) then simplifies to

$$\nabla \cdot (\bar{\rho}\mathbf{u}) = 0, \tag{13}$$

where $\bar{\rho}(z)$ is a horizontally averaged density: the flow is constrained by the mean density variation.

Suppose now that convection occurs in cells with a horizontal dimension L. We expect that L is comparable with the density scale height

$$H = -(\mathrm{d} \ln \bar{\rho}/\mathrm{d}z)^{-1} \tag{14}$$

at some level $z = z_0$ near the base of the cell. But if the cell penetrates to a level $z \ll z_0$, the local scale height H may be small compared with L. From (13) we see that $\bar{\rho}u/L \sim \mathrm{d}(\bar{\rho}w)/\mathrm{d}z$, where u is the horizontal component of $\mathbf{u}$. If, for the moment, we neglect the variation of w with z then

$$u \sim \left(\frac{L}{H}\right)w . \tag{15}$$

Thus $u \gg w$ if $H \ll L$ and the buoyancy force is spent in driving rapid horizontal motions, which do not contribute to the heat transport. This inefficiency favours convection on a smaller scale. More precisely, we might expect local instabilities to develop, deriving their energy either from the sheared velocity or from the

superadiabatic stratification itself. At some level these instabilities will grow fast enough to form smaller convection cells that are able to transport energy more effectively than the original, larger cells.

Somewhat more generally, we can consider a polytropic atmosphere, such that the horizontally averaged temperature and density are given by

$$\bar{T} = \beta_0 z, \qquad \bar{\rho} = \rho_0 z^m, \tag{16}$$

where the polytropic index $m = g/(\mathcal{R}\beta_0) - 1$ and $\mathcal{R}$ is the gas constant. If we suppose that $w \propto z^n$ then, from (13), $u \propto w/z \propto z^{n-1}$ and the horizontal component of the velocity increases upwards for $n < 1$. Moreover, if all the energy is carried by convection then, from (6),

$$\theta \sim \frac{F}{c_p \bar{\rho} w} \propto z^{-(m+n)} \tag{17}$$

and so the superadiabatic gradient $\beta \propto z^{-(m+n+1)}$. For the conventional mixing length theory of Section 2, with l everywhere proportional to the local scale height H, $n = -\frac{1}{3}m$, from (8). If the cell extends over many scale heights, so that dissipation of energy is dominated by horizontal motions, Equation (8) must be replaced by

$$\frac{u^3}{L} \sim g\alpha\theta w \sim \left(\frac{gF}{c_p}\right)\frac{1}{\bar{\rho}\bar{T}} \tag{18}$$

and it follows that $n = \frac{1}{3}(2 - m)$ and hence that $u \propto z^{-(m+1)/3}$. (In particular, for $m = \frac{3}{2}$, as in the deep solar convection zone, $w \propto z^{1/6}$ and $u \propto z^{-5/6}$, so that the vertical velocity varies only slightly while the horizontal velocity increases upwards.) The ratio of the superadiabatic gradient for a cell extending over many scale heights to that for a local cell is proportional to $z^{-2/3}$. Eventually, therefore, small scale convection should take over (Simon and Weiss, 1968).

This crude discussion needs to be supported by a proper calculation. Unfortunately, computations have so far provided no evidence for the existence of any vertical scale other than the layer depth, even in a compressible atmosphere. The solution of the linearized marginal stability problem for a polytropic atmosphere (Spiegel, 1965; Gough et al., 1976) shows that, for any cell width, instability occurs first for the fundamental vertical mode, with no internal zeros in the eigenfunctions θ and w. (Vickers' (1971) solutions showing a reversal in θ at the upper boundary are wrong, apparently owing to a numerical error in treating the boundary conditions.) Growth rates have been calculated for small perturbations to models of the convective zone computed using mixing length theory (Böhm, 1963, 1967; Vickers, 1971; Vandakurov, 1975): the highest growth rates are shown by small scale modes with greatest amplitudes near the surface, where the superadiabatic gradient is largest, but all modes extend throughout the region and there is no direct evidence for smaller cells. Of course, linear modes are solutions to a simplified problem. The nonlinear constraint of constant heat flux is not included (for example, both w and θ are relatively small near $z = 0$ in the eigenfunctions for an infinite polytropic atmosphere, so that (17) could not be satisfied). Do non-linear models allow multiple cells to develop in a compressible layer?

The only non-linear solutions are those of Graham (1975) for two-dimensional convection in a fully compressible atmosphere. His numerical experiments, with densities varying by a factor up to 10 (4 pressure scale heights) across the layer and Rayleigh numbers up to 100 times the critical value, all gave cells that filled the entire convecting region. To demonstrate the development of smaller cells it may be necessary to have a much greater density variation, or to proceed to time-dependent three-dimensional models. An alternative possibility is that Graham's results (like the linear solutions of Gough *et al.*) are affected by the assumption of a constant molecular viscosity $\mu = \rho\nu$. At the top of the layer, where the density is small, the viscous term dominates the equation of motion and (for stress-free boundary conditions) forces a horizontal velocity u that is independent of z. If u is constant, $w \propto 1/z$ and this rather unrealistic constraint may inhibit the growth of instabilities and so stabilize cells extending over many scale heights. Turbulent viscosity in the Sun is better represented by a constant diffusivity ν: calculations with $\mu \propto 1/\bar{\rho}$ might allow greater variation in u and so permit the development of other scales of motion (cf. Parker, 1973).

The pattern of cellular motion is not the only feature of compressible convection that is poorly understood. It is not obvious that the functional dependence of N on R and p will be the same as for a Boussinesq fluid: once the temperature scale height H_T becomes comparable with the layer depth the rate of working against pressure and viscous forces makes a significant contribution to the energy equation and the dissipation rate over a cell $(\bar{\rho}w^3 l^2)$ becomes comparable with the energy flux (Fl^2). From (9) and (10),

$$\bar{\rho}\frac{w^3}{F} \sim \frac{g\alpha l}{c_p} = \frac{l}{H_T} \tag{19}$$

and for a polytrope $H_T = mH$. Graham's (1975) numerical experiments already show many details and further computations are badly needed.

Fully compressible computations, especially in three dimensions, require vast amounts of computer time if it becomes necessary to follow sound waves in regions where the Mach number is small. Hence it seems advisable to use the anelastic approximation, with the simplified form (13) of the continuity equation and corresponding modifications to the momentum and energy equations (Gough, 1969). The modal approximation (with a fixed horizontal ground plan) has been adapted to the anelastic approximation by Latour *et al.* (1975) and used to compute the extent of overshooting from convective zones in A-type stars. Unfortunately this model does not allow the development of smaller cells. However, a two-dimensional anelastic code is being developed at Cambridge. This, combined with Graham's recent three-dimensional compressible calculations may allow us to carry out a systematic study of compressible convection.

5. Cellular Convection in the Sun

It is clear that there is no firm theoretical basis for assuming that energy is everywhere carried by cells with a scale comparable with the local density scale height, as in

normal mixing length theory. The crude argument outlined above suggests that cells might extend over about three density scale heights (Simon and Weiss, 1968) but there are still no reliable calculations to support this estimate. On the other hand, observations show the presence of at least two distinct scales of motion at the surface of the Sun. Photospheric granules have a radius (1000 km) about twice the local density scale height. Supergranules are intimately associated with strong sub-photospheric magnetic fields and must therefore correspond to motion below the surface of the Sun; their radius (15 000 km) corresponds to the density scale height at about 15 000 km depth. Bumba (1967) inferred that there should be a third scale, around 150 000 km, corresponding to giant cells extending throughout the whole convective zone. There is some observational evidence for such giant cells from Doppler measurements of azimuthal velocities (Howard, 1971; Howard and Yoshimura, 1976) and the distribution of magnetic features. Indeed, it seems likely that their presence will be demonstrated from observations before we can succeed in providing a proper theoretical description.

No one has suggested that eddies far from a boundary should be limited by a length-scale smaller than the local scale height and there is general agreement that energy must be carried by large scale cellular motions over the bulk of the convective zone. There is less unanimity over the relation between these giant cells and the observed smaller scales of supergranules and granules. Spiegel (1968) has suggested that they are formed as a result of shear instabilities in thermal boundary layers. In a normal convecting layer, large-scale motions can easily carry energy except near the upper and lower boundaries, where an enhanced temperature gradient develops. In the Sun (or any star with a convection zone produced by ionization of hydrogen) the superadiabatic gradient is high over a narrow region near the surface, whose depth is comparable with the local scale height. At the base of the zone, convection becomes less efficient; Böhm and Stückl (1967), using a non-local mixing length theory, found an enhanced superadiabatic gradient over the bottom 30 000 km of the convective zone. One possibility is that the granules result from small scale turbulence in the upper superadiabatic boundary layer, while the supergranules are similarly generated in the lower boundary layer and somehow penetrate to the upper surface.

The alternative hypothesis, which I myself prefer, is that the giant cells develop smaller scale instabilities (supergranules) which take over the energy transport until they themselves become unstable, allowing granules to carry energy towards the solar surface. This description is an obvious oversimplification. The velocity pattern of supergranules, and apparently that of giant cells too, penetrates into the photo-sphere, though no corresponding temperature variation has been observed. (Hori-zontal temperature variations, unless they are associated with energy transport, seem to be eliminated over a height comparable with the local scale height H, presumably by rapid horizontal motions). Moreover, other scales of motion must also be present: at any level we might expect smaller parasitical eddies, carrying little energy but affecting, for example, the diffusion of magnetic fields. Indeed, small scale motions in a region where energy transport is dominated by supergranules appear to be necessary in order to explain the slow decay of sunspots (Meyer et al., 1974). It is not yet possible to determine observationally whether there is a short wavelength cut-off in the photospheric velocity spectrum (Harvey and Schwarzschild, 1975) and the

spectrum at the surface may be a distorted form of that at greater depths. Experiments by Busse and Whitehead (1974) on convection at high Rayleigh number in a viscous fluid showed the development of a semi-permanent large scale cellular structure within which irregular motions on a smaller scale (comparable with the layer depth) could be seen. If such a pattern were present on the Sun, and could be observed only indirectly through its effect on the magnetic field, only the large scale structure would be seen.

6. Effects of Rotation

The interaction between convection and rotation has been discussed in several reviews (Spiegel, 1972; Gilman, 1974, 1976) and some recent calculations have already been described by Durney (1976) and by Gilman (1976). The generally acepted recipe for a solar dynamo (Parker, 1955) has two essential ingredients, differential rotation and helicity. Helicity is generated by the Coriolis force, acting on cellular convection, and individual dynamo models prove sensitive to the assumed variation of the angular velocity Ω with position in the convective zone. I shall therefore attempt to summarize the possible effects of rotation on the cellular pattern that I have described above.

The importance of rotation in a convection cell can be estimated from the parameter

$$\sigma = 2\Omega l/w = 4\pi\tau/\tau_{\rm rot} , \tag{20}$$

where τ is the turnover time and $\tau_{\rm rot}$ the period of rotation. This parameter (the reciprocal of the Rossby number) measures the ratio of the Coriolis force to the inertial term in the equation of motion, and so the extent to which a fluid element, conserving its angular momentum, is deflected as it traverses a cell. For granules, with a lifetime of minutes, $\sigma \approx 5 \times 10^{-3}$; for supergranules, lasting for a day, $\sigma \approx 0.4$ and for giant cells, with a turnover time of a month, $\sigma \approx 20$. Hence any effect of rotation on granules must be imperceptible. Supergranules will be significantly affected and Coriolis forces will dominate the motion in any giant cell.

When σ is large, two different effects can be distinguished. The first is a consequence of the Proudman-Taylor theorem. In a uniformly rotating system, the rate of generation of vorticity by the Coriolis force is, in the anelastic approximation,

$$\nabla \wedge (2\bar{\rho}\Omega \wedge \mathbf{u}) = 2\Omega \cdot \nabla(\bar{\rho}\mathbf{u}), \tag{21}$$

which vanishes if $\bar{\rho}\mathbf{u}$ does not vary in the direction parallel to the axis of rotation. For a Boussinesq fluid, with $\bar{\rho}$ constant, the constraint imposed by rotation disappears provided $\mathbf{u}$ itself does not vary in this direction. Consider for the moment the simplified problem of convection in an infinite self-gravitating cylinder, rotating about its axis. If convection is everywhere in rolls parallel to the axis of rotation then the Coriolis force can be balanced by a pressure gradient and the motion is unaffected by rotation (except insofar as the density perturbation is coupled to the pressure through the equation of state) even in a compressible fluid.

Convection in a sphere is more complicated. Let (r, θ, φ) be spherical polar co-ordinates, with Ω along the axis $\vartheta = 0$. In a Boussinesq fluid, instability first

appears as a ring of rolls parallel to the rotation axis (Roberts, 1968; Busse, 1970a) and Busse (1975) has demonstrated experimentally that this pattern persists into the non-linear regime. In a spherical shell the eigenfunctions of the marginal stability problem are specified by the spherical harmonics $P_n^m(\cos \theta) \, e^{im\varphi}$. In the absence of rotation, the problem is degenerate with respect to m; Busse (1970b, 1973) used a double perturbation expansion to show that rotation favoured the sectorial harmonics with $m = n$. These sectorial modes (banana cells) show the effect of the Proudman-Taylor constraint even with spherical geometry. Similar results for the non-linear regime were obtained by Durney (1970, 1971), using the mean field approximation, though Gilman's (1976) recent computations show a more complicated pattern of behaviour.

The second effect is the redistribution of angular momentum. The Coriolis force expresses the conservation of angular momentum; with differential rotation such that $r^2 \sin^2 \vartheta \Omega$ is everywhere constant, this constraint is relaxed. If the viscosity is sufficiently small, convection itself can alter the angular velocity distribution so that the angular momentum is nearly uniform except in narrow boundary layers. This redistribution, which allows the development of subcritical instabilities (Veronis, 1959), was found in two-dimensional computations by Veronis (1968). Weir's (1975, 1976) numerical experiments on axisymmetric Boussinesq convection in a sphere show large regions of constant angular momentum, with a sharp gradient near the axis, where Ω is finite but the angular momentum drops to zero.

The Sun's convective zone, with a thickness $\Delta r \approx 0.2 \, R_\odot$ can be divided schematically into two regions, separated by the cylindrical surface parallel to the axis of rotation that encloses the radiative zone. The equatorial region, spanning latitudes less than 35°, resembles the cylindrical model discussed above: $\mathbf{g}$ and Ω are almost perpendicular, though $\bar\rho$ is no longer constant on cylinders. Convection should be dominated by the Proudman-Taylor constraint, favouring cells elongated parallel to the rotation axis. Motion in the plane perpendicular to this axis is effective at transporting heat and need not violate the constraint. In the polar region Ω and $\mathbf{g}$ are nearly parallel and $\Omega \cdot \nabla(\bar\rho \mathbf{u})$ cannot be small if convection is effective in radially transporting heat. (For the linear eigensolution with $m = n$ there is no convective heat flux at the poles.) In this region we might expect to find normal cellular convection, redistributing angular momentum in the radial direction, so that $r^2\Omega$ is approximately constant.

This simplified description suggests that giant cells in the equatorial region will be elongated, like a ring of truncated bananas. It is then tempting to identify this region with the sunspot zone (Hide, 1960, private communication) and to relate the elongated cells to the velocity variations described by Howard and Yoshimura (1976). The angular velocity in this region would be approximately uniform and the rotation period of the convection pattern (not necessarily equal to that of the gas itself) would then be the familiar sidereal period of 25 days. This model is consistent with the existence of active longitudes and the sector structure of magnetic fields. In the polar regions, giant cells would form a tesselated pattern and redistribution of angular momentum could reduce angular velocity at the surface by up to one third; this is consistent with the measured rotation period of 37 days at the poles. Of course, the two regions must merge smoothly together. It is clear from this brief discussion

that a full description of solar convection requires nonlinear, nonaxisymmetric solutions for anelastic convection in a thick spherical shell, with a uniform heat flux maintained at the inner boundary and, say, a fixed temperature at the outer surface. Such an ambitious calculation must be approached by gradual stages. Gilman's (1976) computations already show the range of complicated behaviour that can result as more sophisticated models are investigated. It is only from systematic numerical experiments that convection, and its interaction with rotation, will ultimately be understood.

The chromospheric network does not vary between the poles and the equator, nor should the pattern of convection in supergranules be affected by rotation. Nevertheless, some redistribution of angular momentum will occur. This would suffice to account for the difference of 5% between the rotation rates of deep-seated magnetic features and those directly measured in the photosphere (Foukal, 1972; Foukal and Jokipii, 1975). Both in supergranules and in giant cells motion is dominated by rising columns at the centres of the cells, which spread outwards over most of their height. The rising and expanding gas will be acted upon by Coriolis forces, so as to produce cyclonic motions and the helicity needed to maintain a dynamo. In supergranules, where this effect is relatively small, we might expect the helicity $\overline{\mathbf{u} \cdot \nabla \wedge \mathbf{u}}$ to vary as $\cos \vartheta$ but the distribution of helicity in giant cells must be obtained from a numerical solution. In the equatorial region the α-effect would be reduced by the formation of elongated cells.

7. Cellular Convection and Magnetic Fields

The kinematic effects of convection on a weak magnetic field can be summarized briefly.

(i) Toroidal fields are drawn out from poloidal fields by differential rotation. In an axisymmetric configuration the local rate of production of the toroidal field is given by $\mathbf{B} \cdot \nabla \Omega$ and there are contributions from both radial and latitudinal components of the gradient of the angular velocity. The relative importance of these components varies with position for any choice of $\Omega(\mathbf{r})$. The discussion above suggest that $|\partial \Omega / \partial \vartheta|$ should be large near the boundary of the sunspot zone and that $-\partial \Omega / \partial r$ should be largest in the polar region.

(ii) Rising fluid expands and rotates, dragging up toroidal flux and generating from it a reversed poloidal field, at a rate depending on the local helicity.

(iii) Horizontal velocities rapidly concentrate the magnetic field between cells. This process is seen in high resolution observations of intergranular magnetic structures (Dunn and Zirker, 1973; Mehltretter, 1974; Stenflo, 1976). Magnetic flux is expelled from most of a convection cell, though the lifetime of an eddy is too short for flux to be eliminated from its centre.

(iv) Magnetic flux ropes, concentrated at the boundaries of convection cells, are brought sufficiently close together for reconnection to occur, with the annihilation of opposing fields. The reconnection process itself must be dynamically driven, at a velocity comparable with the Alfvén speed (Priest and Soward, 1976).

(v) In three-dimensional convection cells, sinking fluid forms a continuous network while rising fluid is confined to an array of isolated columns. This topological

difference allows the motion to pump horizontal field downwards, as demonstrated by Drobyshevski and Yuferev (1974). Topological pumping would not occur for elongated two-dimensional cells but should be effective in the polar region. Differences in horizontal velocities caused by compressibility could also concentrate fields at the base of the convective zone. This geometrical pumping has been investigated by Moore and Proctor (1976). Both these mechanisms act in the opposite direction to the well known buoyancy of magnetic flux ropes (Parker, 1955, 1975).

Strong magnetic fields are no longer passively distorted. The concentration of magnetic flux is limited by forces exerted by the field, though the details of this process are not yet properly understood. The concentrated fields are strong enough locally to hinder convection, and the α-effect is quenched either by excluding flux from regions of strong helicity or by suppressing the cyclonic motion. In addition, magnetic fields may affect the mean flow and, in particular, the extent of differential rotation.

All these processes must be included in a proper treatment of the solar cycle. As simplified models the kinematic dynamos of mean field electrodynamics seem convincing. Now we need to see more elaborate treatments, including large scale cellular motions and discontinuous flux ropes. As our understanding of convection gradually improves, the results should be incorporated into more sophisticated dynamo models, which may ultimately provide an accurate and detailed picture of magnetic fields in the Sun.

Acknowledgments

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DISCUSSION

Giovanelli: Since there are differences in the convective behaviour in polar and equatorial regions, could you predict any observable differences to be expected between supergranules in these regions?

Weiss: Supergranules are only slightly affected by rotation, so I do not expect that their shapes or velocities would show any observable variation with latitude. Have any differences been observed?

Schröter: Since observers are asked to comment on Dr Giovanelli's question, I shall try to summarize our experiences during our observations last year and this year (Dr Wöhl and myself). As I reported yesterday, we had every day to preselect 10–15 fine Ca^+-mottles in different solar latitudes for our differential rotation program. My experience (and Dr Wöhl reported to me a very similar impression) is that there was no problem in finding well defined tiny Ca^+-mottles showing an arrangement similar to supergranules in latitudes near the equator. When searching for well defined bright Ca^+-mottles in medium latitudes, we had some problems. In rather high latitudes I easily found tiny bright Ca^+-mottles again, but this time they looked like single, not specifically aligned features. In interpreting this, please do not forget two facts:

(a) These observations refer to a time close to the solar activity minimum (e.g. the Ca^+-mottles close to the equator may well reflect the solar activity belt, the polar Ca^+-mottles the polar faculae, as investigated by Waldmeier).

(b) Our observations refer to Ca^+-structures and not to a velocity pattern which defines supergranules. However, we know the close correlation between both phenomena.

Stenflo: Supergranulation is defined by its velocity pattern, which is associated with magnetic fields and brightness enhancements. The network seen in magnetograms and Ca spectroheliograms varies strongly with latitude and with the solar cycle, but there seems to be no observational evidence that the velocity pattern associated with supergranulation varies with heliographic latitude.

Giovanelli: Some years ago Dr Beckers mentioned to me that he was unable to identify supergranule cells in the chromosphere well away from equatorial regions; this is certainly associated with magnetic differences between the two regions. As far as I am aware, there have been no differences observed in the line-of-sight velocities. Therefore, Dr Weiss' expectation seems to be confirmed

Roxburgh: Were you suggesting that there are steady convective cells extending over several scale heights and if so why does motion not become turbulently unstable since the Reynolds number is very high, of the order of 10^{13}.

Weiss: I certainly do not imagine that there are steady convection cells in the Sun. However, cells lasting for about one turnover time may extend over several scale heights without being prevented by shear instabilities, regardless of the Reynolds number.

Durney: I think that you said that Vickers' results were difficult to understand. Heard has obtained results that are somewhat similar; the large-scale convective motions are large in the lower part of the convection zone and very small in the upper part.

Weiss: I said that Vickers' result, that the temperature perturbation could change sign without a corresponding change in the velocity perturbation was incorrect. Indeed, one can show that such an eigenfunction cannot satisfy the temperature equation. However, it seems a fairly general result that large scale convective motions have lower velocities in regions where the scale-height is small compared with the scale of the motion. This is found in our linear solutions as well as in Vickers' and Heard's results.

Gilman: You have pointed out two mechanisms for convection giving net *inward* transport of magnetic flux. Dr Parker has pointed out that magnetic buoyancy should cause a net *outward* movement of flux. Would you care to speculate on the relative importance of these two effects?

Weiss: It would be rash to make a prediction without any proper calculation. However, my guess is that topological pumping may be able to keep flux ropes deep in the convective zone.

Parker: The network of downflow represented by the supergranule boundaries blocks the escape of magnetic lines of force from the interior of the Sun provided that the downflow velocity v exceeds the rate of rise of the field. Magnetic buoyancy causes a horizontal magnetic flux tube in a region of convective instability to rise at a rate of the general order of magnitude of the Alfvén speed. Thus, very roughly, the lines of force can be blocked from rising to the surface if $V_A < v$. Strong fields come up through regardless of the downflow, as we know from observation.

We must not overlook the fact, however, that any field – weak or strong – can come up in the rising currents at the center of a supergranule, forming a local bipolar region there.

Deinzer: Are there theoretical arguments for the occurrence of three distinct linear scales in the convection zone, represented by granulation, supergranulation and giant cells?

Weiss: There is still nothing more substantial than the crude arguments that I have repeated in my paper. The exact computations so far carried out all show a single cell across the entire unstable layer.