THE POSSIBILITY OF MAGNETIC FIELD ORIGIN IN FINE STRUCTURE ELEMENTS OF SOLAR FEATURES

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Abstract. It is demonstrated that in principle the magnetic field may originate in the fine structure elements of the photosphere and the sunspots as a result of lack of coincidence of isobaric and isothermic surfaces in those elements.

In recent years a large number of papers have appeared in which the fine structure of solar features was considered. Fine structure elements are observed in the photosphere, in sunspots, in faculae etc. They are connected closely with the structure of magnetic fields. The estimations of the magnetic field decay time show that the time scale is comparable with the lifetime of these elements. Naturally the magnetic field must be regenerated, or else the magnetic fields of larger scale may decay during the time interval of the same order, if we consider them as only chance collections of the fine structure elements.

Among various conceivable mechanisms for magnetic field origin conforming to the fine structure elements, an assumption about the magnetic field origin in the stars proposed by Biermann and Schlüter is the most attractive one. If surfaces of equal electronic pressure and concentration do not coincide then a priming magnetic field may appear and may be amplified by any mechanism, for instance by turbulence. Therefore in any inhomogeneous layer of the Sun where suitable conditions exist, the origin of the magnetic field is possible in principle. Maybe this mechanism works in the subphotospheric layers as well as in the outer solar atmosphere.

Here we shall estimate the possibility of magnetic field origin in the fine structure elements of the photosphere and of the sunspots. The cause of currents generating the magnetic field is an electric field connected with the pressure gradient. It is very difficult to consider the whole evolution of the magnetic field because we must solve the full system of nonlinear equations of hydromagnetics. This is accompanied by great computational difficulties and with uncertainties in values of a number of the physical parameters and in the fine structure element models. Earlier we obtained the equations for the electric field caused by the pressure gradient (Kopecký and Kuklin, 1967). One may conclude that in the presence of a magnetic field, there is no full compensation of the pressure gradient drift by the electric field because in a coordinate system connected with different components of the plasma, uncomparable expressions for the electric field are obtained.

So we shall try to estimate the magnetic field origin rate at an initial moment when

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there is no magnetic field either in the fine structure elements or in the environment. Then the expression for the electric field is independent of the coordinate system,

$$\mathbf{E}^* = \frac{1}{en_e}(\varepsilon\mathbf{G} - \nabla p_e). \tag{1}$$

The initial equation of the magnetic field dynamics

$$\frac{\partial \mathbf{H}}{\partial t} = c \operatorname{rot} \mathbf{E}^* + \operatorname{rot}(\mathbf{v} \times \mathbf{H}) + \frac{c^2}{4\pi\sigma} \Delta\mathbf{H} + \frac{c^2}{4\pi\sigma^2}(\nabla\sigma \times \operatorname{rot}\mathbf{H}) \tag{2}$$

becomes

$$\frac{\partial \mathbf{H}}{\partial t} = \frac{c}{e} \operatorname{rot} \frac{\varepsilon\mathbf{G} - \nabla p_e}{n_e} \tag{2'}$$

where

$$\mathbf{G} = \xi_n \nabla(p_e + p_i) - \xi_i \nabla p. \tag{3}$$

Here $\xi_i \approx 1 - \xi_n$ is the relative ion mass and ξ_n is the relative neutral mass. Assuming

$$T_e = T_i = T_n = T = \frac{5040}{\theta}, \quad n_e = n_i = \frac{p_e\theta}{5040\,k}, \quad p_e = p_i = sp$$

we obtain

$$\frac{\partial \mathbf{H}}{\partial t} = -\,5040\,\frac{ck}{e} \operatorname{rot}\left[\frac{\varepsilon(1-\xi_n)}{\theta p_e}\nabla p + \frac{1-2\varepsilon}{\theta p_e}\nabla p_e\right] \tag{4}$$

or introducing

$$\psi_0 = \frac{\varepsilon(1-\xi_n)}{s\theta}, \quad \psi_2 = \frac{1-2\varepsilon}{\theta}$$

$$\frac{\partial \mathbf{H}}{\partial t} = 5040\,\frac{ck}{e}\left[\nabla \ln p \times \nabla\psi_0 + \nabla \ln p_e \times \nabla\psi_2\right]. \tag{4'}$$

The auxiliary values may be computed with the help of the following formulae

$$1 - \xi_n = \frac{sM}{1-2s}, \quad \varepsilon = (1 + Cr_{in}^2)^{-1}, \quad C = \frac{42.69}{r_{en}^2}\sqrt{\frac{\mu_i}{1+M}},$$

where $M = \mu_i/\mu_n$ and μ_i, μ_n are the ion and neutral masses. Let the fine structure element have an axial symmetry and then considering ψ_0 and ψ_2 as functions of θ, $\lg p$ and $\lg p_e$ we obtain

$$\begin{aligned}\frac{\partial H_\varphi}{\partial t} &= 10^8\left[\frac{\partial\psi_0}{\partial\theta}\left(\frac{\partial\theta}{\partial r}\frac{\partial \lg p}{\partial z} - \frac{\partial\theta}{\partial z}\frac{\partial \lg p}{\partial t}\right) + \right.\\ &\quad \left. + \frac{\partial\psi_2}{\partial\theta}\left(\frac{\partial\theta}{\partial r}\frac{\partial \lg p_e}{\partial z} - \frac{\partial\theta}{\partial z}\frac{\partial \lg p_e}{\partial r}\right)\right] = \\ &= 10^8\left[\frac{\partial\psi_0}{\partial\theta}(\nabla \lg p \times \nabla\theta)_\varphi + \frac{\partial\psi_2}{\partial\theta}(\nabla \lg p_e \times \nabla\theta)_\varphi\right].\end{aligned} \tag{5}$$

This expression may be transformed also in such a way

$$\frac{\partial H_\varphi}{\partial t} = 10^8\, \psi(\theta, p)\, (\nabla \lg p \times \nabla\theta)_\varphi \tag{5'}$$

where

$$\psi(\theta, p) = \frac{\partial\psi_0}{\partial\theta} + \frac{\partial\psi_2}{\partial\theta}\left(1 + \frac{\partial \lg s}{\partial \lg p}\right) - \frac{\partial\psi_2}{\partial \lg p}\,\frac{\partial \lg s}{\partial\theta}. \tag{6}$$

In order to make numerical estimations we need information about the geometry of the fine structure elements, their models and tables of functions ψ_0, ψ_2, and ψ. We shall not discuss defects of existing models or an accurate form of these elements because we are interested in making estimations with an accuracy of an order of magnitude, solving principally to see if magnetic field origin is possible in such conditions. Thus we selected the inhomogeneous photosphere model by de Jager (1959) and the inhomogeneous sunspot model by Obridko (1968) assuming the hot elements to be vertical cylindrical columns in the cold medium which are some hundreds of km in length. The adopted geometry of the physical parameter distribution is presented in Figure 1. If we are wrong then the errors in the gradient computation do not exceed one order and most probably we are close to the lower limit of the values.

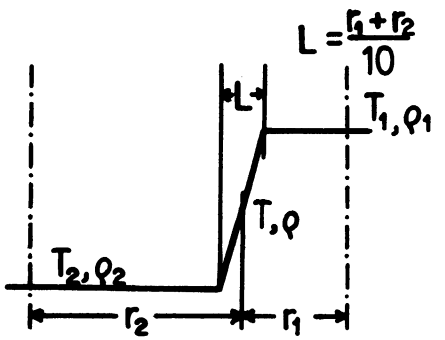

Fig. 1. The adopted distribution of physical parameters in hot and cold elements of the fine structure.

We have computed tables of ψ_0, ψ_2, and ψ using the data published in our previous papers (Kuklin, 1966; Kopecký and Kuklin, 1969) for the same composition of the solar atmosphere. Only the selection of the ion-neutral interaction cross-section value r_{in}^2 is uncertain. We made computations of tables for r_{in}^2 equal to 10^{-16}, 10^{-14} cm^2 but all estimations of the magnetic field origin rate are made for $r_{in}^2 = 10^{-15}$ cm^2.

We used all 3 variants of the inhomogeneous sunspot model by Obridko and by this for the initial model of Michard we considered the value α in computations of dark element sizes assuming for bright elements $r_1 = 150$ km. In the case of the initial model by Fricke-Elsässer we took a distance between centers of bright and dark elements $r_1 + r_2 = 1000$ km. The results of our computations are presented in Figure 2 (1: the initial model by Fricke-Elsässer; 2: initial model by Michard,

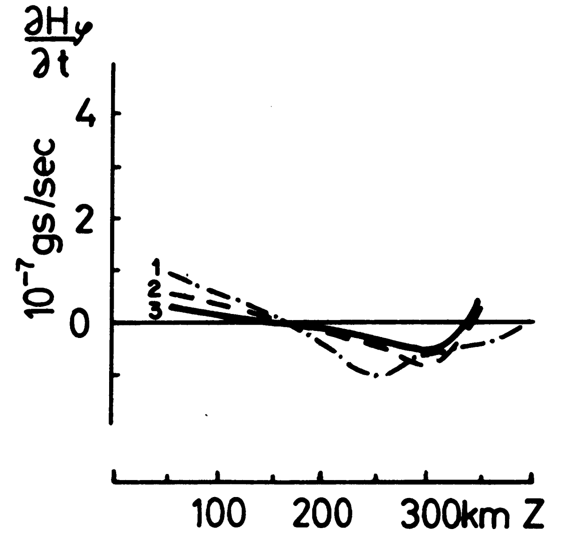

Fig. 2. The rate of magnetic field origin in the fine structure elements of the two-component umbra models: (1) Initial model by Fricke and Elsässer; (2) Initial model by Michard $\alpha = 0.10$; (3) Initial model by Michard $\alpha = 0.05$.

TABLE I

Values of $1 - \xi_n$

$\lg p$ \ θ	0.8	1.1	1.4	1.7
3.0	3.57×10^{-3}	1.057×10^{-3}	3.49×10^{-4}	7.76×10^{-5}
4.5	1.16×10^{-3}	7.04×10^{-4}	1.23×10^{-4}	3.33×10^{-5}
6.0	6.85×10^{-4}	2.64×10^{-4}	4.58×10^{-5}	9.41×10^{-6}

TABLE II

Values of ε

r_{in}^2	$\lg p$ \ θ	0.8	1.1	1.4	1.7
	3.0	1.45×10^{-1}	1.22×10^{-1}	1.30×10^{-1}	1.26×10^{-1}
10^{-16}	4.5	1.30×10^{-1}	1.22×10^{-1}	1.26×10^{-1}	0.97×10^{-1}
	6.0	1.16×10^{-1}	1.17×10^{-1}	0.93×10^{-1}	0.67×10^{-1}
	3.0	1.66×10^{-2}	1.38×10^{-2}	1.47×10^{-2}	1.42×10^{-2}
10^{-15}	4.5	1.48×10^{-2}	1.37×10^{-2}	1.42×10^{-2}	1.06×10^{-2}
	6.0	1.29×10^{-2}	1.31×10^{-2}	1.02×10^{-2}	0.71×10^{-2}
	3.0	1.69×10^{-3}	1.39×10^{-3}	1.49×10^{-3}	1.44×10^{-3}
10^{-14}	4.5	1.50×10^{-3}	1.39×10^{-3}	1.44×10^{-3}	1.07×10^{-3}
	6.0	1.31×10^{-3}	1.32×10^{-3}	1.03×10^{-3}	0.72×10^{-3}

TABLE III

Values of ψ_0

r_{in}^2	$\lg p$ \ θ	0.8	1.1	1.4	1.7
	3.0	1.82×10^{-1}	1.78×10^{0}	1.78×10^{0}	1.45×10^{0}
10^{-16}	4.5	2.85×10^{-1}	2.18×10^{0}	1.72×10^{0}	7.92×10^{-1}
	6.0	7.18×10^{-1}	1.99×10^{0}	9.67×10^{-1}	4.26×10^{-1}
	3.0	2.09×10^{-2}	2.00×10^{-1}	2.01×10^{-1}	1.63×10^{-1}
10^{-15}	4.5	3.24×10^{-2}	2.41×10^{-1}	1.94×10^{-1}	8.68×10^{-2}
	6.0	8.02×10^{-2}	2.23×10^{-1}	1.06×10^{-1}	4.53×10^{-2}
	3.0	2.12×10^{-3}	2.03×10^{-2}	2.04×10^{-2}	1.65×10^{-2}
10^{-14}	4.5	3.28×10^{-3}	2.44×10^{-2}	1.97×10^{-2}	8.77×10^{-3}
	6.0	8.11×10^{-3}	2.25×10^{-2}	1.07×10^{-2}	4.56×10^{-3}

TABLE IV

Values of $10\psi_2$

r_{in}^2	$\lg p$ \ θ	0.8	1.1	1.4	1.7
	3.0	8.89	6.86	5.29	4.40
10^{-16}	4.5	9.24	6.88	5.34	4.74
	6.0	9.60	6.96	5.81	5.09
	3.0	12.08	8.84	6.93	5.72
10^{-15}	4.5	12.13	8.84	6.94	5.76
	6.0	12.18	8.85	7.00	5.80
	3.0	12.46	9.07	7.12	5.87
10^{-14}	4.5	12.46	9.07	7.12	5.87
	6.0	12.47	9.07	7.13	5.87

TABLE V

Values of $\partial\psi_0/\partial\theta$

r_{in}^2	$\lg p$ \ θ	0.8	1.1	1.4	1.7
	3.0	9.42	1.95	-1.27	-0.24
10^{-16}	4.5	12.37	1.34	-3.37	-1.77
	6.0	11.15	-1.13	-4.15	2.09
	3.0	1.050	0.223	-0.139	-0.038
10^{-15}	4.5	1.341	0.161	-0.366	-0.240
	6.0	1.257	-0.133	-0.481	0.244
	3.0	0.1063	0.0226	-0.0141	-0.0037
10^{-14}	4.5	0.1357	0.0163	-0.0371	-0.0246
	6.0	0.1273	-0.0135	-0.0477	0.0248

TABLE VI

Values of $\partial\psi_2/\partial\theta$

r_{in}^2	θ / $\lg p$	0.8	1.1	1.4	1.7
	3.0	−0.723	−0.612	−0.424	−0.159
10^{-16}	4.5	−0.915	−0.656	−0.362	−0.032
	6.0	−1.246	−0.574	−0.253	−0.282
	3.0	−1.376	−0.822	−0.485	−0.363
10^{-15}	4.5	−1.401	−0.828	−0.477	−0.347
	6.0	−1.422	−0.818	−0.464	−0.380
	3.0	−1.457	−0.847	−0.491	−0.389
10^{-14}	4.5	−1.459	−0.848	−0.490	−0.387
	6.0	−1.464	−0.847	−0.489	−0.390

TABLE VII

Values of $\psi(\theta, p)$

r_{in}^2	θ / $\lg p$	0.8	1.1	1.4	1.7
	3.0	9.30	1.46	−1.59	−0.34
10^{-16}	4.5	12.06	0.84	−3.60	−1.77
	6.0	10.57	−1.50	−4.28	1.98
	3.0	0.508	−0.436	−0.482	−0.335
10^{-15}	4.5	0.633	−0.485	−0.715	−0.522
	6.0	0.438	−0.695	−0.848	0.026
	3.0	−0.490	−0.655	−0.358	−0.326
10^{-14}	4.5	−0.621	−0.645	−0.399	−0.342
	6.0	−0.719	−0.597	−0.443	−0.205

TABLE VIII

Values of $10^7\ \partial H\varphi/\partial t$ G/s

z	Initial sunspot umbra models by			Photosphere model by de Jager
	Michard $\alpha = 0.05$	Michard $\alpha = 0.10$	Fricke-Elsässer	
50	+0.29	+0.53	+0.94	
100	+0.17	+0.32	+0.54	
150	+0.06	+0.12	+0.17	−0.05
200	−0.12	−0.16	−0.40	+0.50
250	−0.31	−0.43	−0.98	+0.64
300	−0.49	−0.78	−0.59	
350	+0.49	+0.30	−0.39	
400			+0.09	

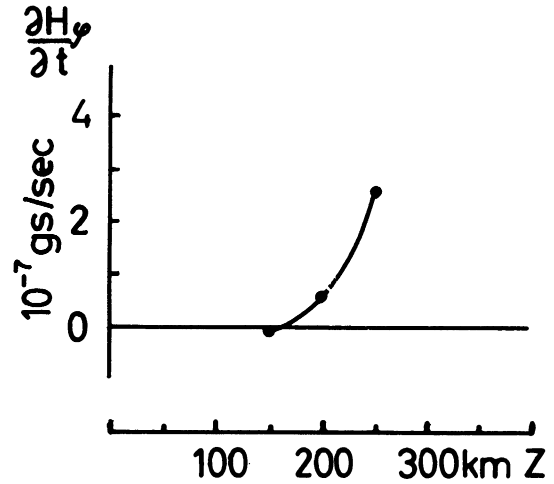

Fig. 3. The rate of the magnetic field origin in the fine structure elements of the inhomogeneous photospheric model by de Jager.

$\alpha = 0.10$; 3: $\alpha = 0.05$). We used here the expression (5′). In the case of the inhomogeneous photosphere model by de Jager we took $r_1 = 0''.8 \approx 580$ km for hot elements and $r_2 = 0''.4 \approx 290$ km for cold elements. In computations the expression (5) was used but strictly speaking this is not sufficiently correct because the solar atmosphere composition differs from that for which $\partial\psi_0/\partial\theta$ and $\partial\psi_2/\partial\theta$ are computed. For estimations with accuracy up to an order of magnitude we can neglect this contradiction. The results are given in Figure 3.

One may see that according to Figure 2 and 3 and Table VIII at the initial moment when the external magnetic field is absent in the fine structure elements, the toroidal magnetic field originates with a rate close to 10^{-7} G/s. Usually above the depth level 150 km its direction is opposite to that under this level. For rough estimation, using the life time of the order of 10^3 s, we obtain a priming magnetic field of the order of 10^{-4} G which is sufficient for the action of the amplifying mechanisms. It is difficult to estimate how long this effect will work because as the magnetic field arises so the complex processes of plasma component diffusion and magnetic field dissipation begin to act. Then the velocity field is important which may be neglected if $H = 0$. Returning to the sunspot models we may find that the difference of the gas pressures in dark and bright elements is of such an order that the corresponding difference in the magnetic field intensities is of the same order. Naturally then our computations made for boundary layers without magnetic field are very conditional ones. Finally according to Wilson (1969) and Teplitzkaya (1970) the hot fine structure elements cannot stretch deeper than 230–250 km. Therefore with the adopted distribution geometry at the bottom of the elements we reach the maximum values 3–10 times larger than at 150 km level.

In spite of these objections, we presume to declare that in general the magnetic field may originate in the fine structure elements of the photosphere and the sunspots as a result of lack of coincidence of isobaric and isothermic surfaces in those elements. In the future we hope to succeed in numerical simulation of the evolution of magnetic fields appearing in such a way.

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