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A conceptual model for pancake-ice formation in a wave field

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ABSTRACT. It is well known that waves jostle frazil-ice crystals together, providing opportunities for them to compact into floes. The shape of these consolidated floes is nearly circular, hence the name: pancakes. From field and laboratory observations, the size of these pancakes seems to depend on the wavelength. Further aggregation of these individual pancakes produces the seasonal ice cover. Pancake-ice covers prevail in the marginal ice zone of the Southern Ocean and along the edges of many polar and subpolar seas. This theoretical paper describes conceptually the formation process of a pancake-ice cover in a wave field. The mechanisms that affect the evolution of the entire ice cover are discussed. Governing equations for the time-rate change of the floe diameter, its thickness and the thickness of the ice-cover are derived based on these mechanisms. Two criteria are proposed to determine the maximum floe size under a given wave condition, one based on bending and the other on stretching. Field and laboratory observations to date are discussed in view of this theory.

INTRODUCTION

Pancake ice is commonly found in polar regions where waves are present. Lange and others (1989) described their observation of this phenomenon in the Weddell Sea. Wadhams (1991) estimated the areal coverage of pancake ice in the Southern Ocean to be 6×10^6 km^2. In many marginal ice zones of the Northern Hemisphere, pancake ice is also commonplace, including the Odden ice tongue of the Greenland Sea, the Okhotsk Sea and the Bering Sea. This paper provides a theoretical description of the formation of pancake ice, its interaction with the wave field, and compares the theory with some existing field and laboratory data.

Based on field and laboratory observations, we propose a conceptual model for pancake-ice formation as described in Figure 1. The three stages of the formation process are illustrated below.

(1) *Grease-ice stage*: As the water becomes supercooled, frazil crystals are formed. These undergo rapid dendritic growth. Collisions in the turbulent water cause the frazil crystals to erode the sharp spikes of the dendrites and to adhere to each other to form millimeter-size discoid frazils (Ashton, 1986). The discoid frazils float to the water surface broad side up and form a thin layer. The high-frequency part of the wave spectrum herds the discoid frazils together. The viscosity of this frazil slurry and the inelastic collisions and rafting between the tiny discoid frazils consume the high-frequency wave energy, resulting in a smooth surface, hence the name "grease ice".

(2) *Pancake formation stage*: As discoid frazils continue to form beneath the cover, the increasing buoyancy of the accumulation exposes the surface to cold air. This exposure freezes the cover to form an ice sheet. At the same time, bending and stretching from waves limit the lateral growth of the pancakes. Collisions between pancakes form the characteristic raised edges. The size of the pancakes depends on the high-frequency (short-wavelength) part of the spectrum that produces the most surface curvature. As rafting and collisions consume the higher-frequency wave energy, the process is driven by progressively longer wavelengths, resulting in larger pancakes.

(3) *Composite pancake stage*: The longer wavelengths work on the pancakes in exactly the same way as the short wave-

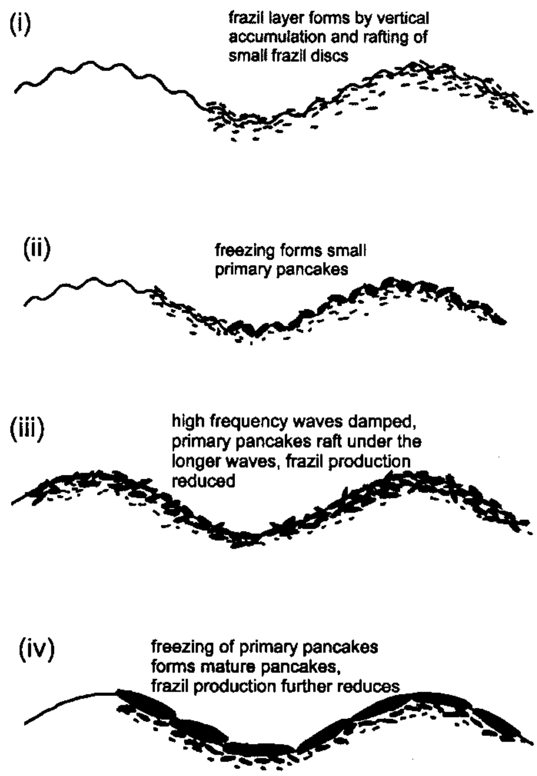

Fig. 1. Schematics of the pancake-ice formation process.

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lengths work on the discoid frazils. Thus, differential drift herds and rafts the primary pancakes together. Freezing between these neighboring pancakes forms composite floes. As the floes grow in size, the ice field acts like a low-pass filter with a decreasing low-frequency cut-off. The growth process is a competition between the vertical thickening of the floes due to rafting and the lateral growth of the floes by freezing with neighboring floes. The floes continue to form ever larger composites until either the wave field is attenuated so much that all floes stop jostling or the wave field is so strong that it keeps the floes from further lateral growth.

Three laboratory tests were carried out to more closely observe the formation of pancake ice in a wave field. Two of these tests were done at the U.S. Army Cold Regions Research and Engineering Laboratory (CRREL) and one at the Hamburg Ship Model Basin (HSVA). The latter was part of a large interdisciplinary experiment INTERICE (Eicken and others, 1998). The CRREL experiments are summarized in Shen and Ackley (1995) and Ackley and Shen (1996). A preliminary report of the HSVA experiment is given in Leonard and others (1999b), with more details in Leonard and others (1999a). We will discuss these tests after we formulate the theory in the following sections.

THEORY

As described in the Introduction, there are four stages in the formation process. Each stage may be the final one if the wave energy is not sufficient to allow further development into the next stage. For example, a frazil slick may be the final stage, and no identifiable pancakes would form before the wave attenuates from the rapid frazil production. The final stage may be dispersed pancakes with uniform circular shape, and no further formation of composite floes will take place because additional freezing between pancakes fails to occur due to a warming air temperature.

The grease-ice stage is marked by rapid production of frazil ice. Formation of frazil and grease ice is attributed to turbulence in supercooled water. Field observations of this stage in the marginal sea-ice zone are given in Wadhams (1980), Martin and Kauffman (1981) and Weeks and Ackley (1982). Tsang and Hanley (1985) conducted laboratory studies with sea water. The morphology and dynamics of frazil-ice formation are described in the review report of Daly (1984). Omstedt (1985) derived a mathematical model for the formation process. More details of these and other related studies can be found in Daly (1994). Some recent observations of this initial stage are reported in Leonard and others (1999a) and Smedsrud (2000).

As described in these reports, frazil crystals are initially irregular in shape. As they grow in size, they mature into disk shape with diameter 1–2 mm and thickness 10–100 μm. It takes about 1 h from the appearance of initial irregular-shaped crystals for these circular discoid frazils to appear. The discoid frazils grow unstable notches at the edges. These irregular protuberances can be easily eroded from collision interactions. Hence, the mature discoid frazils do not grow in size despite further freezing. These discoids freeze together to form flocs on the water surface. The World Meteorological Organization (1970) calls this type of ice "grease ice", due to the smooth appearance of the water surface after these ice flocs form. These observations were reconfirmed in all three of our laboratory tests. The rate of frazil- and grease-ice production is dependent on the turbulence level and the air temperature, as shown in Omstedt (1985) and Daly (1994). In this study we concentrate on the stages after the grease ice is formed.

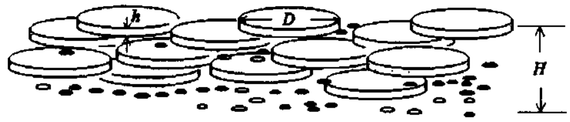

Fig. 2. Definition sketch of the pancake-ice cover.

At any given stage of the formation process, there are three variables that may be used to define the pancake-ice cover. As shown in Figure 2, we denote these three variables by D, h and H. The pancake diameter is D, its thickness is h and the thickness of the pancake-ice cover is H. In the grease-ice stage, no pancake floes have yet been formed. In this initial stage, D is the discoid frazil diameter, h is its thickness and H is the thickness of the entire grease-ice cover. After the formation of pancake floes, as observations show, pancake-ice cover is a mixture of well-defined nearly circular floes and discoid frazils. The interstices in the floe aggregates as well as the water body below the pancake-ice cover are occupied by frazil ice.

Rafting among individual pancake floes is one source for H to be multiples of h. The growth due to freezing, from either frazil accumulation underneath, or columnar growth after frazil production stops, is another. The central question in this paper is to formulate the time evolution of these three variables. We will write them down as follows.

The first equation is for floe diameter,

$$\frac{\partial D}{\partial t} + \nabla \cdot (D\vec{V}_{\mathrm{drift}}) = f_{\mathrm{f}}^{\mathrm{D}} + f_{\mathrm{c}}^{\mathrm{D}}\,, \tag{1}$$

where $f_{\mathrm{f}}^{\mathrm{D}}$ is the rate of growth of the diameter due to frazil incorporation on the floe perimeter, and $f_{\mathrm{c}}^{\mathrm{D}}$ is the rate of growth of the diameter due to freezing between contacting neighboring floes.

The second is for floe thickness,

$$\frac{\partial h}{\partial t} + \nabla \cdot (h\vec{V}_{\mathrm{drift}}) = f_{\mathrm{f}}^{\mathrm{h}} + f_{\mathrm{c}}^{\mathrm{h}}\,, \tag{2}$$

where $f_{\mathrm{f}}^{\mathrm{h}}$ is the rate of growth of floe thickness due to freezing of frazil crystals from the bottom of the floes, and $f_{\mathrm{c}}^{\mathrm{h}}$ is the rate of growth of floe thickness due to freezing between top and underlying floes.

The third is for the ice-cover thickness,

$$\frac{\partial H}{\partial t} + \nabla \cdot (H\vec{V}_{\mathrm{drift}}) = f_{\mathrm{f}}^{\mathrm{H}} + f_{\mathrm{r}}^{\mathrm{H}}\,, \tag{3}$$

where $f_{\mathrm{f}}^{\mathrm{H}}$ is the rate of growth due to thermal/turbulence production of frazil ice that rises under the ice cover, and $f_{\mathrm{r}}^{\mathrm{H}}$ is the rate of growth due to rafting of existing disks of size D.

Suppose we take a Lagrangian reference frame that moves with the drift velocity of the ice floes. In this reference frame the evolution of the ice cover is entirely defined by the right-hand side of Equations (1–3). We now discuss the physical meaning of terms on the righthand side of these equations.

Floe diameters can grow due to freezing between contacting neighboring floes. The rate of this growth is indicated by the term $f_{\mathrm{c}}^{\mathrm{D}}$. This phenomenon occurs at any stage, including the frazil/grease-ice stage, in which frazil flocs or grease ice are formed from contacting frazil crystals. In salt water, however, bonding between frazil crystals is

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weak and continuously broken by mechanical forces. There are two sources for these mechanical forces: the wave field and water turbulence. In the absence of these mechanical forces, surface frazil crystals freeze into a smooth continuous ice sheet, as can be observed in quiescent lakes.

After pancake floes are formed, in the absence of freezing between two neighbors, floe diameter grows purely from horizontal incorporation of frazil ice. The supply of frazil ice to the water surface continues as long as the water column is supercooled. The term f_f^D accounts for this growth. This term is a function of the heat loss from the water surface, the water-column temperature and the turbulence distribution in the water column. The first and second factors determine the frazil production in the water column, and the third determines the frazil concentration distribution. Only the near-surface concentration is related to the pancake growth. The frazil crystals not only adhere to the perimeter of the pancake, but they also pile up at the edge when neighboring floes collide and exert compressive stress to form raised edges. The height of these raised edges is a function of the compliance of the frazil accumulation. The raised edges are exposed to colder air temperature. With the salt water drained away from the exposed edges and the colder air temperature within the raised edges, frazil/frazil bonds become stronger and the accumulation hardens. The height of the raised edge stops increasing. Raised edges provide a larger surface of contact between neighboring floes and enhance the formation of the frozen bonds.

The part of a floe that is exposed to air is more likely to form strong bonds. That is why pancake ice always has a hard frozen top and a slushy bottom. With each collision between two neighboring pancakes, there is the opportunity to form a frozen bond at the contact. Having a raised edge, neighboring floes not only have a larger area of contact, but the contact area that is exposed in air also increases. Bonding between floes becomes more likely. The strength of the bond against external forces determines whether the bonding is successful. If successful, the floe diameter will increase. These considerations will be used to formulate the f_c^D term later.

A pancake floe has an ill-defined thickness, due to the slushy bottom of the floes as described above. The slushy bottom is an accumulation of frazil ice continuously formed in the turbulent water column. As the slush accumulates, the buoyancy increases to lift the existing pancakes up into the cold air. Freezing goes deeper down into the pancakes, and the hardened layer thickens. We define h as the thickness of the frozen layer, excluding the slushy part. This rate of growth of h as indicated by f_f^h is a function of buoyancy and the air temperature. Buoyancy is a function of the thickness of the slushy layer, or the production rate of the frazil and its vertical distribution.

Pancake floes can also freeze together in the vertical direction. If the ice cover consists of several layers of stacked-up pancake floes, the temperature locally is supercooled and the wave action between the top and underlying floes is not too strong, then bonding can be established between top and underlying floes. The conditions required for the f_c^h and f_c^D terms are identical. However, because the local temperature during the ice-formation process is always colder at the water surface than underneath, and the freezing bonding is stronger in air than in salt water, more favorable conditions exist for f_c^D than for f_c^h.

The thickness of a pancake-ice cover H includes both the surface layer of hardened pancakes and the slushy bottom layer. If rafting occurs among the hardened pancakes, then we may have layers of pancakes stacked up together with a slushy bottom. The interstitial frazil collection between pancakes is negligibly small. Hence, the growth of H includes both rafting and the growth of the slush. The former is included in f_r^H and the latter in f_f^H.

FUNCTIONAL FORMS OF THE SOURCE TERMS

The source terms in the growth equations are f_f^D, f_c^D, f_f^h, f_c^h, f_f^H and f_r^H. We will obtain functional forms of these terms below.

Let V be the drift velocity of a pancake floe relative to the surrounding fluid. Assuming that frazil crystals move with the local water velocity, then the volume swept by one floe is Vdh. Let c_{fs} be the frazil concentration at the water surface, then

$$f_f^D = c_{fs} V D h K_1 \,, \tag{4}$$

where K_1 is the probability of freezing incorporation upon contact between a frazil crystal and the existing floe. K_1 is a function of at least the air temperature and the water surface temperature.

Collisions among neighboring floes provide opportunities to freeze two floes together. Hence,

$$f_c^D = F K_2 D \,, \tag{5}$$

where F is the collision frequency per floe with its neighbors, and K_2 is the probability of successful freezing together upon contact. F is a function of the wave amplitude and wavelength, as well as the floe concentration and their material properties. This term is investigated in Shen and Ackley (1991), Frankenstein and Shen (1993) and Shen and Squire (1998). K_2 is a function of at least the air temperature and the height of the floe edges.

The thickness of the floes can increase by extending the frozen front downward. This is accomplished by the increasing buoyancy due to accumulation of frazil crystals at the bottom of the ice cover. In the absence of such accumulation, conductive heat loss through the ice-cover surface can also increase the frozen depth, at a much slower rate. Hence, if we ignore the conductive effect and consider only dynamic growth, then the rate of ice production contributes to the rising rate of ice cover as follows:

$$\begin{aligned} f_h^f &= q_{\mathrm{frazil}} \frac{1}{\nu} \frac{\rho_{\mathrm{slurry}}}{\rho_w} = q_{\mathrm{frazil}} \frac{1}{\nu} \frac{\nu \rho_w + (1-\nu)\rho_{\mathrm{ice}}}{\rho_w} \\ &= q_{\mathrm{frazil}} \frac{1}{\nu} (0.9 + 0.1\nu) \,, \end{aligned} \tag{6}$$

where q_{frazil} is the volume rate of frazil production per unit volume, ν is the porosity of the frazil/water slurry at the bottom of the ice cover, and $\rho_{\mathrm{ice}} \cong 0.9\rho_w$. We assumed all frazil production rises to the underside of the ice cover. This is a good approximation if the turbulence in the water column is low.

The vertical incorporation of neighboring floes is essentially the same mechanism as the incorporation of frazil crystals. Whenever the contact point between two vertically adjacent floes is exposed to a supercooled condition, bonding will occur. The bonding can be ruptured again by subsequent wave agitation. The deepening of the frozen front due to thermodynamics alone can also freeze vertically adjacent floes together, even when the contact point is submerged. This situation requires low wave activity, so that the contact

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point is not mechanically stressed. Hence, to the leading order, f_c^h is a pure thermodynamic function

$$f_c^h = \text{function}(T_{\text{air}}, T_{\text{water}}, k_{\text{ice}}, c_{\text{ice}}, H)\,. \qquad (7)$$

Temperatures T_{air}, T_{water}, heat conductivity k_{ice}, specific heat c_{ice} and the total ice-cover thickness H affect the temperature-profile evolution in the ice cover (Maykut and Untersteiner, 1971), and hence determine the movement of the frozen front.

The rate of thermal growth of the pancake-ice cover due to the net frazil production rate is

$$f_f^H = q_{\text{frazil}} \frac{1}{\nu}\,. \qquad (8)$$

The rafting contribution f_r^H is a pure mechanical term given by

$$f_r^H = \text{function}(A, L, D, h, E, \mu, \eta)\,, \qquad (9)$$

where A is the wave amplitude, L is the wavelength, E is the Young's modulus of the ice floes, μ is the friction between floes, and η is the viscosity coefficient of the floes. Details on how to model this term are discussed in Hopkins and Shen (2001).

So far we have only considered the influence of waves on ice. There is a strong feedback from the ice to the wave that must be included to complete the theory. Wave attenuation has been commonly observed in the field (e.g. Wadhams and others, 1988). Laboratory studies also reported this phenomenon in the grease-ice regime (Martin and Kauffman, 1981; Newyear and Martin, 1997). For the pancake-ice regime, the attenuation process can be formulated in the following way. Let the wave field be described by the spectrum function $f(A, L)$. This spectrum can change due to damping in the system,

$$\frac{\mathrm{d}f}{\mathrm{d}t} = \varepsilon_c + \varepsilon_\nu + \varepsilon_w\,. \qquad (10)$$

In the above, the lefthand side is the Lagrangian derivative of the power spectrum, and the righthand side consists of three dissipation functions. The first comes from floe–floe collisions. The second is from the viscoelastic (or pure viscous) properties of the ice cover as a whole. The third is from the turbulence eddy in the water column. These three and a fourth term from wave reflections are discussed in Shen and Squire (1998). Here we ignore the term from wave reflection because floe–floe gaps in a pancake-ice cover are very small and usually filled with frazil ice. Hence no internal reflections are possible.

Each of the dissipation terms in Equation (10) is a function of the wave amplitude and wavelength, as well as parameters from the ice cover itself. Therefore, as the ice cover evolves, so does the wave spectrum. Equations (1–3) must be solved simultaneously with Equation (10). Together, Equations (1–3) and (10) represent a complete formulation for the pancake-ice formation in a wave field.

The above theoretical formulation includes a large number of parameters and unknown functions. A comprehensive and quantitative model for pancake-ice formation under a given wave and weather condition awaits careful determination of these parameters and functions. However, some postulates may be made concerning the limiting size of pancake-ice floes in a given wave field, as will be discussed below.

LIMITING SIZE OF PANCAKE FLOES

As the pancake floes grow in diameter through freezing with neighboring floes, we believe that there are two modes under which ice floes can break their bonding with neighboring floes. The first is bending failure and the second is stretching failure. Both are due to tensile stress created by differential wave force acting on two floes joined by freezing. This differential force is a result of the wave curvature, as we will see below. Each failure mode determines a maximum size above which frozen bonds break. The limiting size of the pancake diameter is the smaller of these two maximum sizes.

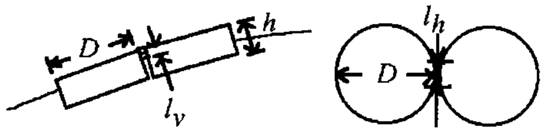

Fig. 3. Two floes frozen together on top of a wave field. The one on left is the side view and the one on right is the top view. The frozen area has a vertical dimension of l_ν and horizontal dimension of l_h. Both floes can be composites of smaller floes formed at an earlier stage.

First we consider the possibility of a bending failure. Let two pancake floes be temporarily frozen together as shown in Figure 3. These floes may be composites of smaller floes formed at an earlier stage.

If we approximate the two-floe unit by an elastic plate, as the wave field passes by, the maximum bending strain in the plate is (Fox and Squire, 1991)

$$\varepsilon_{\max} \approx h \left.\frac{\partial^2 \eta}{\partial x^2}\right|_{\max} T = hAk^2T = hA\frac{(2\pi)^2}{L^2}T\,, \qquad (11)$$

where h is the ice-cover thickness, T is the transmission coefficient and the wave field is assumed to be monochromatic with the water surface defined as

$$\eta = A\cos(kx - \omega t)\,, \qquad (12)$$

where k is the wavenumber and ω is the angular frequency. We assume that the floe diameter is much smaller than the wavelength and hence T is nearly 1. The resulting tensile stress due to this bending is $E\varepsilon_{\max}$. If we assume a linear distribution of the tensile stress through the depth of the frozen bond, the total tensile force is $E\varepsilon_{\max}(l_\nu l_h/2)$. Gold (1971) reported that the bearing capacity of an ice cover is proportional to h^2. We thus estimate that maximum bending strength of the pair of ice floes is also proportional to h^2. Hence the criterion for breaking a frozen bond is

$$EhA\frac{(2\pi)^2}{L^2}\frac{l_\nu l_h}{2} = C_1 h^2\,, \qquad (13)$$

where C_1 likely increases as temperature decreases. However, no data are available at this point.

It is reasonable to estimate that $l_\nu \approx h$ and $l_h \approx D$. Substituting these into Equation (13), the maximum diameter is obtained as

$$D_{\max} \approx \frac{C_1 L^2}{2\pi^2 EA}\,. \qquad (14)$$

Next we examine the stretching-failure mode. Consider two identical pancake floes that have formed a frozen bond as shown in Figure 4. The acceleration of floe 1 is indicated by $\vec{a}_1$ and that of floe 2 is $\vec{a}_2$.

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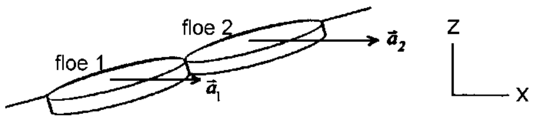

Fig. 4. Two floes frozen together in a wave field. Both floes can be composites of smaller floes formed at an earlier stage.

Assuming $m_1 = m_2$, the dynamic equations for the two floes are

$$\begin{aligned}(1+C_\mathrm{m})m\vec{a}_1 &= (\vec{F}_\mathrm{p}+\vec{F}_\mathrm{g}+\vec{F}_\mathrm{d}+\vec{F}_\mathrm{f}+\vec{F}_\mathrm{c})_1\,, \text{ and}\\(1+C_\mathrm{m})m\vec{a}_2 &= (\vec{F}_\mathrm{p}+\vec{F}_\mathrm{g}+\vec{F}_\mathrm{d}+\vec{F}_\mathrm{f}+\vec{F}_\mathrm{c})_2\,,\end{aligned} \tag{15}$$

where C_m is the added mass coefficient, $\vec{F}_\mathrm{p}$ is the buoyancy, water-pressure induced force, $\vec{F}_\mathrm{g}$ is the gravity force, $\vec{F}_\mathrm{d}$ is the water drag, $\vec{F}_\mathrm{f}$ is the cohesive force due to freezing, and $\vec{F}_\mathrm{c}$ is the compressive force at the contact.

At the moment of separation, $\vec{F}_\mathrm{c} = 0$ and the two right-hand sides of Equations (15) become equal. Hence, $\Delta\vec{F}_\mathrm{p} + \Delta\vec{F}_\mathrm{g} + \Delta\vec{F}_\mathrm{d} + \vec{F}_{\mathrm{f}_1} - \vec{F}_{\mathrm{f}_2} = 0$, where Δ represents the variation between centers of mass of floes 1 and 2. Since $\vec{F}_{\mathrm{f}_1} = -\vec{F}_{\mathrm{f}_2}$, the above reduces to

$$\Delta\vec{F}_\mathrm{p} + \Delta\vec{F}_\mathrm{g} + \Delta\vec{F}_\mathrm{d} = 2\vec{F}_{\mathrm{f}_2}\,. \tag{16}$$

To simplify the analysis, consider x-direction motion only. We apply

$$\Delta F = \frac{\partial F}{\partial x}\Delta x = D\frac{\partial F}{\partial x}\,. \tag{17}$$

Then, since $\Delta F_{\mathrm{p}x} \cong (\partial\eta/\partial x)mg$,

$$\begin{aligned}\Delta F_{\mathrm{p}x} &\cong D\frac{\partial}{\partial x}\left(\frac{\partial\eta}{\partial x}\right)mg = D\frac{\partial^2\eta}{\partial x^2}\frac{\pi}{4}D^2h\rho_\mathrm{ice}g\\&= \frac{\pi}{4}\rho_\mathrm{ice}D^3hAgk^2\cos(kx-\omega t)\\&\approx \frac{\pi}{4}\rho_\mathrm{ice}D^3hAgk^2\,.\end{aligned} \tag{18}$$

Because of the $(\partial^2\eta/\partial x^2)$ term, this force is wave-curvature dependent. The gravity forces acting on the two floes are identical and in the z direction only, hence they drop out of this analysis. The drag force in the x direction is

$$F_{\mathrm{d}x} = \frac{\pi}{4}D^2C_\mathrm{d}\rho_\mathrm{w}|\vec{\nu}_\mathrm{w} - \vec{\nu}_\mathrm{ice}|(\nu_{\mathrm{w}x} - \nu_{\mathrm{ice}x})\,, \tag{19}$$

where $\nu_{\mathrm{w}x} = A\omega\sin(kx-\omega t)$. At the separating moment, $\Delta\vec{\nu}_\mathrm{ice} \cong 0$, hence

$$\begin{aligned}\Delta F_{\mathrm{d}x} &\cong \frac{\pi}{2}D^2C_\mathrm{d}\rho_\mathrm{w}|\vec{\nu}_\mathrm{w} - \vec{\nu}_\mathrm{ice}|\Delta\nu_{\mathrm{w}x}\\&\cong \frac{\pi}{2}D^2C_\mathrm{d}\rho_\mathrm{w}|\vec{\nu}_\mathrm{w} - \vec{\nu}_\mathrm{ice}|DA\omega k\cos(kx-\omega t)\\&\cong \frac{\pi}{2}C_\mathrm{d}\rho_\mathrm{w}D^3A^2\omega^2k\cos(kx-\omega t)\sin(kx-\omega t)\\&\cong \frac{\pi}{4}C_\mathrm{d}\rho_\mathrm{w}D^3A^2\omega^2k = \frac{\pi}{4}C_\mathrm{d}\rho_\mathrm{w}gD^3A^2k^2\,.\end{aligned} \tag{20}$$

In the above the deep-water dispersion relation $\omega^2 = gk$ has been adopted. Furthermore, we assumed that $\vec{\nu}_\mathrm{ice}$ is a fraction of $\vec{\nu}_\mathrm{w}$ in order to simplify $|\vec{\nu}_\mathrm{w} - \vec{\nu}_\mathrm{ice}|(\vec{\nu}_\mathrm{w} - \vec{\nu}_\mathrm{ice})$. This force is also wave-curvature dependent because it contains the Ak^2 term.

Thus the critical condition for separation occurs when

$$\frac{\pi}{4}\rho_\mathrm{ice}D^3hAgk^2 + \frac{\pi}{4}C_\mathrm{d}\rho_\mathrm{w}D^3A^2\omega^2k = 2F_\mathrm{f}\,. \tag{21}$$

The lefthand side of this equation is the resultant force of the slope introduced by the wave profile and the water drag created by the wave motion. The freezing force on the right-hand side of the equation counteracts this differential force. When the freezing force is less than the stretching imposed by the differential wave force, the frozen bond breaks. This condition yields

$$D_\mathrm{max} = \left(\frac{2F_\mathrm{f}L^2}{\pi^3Ag(\rho_\mathrm{ice}h + C_\mathrm{d}\rho_\mathrm{w}A)}\right)^{1/3}. \tag{22}$$

It is reasonable to expect that the freezing force is proportional to the contact surface area, i.e. $F_\mathrm{f} = C_2Dh$ where C_2 is the bonding strength. Substituting this into Equation (22), we obtain the maximum diameter from the stretching failure

$$D_\mathrm{max} = \left(\frac{2C_2L^2}{\pi^3Ag(\rho_\mathrm{ice} + C_\mathrm{d}\rho_\mathrm{w}A/h)}\right)^{1/2}. \tag{23}$$

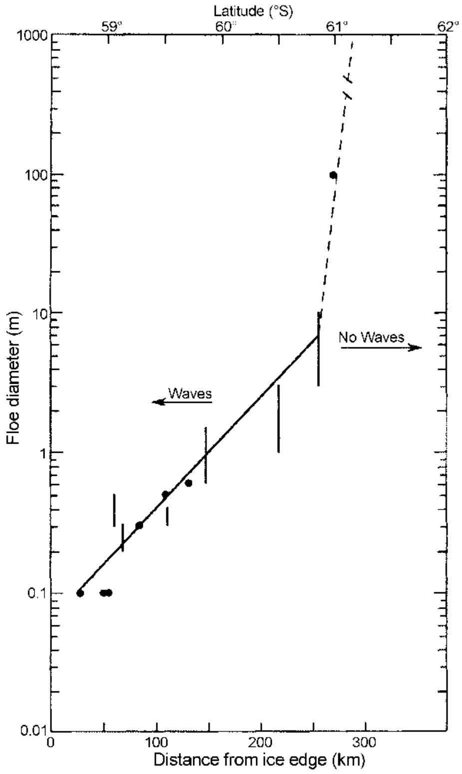

Fig. 5. Pancake-ice floe size vs distance from the ice edge. The dots represent the estimated average diameters and the bars represent the spread of data (from Shen and Ackley, 1991).

Both the bending-failure mode predicted by Equation (14) and the stretching-failure mode predicted by Equation (23) suggest that the maximum pancake size decreases with wave amplitude and increases with wavelength. The diameter increases slowly with increasing thickness under stretching-failure mode and is independent of thickness in the bending-failure mode. If we may assume that both C_1 in Equation (14) and C_2 in Equation (23) increase as temperature decreases, then Equations (14) and (23) also show that pancake size increases when the air temperature is colder, given that everything else remains the same.

The above is an estimate of the maximum floe diameter under two different failure modes. We believe that both modes are operating. The smaller of the critical diameters as determined by these two modes determines the final size of the pancake floe.

FIELD AND LABORATORY OBSERVATIONS

Very few quantitative descriptions of the pancake-ice cover currently exist. Although pancake-ice fields have been sighted in many field experiments as research vessels cross the marginal ice zone, a systematic observation has not been made. Better-designed field observations will become possible as our understanding of the formation process improves. In the 1986 Winter Weddell Sea Project, pancake-ice floes and their composites were clearly identified during airborne surveys of the ice cover. As shown in Shen and Ackley (1991), the ice cover clearly consists of small floes frozen into larger ones. The size of the floes grows as the distance from the ice edge increases. Figure 5 shows an approximate exponential growth of the floe diameter as observed in this field experiment.

If we adopt Equation (23) obtained from stretching failure as an estimate of the floe size in a wave field, and assume exponentially decaying wave amplitude vs distance (Wadhams and others, 1988), the predicted floe diameter increases exponentially from the ice edge:

$$D \approx K_2^{1/2} L A_0^{-1/2} \mathrm{e}^{\alpha x/2} \,. \tag{24}$$

In the above, we dropped the C_d term in Equation (23) which is typically much smaller than 1, and let the wave amplitude decay as $A = A_0^{-\alpha x}$. Moreover, the ice thickness is assumed to be constant for all x. In the field, ice thickness generally increases with distance from the edge. Thickness data associated with Figure 5 are not available. Realizing this is only an estimate, Equation (24) predicts an exponential growth of floe size as shown in Figure 5. The slope of the exponential growth in floe diameter given in Figure 5 suggests $\alpha \approx 3.4 \times 10^{-5}\,\mathrm{m}^{-1}$.

If instead we use the bending-failure mode as given in Equation (14), then

$$D \approx K_1 E^{-1} L^2 A_0^{-1} \mathrm{e}^{\alpha x} \,. \tag{25}$$

The corresponding estimate from Figure 5 yields $\alpha \approx 1.7 \times 10^{-5}\,\mathrm{m}^{-1}$. The estimated decay coefficient from either failure mode is reasonable according to Wadhams and others (1988).

As mentioned in the Introduction, three laboratory tests were conducted to closely observe the formation of the pancake-ice cover. The purpose of these experiments was to ascertain the possibility of producing pancake ice in a laboratory-scale wave field. The range of the parameters in these laboratory studies was not sufficient to test the current theory. Nevertheless, some trends were consistently observed, as reported in Leonard and others (1999a). Highlights of the observations are:

1. The diameters of the pancakes are in the order of 1/100 of the dominant wavelength.
2. The evolution of a pancake-ice cover from grease ice is dependent on the wave condition. Gentle waves produce pancakes that will evolve into composites. Energetic waves produce more irregular and thicker pancakes that remain dispersed.
3. Wave attenuation by an ice cover depends on the type of ice cover. The attenuation efficiency of grease ice increases with ice thickness. The onset of pancakes lessens the attenuation, and the onset of composite pancakes increases the attenuation.
4. The ice temperature begins to decrease at a higher rate when composites first form. This is most likely due to a combination of reasons. First, there is very little frazil ice being produced at this stage, so the heat being extracted is concentrated in cooling the existing ice floes. Second, as the pancakes thicken, more of the pancake surface area is exposed to the air, so the rate of cooling increases.

We may discuss these findings in light of the theoretical formulations given in this paper.

First let us estimate the pancake floe diameter. Since it is easier to estimate the total freezing force F_f, we will use Equation (22) instead of Equation (23). We use the following set of parameters as typical cases reported in Leonard and others (1999a): $A = 0.2$ m, $h = 0.05$ m, $C_\mathrm{d} = 0.03$, $L = 3$ m. We calculate the resulting maximum diameter to be 8 cm if $F_\mathrm{f} = 0.01$ N, and the maximum diameter would be 19 cm (4 cm) when $F_\mathrm{f} = 0.1$ N (0.001 N). These values are close to the data reported in Leonard and others (1999a). However, we have to assume a bonding force due to freezing to obtain the above estimates. At present, we have no data on direct measurement of the bonding force F_f. Using the above data and Equation (14), and assuming that C_1 is an order 1 constant, then the bending failure dictates a maximum diameter to be $1/(2 \times 0.02E)$ in meters. The Young's modulus of a pancake-ice plate is low. If we use $E \approx 10^2 \sim 10^3$ Pa, the resulting floe diameter is 2.5–25 cm. This range is reasonable. Again, the Young's modulus of a pancake-ice cover has not been measured.

Next, if either Equation (14) or Equation (23) is correct, then they predict larger-size pancake floes if the wave amplitude is lower. This agrees with the second point given above. Thicker ice also tends to produce larger floes according to Equation (23). However, this dependence is rather insensitive due to the functional form between the maximum diameter and the ice thickness. More data are needed to verify this point.

The third point observed is difficult to explain without careful analysis of the decay mechanisms in wave–ice interaction. This study will be left for future work.

The fourth point is a result of the thermodynamic nature of the ice-cover growth. While dynamics play an active role in forming the pancake-ice morphology, there is a strong interaction with the thermodynamics.

Lastly, it is interesting to note that in all experiments we have done so far, pancake ice always begins with an elemental size a couple of centimeters in diameter. The shortest water wave, the capillary wave, is 16.5 mm at 0°C and 1030 kg m^{-3} density.

CONCLUSIONS

A theoretical framework is given in this study. In this framework we define the thermodynamic and dynamic growth terms. The interaction between the wave and ice field is an important integral part of the pancake-ice formation theory. Many parameters await further quantification. A complete mathematical model to describe the evolution of the pancake-ice cover may be obtained after these parameters are determined. We expect the continuation of this study will allow us to answer the following question: given a wind and wave field associated with a storm, can we predict the morphology of the resulting pancake-ice cover? Although at present we cannot answer this question, we have some estimates of the mature pancake size for a given wave field. These estimates agree with limited field and laboratory observations. Extensive laboratory experiment is needed to complete and validate the theory proposed here.

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