## CORRESPONDENCE.

## MANCHESTER UNITY EXPERIENCE 1893-7.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—The author of the review of the "Manchester Unity Experience 1893-7" has devoted part of his most interesting article to a brief criticism of the method of obtaining the "Exposed to Risk of Sickness", which, as he points out, differs from that usually followed. In order that my purpose in departing from the common plan may be fully apprehended by students of the subject, I venture to ask that space may be found in the Journal for the following explanation.

The Sickness rates of the Manchester Unity Investigation may, by analogy with ordinary practice, be called "central sickness rates." Symbolizing such rate at age x by  $s'_x$  the following identity is established in cases in which the same body of data have supplied both the sickness and mortality rates.

$$s_x = \frac{1}{2} p_x s'_x$$

The expression  ${}_{k}p_{x}s'_{x}$  shows that in the ordinary rate of Sickness  $(s_{x})$  a certain rate of mortality at the age x is involved, and leads to the conclusion that, although the sickness per unit of actual exposure may not change, the introduction of varying death rates will produce varying rates of sickness. A good example of this is found in Sutton's Tables, page 1171, wherein it is shown that the adjusted sickness rate decreases from 45 weeks per member per annum at age 90 to 26 weeks at age 100. Premising that nearly all the lives at risk at these advanced ages were constantly sick (which appears to have been the case), it is clear that the diminishing "rate of sickness" can only arise from an increasing rate of mortality, and consequent increase in the number of fractions of exposure counted as whole units in the denominator.

Now, without discussing whether this is a satisfactory "rate of sickness", or not, I may point out that for the purpose of the new Manchester Unity Experience it was necessary to separate the two factors  $s'_x$  and  $\frac{1}{2}p_x$ , which make up the ordinary  $s_x$ . The rate of sickness was obtained from data grouped by occupations, but the rates of mortality came from the geographical areas, and thus with one value of  $s'_x$  were combined three values of  $\frac{1}{2}p_x$ , so yielding (what could not have been obtained by any other ready process of calculation) three distinct values of  $s_x$ , responding to the variations in the mortality of the several bodies at risk.

## EXAMPLES.

At age 80, 
$$s'_x=29.777$$
 weeks (p. 183)—

Area 1,  $\frac{1}{2}p_x=.93085$ 
,, 2,  $\frac{1}{2}p_x=.90504$ 
,, 3,  $\frac{1}{2}p_x=.92064$ 
 $\frac{1}{2}p_xs'_x=s_x$  (the ordinary sickness rate)—

Area 1,  $29.777 \times .93085=27.72$  weeks.
,, 2,  $\times .90504=26.95$  ,,  $\times .92064=27.41$  ,,

It will be seen that these differences are not altogether insignificant. Without the complete separation of the mortality and sickness factors it would have been impossible to avoid scientific incongruities in the various combinations of the data.

I am, SIR,

Yours faithfully,

ALFRED W. WATSON.

Nottingham, 19 May 1904.

To the Editor of the Journal of the Institute of Actuaries.

Dear Sir,—My attention has been drawn to the fact that the formulæ I gave on pages 354 and 355 of vol. xxxviii (April 1904) of this Journal, have already been given by Mr. G. F. Hardy in a slightly different form on page 137 of the "British Offices Life Tables, Account of Principles and Methods", which was published in December last year. My result was arrived at quite independently; the part of my note referring to the H<sup>M</sup> Table was written before the publication of the British Offices Life Annuity Tables in Feb. 1903, and on their appearance the remainder was immediately added and the manuscript sent to you in the latter half of Feb. 1903. Unfortunately, at the time of reading over the proofs, it was overlooked that Mr. G. F. Hardy had, in the interim, called attention to the property of the tables mentioned in my note.

I am,

Yours faithfully,

Liverpool, 3 June 1904.

H. W. CURJEL.

[It is due to Mr. Curjel that we should insert the above letter, with the additional explanation that his communication was originally received by us on 27 Feb. 1903, and was unavoidably held over on account of the crowded state of the *Journal*.—Ed. J.I.A.]