

Part I

Introductory Review

The Determination of Cosmological Parameters

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Abstract. The case is made that the 21st century will be the epoch of precision cosmology. It is realistic to expect that many of the standard cosmological parameters will be determined with high precision. This will require detailed and convincing understanding of the astrophysics of the objects used in the determinations.

1. Introduction

At the dawn of the 21st century, I will make the case that we are entering a new epoch in the determination of cosmological parameters, what I will call the epoch of *precision cosmology*. The large-scale properties of the Universe are well described by the standard Big Bang picture and, within the limitations of that picture, we can reasonably hope to be able to determine the values of the standard cosmological parameters with an order of magnitude better precision than can be achieved today. To attain this goal, however, many important questions have to be addressed and I will outline my own manifesto for what has to be done to make this a reality.

In this brief review, I will touch on the following topics: the validation of the cosmological framework within which cosmological studies are carried out, the cosmological parameters and my impression of the current situation regarding their determination, the interplay between astrophysics and cosmology, and some of the great challenges for the new century.

2. A Hierarchy of Descriptions of the Large Scale Structure of the Universe

I find it helpful to think in terms of a hierarchy of descriptions of the different types of model used in astrophysical cosmology – this way of looking at cosmology was first brought to my attention by Rashid Sunyaev in 1968. There are three stages in the development of the standard cosmological models.

- *Zero-order model* These are the standard Friedman world models in which it is assumed that the Universe is uniform, homogeneous and isotropic. All large- and small-scale structure is washed out and the resulting model is described by only a few parameters, which are listed in Section 4. The prime objective of many cosmological experiments is the determination of this limited number of cosmological parameters and these are important

tests of the laws of physics on the largest scales available to us. It is, however, a very boring Universe with no structure, no heavy elements and certainly no astronomers.

- *First-order model – homogeneity with small perturbations* The next step is to include the presence of small perturbations with $\delta\rho/\rho \ll 1$ into the description of the world model. This has been the subject of an enormous amount of study and it has the great advantage that linear approximations can be used to describe the impact of the inclusion of small amplitude perturbations upon the models. This is probably a good approximation on very large scales in the Universe at the present epoch, say, $l \geq 100$ Mpc, but we need to worry about the effects of the huge voids, which can have scale up to 50 Mpc, and whether or not it is really safe to assume that these are small perturbations. Another observable epoch at which it is assumed that the first order perturbation model is a good approximation is at the epoch of recombination at $z \approx 1000$ when the density perturbations in the matter content of the Universe corresponded to $\delta\rho/\rho \sim 10^{-3}$. This is the reason why studies of the power spectrum of fluctuations in the Cosmic Microwave Background Radiation are so powerful – all the important astrophysics is still in the linear regime and the hope is that the radiative transfer problems and the definition of the necessary astrophysics can be so well prescribed that precision predictions can be made. I am sure this will be a major topic of discussion at this meeting.
- *Second-order model – homogeneous overall but with highly non-linear structures* $\delta\rho/\rho \gg 1$ on small scales This is the real world we live in and it is much messier than the idealisation of the zero-order model. In particular, it is no longer safe to make assumptions about small-scale homogeneity. The local dynamics, the distribution of non-baryonic and baryonic matter and the physical properties of the objects used in cosmological tests can all vary from one region of the Universe to another. It is inevitable that local determinations of the values of cosmological parameters may well vary from one region of the Universe to another because of the myriad of non-linear effects, in the broadest sense of the term, which may vitiate a clean determination of the large-scale values of the cosmological parameters. Furthermore, it is no longer safe to adopt the standard expressions relating intrinsic properties to observables when the non-linearities are large – in precision cosmology, we need to take account of the impact of the non-uniform distribution of gravitating matter in the Universe upon observables.

Thus, if we are indeed entering the epoch of precision cosmology, there are several challenges which need to be taken very seriously by observers and theorists. To summarise my own agenda, these include the following:

- To be really convincing, all observational cosmological experiments must be based upon convincing astrophysics. In my view, it is no longer adequate simply to find a standard candle or a rigid rod and find empirically that they do not seem to vary from one region of the Universe to another, or from one cosmic epoch to another. To be really convincing,

we must have excellent astrophysical reasons why this should be so. For example, we need to understand astrophysically why the Type 1a supernovae and Cepheid variables seem to be such good standard objects and to understand in some detail the physics behind correlations such as the width-luminosity relation for the Type 1a supernovae. We really need to understand in some detail the physics of the hot gas in clusters of galaxies to be convinced that the combination of the Sunyaev-Zeldovich effect and the X-ray properties of the clusters are really providing good estimates of physical dimensions at the cluster which are independent of redshift. We need to be convinced that we understand sufficiently well the physics of the primordial mass spectrum to be sure that the acoustic, or Sakharov, peaks in the primordial mass spectrum really are giving us clean information about cosmological parameters. And so on ... These are real challenges and it is essential that they are addressed in depth in order to achieve the sort of precision which I think is a reasonable goal for the present century.

- One of the trends which will certainly be discussed during this meeting will be that it is now possible to acquire really large data-sets in order to make good determinations of cosmological parameters. The technological advances which have made these possible are very impressive indeed and their advancement will certainly be a priority for the future. It is equally important, however, to ensure that *large and independent* projects are undertaken. Time and again in the history of astronomy and physics, the need to confirm the key experiments by independent means has been crucial to establishing the credibility of the science.
- Finally, we need to obtain internal self-consistency. There are many different approaches to the determination of cosmological parameters and, in the end, they all have to agree. The process by which this comes about will undoubtedly involve a symbiosis between astrophysics and cosmology, but there should be sufficient redundancy to enable a clear answer to be given to the question of whether or not all the independent routes to the determination of cosmological parameters hang together.

Before discussing the determination of the parameters themselves, let me make a plea that we all understand exactly what is going on. Many of the aspects of understanding what the tests are really telling us are non-trivial. In the end, there is reasonably simple physics behind the experiments and we should ensure that this understanding is secure. For example, it is a challenge to any research student to explain all the physical phenomena which enter into the determination of the angular power-spectrum of fluctuations in the Cosmic Microwave Background Radiation (for a crib about how I understand what is going on, see my book *Galaxy Formation* (Longair 1998)). Another example of the type of issue which I believe we should be careful about is the extent to which we believe we know anything about the physics of the Universe when it was less than, say, one millisecond old. Some arguments involve reliance upon the idea that the Universe went through an inflationary phase. I would advocate that we issue a clear health warning about the assumptions behind each determination – my preference would be for them to be independent of hypothetical physics. On the other hand, I would not wish to suggest that we

throw the baby out with the bath water and deny that cosmological observations can provide clues to physics at energies much greater than those accessible in particle physics experiments. Time and again astronomical observations have pre-empted laboratory experiments in making key discoveries for physics and there is no reason to believe that that process has come to an end.

3. The Cosmological Infrastructure

It is worthwhile recalling the observational basis for the standard Big Bang framework. The standard model could be rendered invalid by any of the following tests of the Robertson-Walker models. Since my review at the IAU Symposium No. 183 at Kyoto in 1997, I am not aware of any really serious challenges to the following statements (Longair 1999).

- *The isotropy of the Universe* The Cosmic Microwave Background Radiation remains the most remarkable evidence for the overall isotropy of the large-scale structure of the Universe, a conservative upper limit to the anisotropy on the largest scales being $\Delta I/I \leq 10^{-5}$ (Bennett *et al.* 1996). Just as impressive are the final results of the Cambridge APM survey of over 2 million galaxies in the direction of the South Galactic Pole in a solid angle corresponding to about one tenth of the sky. Although there is clear evidence for small-scale fluctuations associated with the ‘cellular’ structure of the distribution of galaxies on the scales greater than clusters of galaxies, the mean number density of galaxies is remarkably uniform averaged over large enough areas of sky.
- *The homogeneity of the Universe* The network of ‘walls’ and ‘voids’ in the distribution of galaxies has now been extended from the local Universe, as represented by the Geller-Huchra survey, to much greater distances by the Las Campanas Redshift Survey and particularly by the 2dF Galaxy Redshift Survey being carried out at the Anglo-Australian Telescope. The 2dF survey extends the mapping of the spatial distribution of galaxies to redshifts $z \sim 0.2$ and so spans a substantial volume of the Universe as it can be observed at the present epoch. Even a superficial inspection of the remarkable 2dF maps, which as of September 2000 contain over 130,000 galaxies, show that qualitatively the same type of ‘cellular’ network which we observe nearby extends to a redshift of 0.2. Thus, provided averages are taken over sufficiently large volumes, the assumption of homogeneity seems secure. On the other hand, these large-scale irregularities should not be neglected when precision cosmological observations are made. We really should take seriously the effect of large-scale inhomogeneities such as the walls and voids upon the expectations of cosmological tests.
- *Time dilation using Type 1a supernovae* One of the splendid by-products of the use of Type 1a supernovae as standard objects is that the characteristic time of the outburst can be used directly as a test of time dilation at large redshifts. In fact, time dilation is built automatically into the method of analysis of the supernova data and so the importance of this aspect of the observations can occasionally be obscured. Nonetheless, it is a key

prediction of the Robertson-Walker models that time dilation must be observed in the standard picture of the expanding Universe and the data now demonstrate this convincingly.

- *The temperature of the Cosmic Microwave Background Radiation at large redshifts* Among the first key cosmological observations made by the Keck Telescope was the determination of temperature of the Cosmic Microwave Background Radiation from hyperfine CI transitions in the absorption spectra of large redshift quasars. The observations of Cowie *et al.* (1994) and Ge *et al.* (1997) showed that the temperature of the background radiation has indeed changed as $(1+z)$ with increasing redshift. This is another example of the type of test which could have undermined the framework of the standard Big Bang, if, for example, a lower radiation temperature had been found than predicted.
- *Tests of General Relativity* In parallel with these observations, we should keep a close eye on tests of general relativity itself. These are very demanding, but so far there is no evidence which is in conflict with the predictions of General Relativity. We should, however, recall that the precision with which we know the theory to be good is only at the level of one part in 10^4 in the coefficients in the post-Newtonian expansion of the metric. There is plenty of scope for improving the precision with which General Relativity is known to be the best-buy relativistic theory of gravity.

4. Cosmological Parameters to be Determined on the Large Scale

It is useful to catalogue the parameters to be determined through the various types of cosmological observations described at this meeting.

- *Hubble's constant*, H_0 , describes the rate of expansion of the Universe at the present epoch t_0 ,

$$H_0 = \left(\frac{\dot{R}}{R} \right)_{t_0} = \dot{R}(t_0), \quad (1)$$

where I use a convention in which the scale factor R is set equal to unity at the present epoch.

- The *deceleration parameter*, q_0 , describes the dimensionless deceleration of the Universe at the present epoch t_0 ,

$$q_0 = - \left(\frac{\ddot{R}}{\dot{R}^2} \right)_{t_0} = - \frac{\ddot{R}(t_0)}{H_0^2}. \quad (2)$$

- The *density parameter*, Ω_0 , is the ratio of the present mass-energy density of the Universe ρ_0 to the critical density $\rho_c = 3H_0^2/8\pi G$

$$\Omega_0 = \frac{\rho_0}{\rho_c} = \frac{8\pi G \rho_0}{3H_0^2} \quad (3)$$

and includes all forms of 'visible' and dark matter.

- The *density parameter in baryonic matter* is Ω_B .
- The *curvature of space* at the present epoch is $\kappa = \mathfrak{R}^{-2}$, where \mathfrak{R} is the radius of curvature of the isotropic geometry of the Universe at the present epoch.
- The *cosmological constant* Λ can be parameterised in terms of the density parameter of the vacuum fields $\Omega_\Lambda = 8\pi G\rho_v/3H_0^2 = \Lambda/3H_0^2$ or $\Lambda = 3H_0^2\Omega_\Lambda$.
- The *age of the Universe* is t_0 and is found from the expression

$$t_0 = \int_0^{t_0} \frac{dR}{\dot{R}}. \tag{4}$$

As shown in all the standard textbooks, these parameters are not independent

$$q_0 = \frac{\Omega_0}{2} - \Omega_\Lambda, \quad \kappa = \frac{1}{\mathfrak{R}^2} = \frac{[(\Omega_0 + \Omega_\Lambda) - 1]}{(c^2/H_0^2)}. \tag{5}$$

These expressions describe different aspects of the cosmological models.

- The first involving q_0 , Ω_0 and Ω_Λ describes the present deceleration (or acceleration) of the Universe under the competing influences of gravity and the vacuum fields.
- The second describes how the curvature of space, $\kappa = \mathfrak{R}^{-2}$, depends upon the total mass density in both the matter and the vacuum fields.

It is worthwhile making a pedantic footnote about these relations. q_0 can be measured *independently* of Ω_0 , Ω_Λ and \mathfrak{R} at small enough redshifts. Purely kinematically, it is straightforward to show that the comoving radial distance coordinate r is

$$r = \frac{c}{H_0} \left[z - \frac{z^2}{2}(q_0 + 1) + \dots \right] \tag{6}$$

(see, for example, Longair (1998)). This would be true even if Friedman’s equations were not correct. q_0 is only independent of Ω_0 , Ω_Λ and \mathfrak{R} at rather small redshifts $z \leq 0.25$. When astronomers claim to measure q_0 , it usually means within the context of the isotropic Friedman world models parameterised by Ω_0 and Ω_Λ and *not* the use of the expression (6). This expression is an important result and enables Einstein’s equations to be tested *on the largest scale we have accessible to us*. It should be emphasised that this test has not yet been carried out and would require, say, very large samples of Type 1a supernovae within $z = 0.25$ to compare with the results obtained at larger redshifts and with the values inferred from the properties of the angular power spectrum of fluctuations in the Cosmic Microwave Background Radiation.

Other parameters appear in the cosmological literature associated with properties of the initial spectrum of perturbations from which large scale structures in the Universe have formed. From the remarkable analysis of the power-spectrum of the large-scale distribution of galaxies carried out by Peacock and

Dodds (1994), it is quite plausible that this initial spectrum had a power-law form and I regard this as excellent *a priori* evidence that we can develop a further list of useful cosmological parameters:

- n , index of primordial scalar fluctuation power spectrum.
- The physical scale at which $\delta\rho/\rho_0 = 1$, or its equivalent.
- b , the bias parameter.

These parameters do not have the same status as the large-scale parameters described above, but are crucial in many determinations of them. The bias parameter b is a particularly worrying quantity since it is easy to imagine circumstances in which it could vary quite markedly from one region of the Universe to another. Indeed, I regard it as one of the most important endeavours of astrophysical cosmology to determine precisely how b and its variance vary with environment.

The other problem lurking in the background is the role of inhomogeneities upon all the classical cosmological tests. There are some very convenient analytic forms for these relations from the early works of Zeldovich (1964), Dashevsky and Zeldovich (1965), Roeder and Dyer (1972, 1973). These show vividly how inhomogeneities in the distribution of gravitating mass can change significantly the expectations of the isotropic models. In these analyses, it is assumed that the inhomogeneities are associated with point-like masses embedded in a uniform sub-stratum, but a more realistic approach would be to consider the types of Swiss-cheese models, for example, the models discussed by Kantowski (1998) (see also the helpful discussion in Chapter 14 of Peebles' *Principles of Physical Cosmology* (1993)). In the limit, we would need to consider ray tracing through the observed distribution of galaxies. Inspection of images such as that of the Abell Cluster A2218 show unambiguously how the effects of gravitational lensing can distort and magnify the images of distant galaxies very markedly. In the case of the lensing of very distant objects, it is inevitable that there will be some deflections of the light paths because of the presence of large-scale inhomogeneities in the distribution of matter. It is now a major industry to invert the observed distortions of the images of large samples of galaxies to find out information about the power spectrum of perturbations on a large scale (see, for example, Kaiser 1992).

5. Estimates of Cosmological Parameters

Granted these concerns, the present state of the determination of cosmological parameters looks surprisingly encouraging. I will simply summarise my interpretation of some recent results which have appeared in the literature, or in preprint form. I am sure these will be significantly updated at this symposium.

5.1. Hubble's Constant

- One of the most pleasing results of the last year has been the completion of the HST Key Project to measure the value of Hubble's constant with an accuracy of 10%. Freedman, Mould, Kennicutt and their colleagues quote

a value of $H_0 = 70 \pm 10\%(1\sigma) \text{ km s}^{-1} \text{ Mpc}^{-1}$. This represents real progress over the situation even a few years ago. From the methodological point of view, one of the most important aspects of this achievement has been the remarkable increase in the precision with which the distances of nearby galaxies are known from the use of a wide range of methods involving the Tully-Fisher relation, Type 1a supernovae, Type 2 supernovae, surface brightness fluctuations, Cepheid variables, and so on.

- Sandage and Tammann now place most weight on the use of Type 1a supernovae and the most recent estimate of H_0 which I could find is $H_0 = 59 \text{ km s}^{-1} \text{ Mpc}^{-1}$.
- The use of gravitational lensing time-delays is a very attractive method of estimating Hubble's constant since it provides a direct route to measuring the distance of the lensing galaxy without the need for a sequence of intermediate distance indicators. The limiting aspect of these procedures is the accuracy with which the gravitational potential at the lensing galaxy can be modelled and so it is an advantage if the gravitationally split image is quite complex. For example, Turner and his colleagues have found $H_0 = 64 \pm 13 \text{ km s}^{-1} \text{ Mpc}^{-1}$ for the quasar 0957+561, where the errors are 95% confidence limits (Kundic *et al.* 1997). A similar analysis for the gravitational lens associated with PKS1830-211 observed by Wiklind and his colleagues gives a value of $H_0 = 59 \pm 10 \text{ km s}^{-1} \text{ Mpc}^{-1}$ (Wiklind, personal communication, 2000).
- The combination of the Sunyaev-Zeldovich effect in X-ray emitting clusters of galaxies combined with the determination of the distribution of emitting gas from X-ray observations and the dynamics of galaxies in the cluster enable its physical dimensions to be determined independent of its redshift. Typically, the results for the best clusters average to values lying in the range $H_0 = 50 - 60 \text{ km s}^{-1} \text{ Mpc}^{-1}$ (Saunders, personal communication, 2000).

Thus, there is a pleasing convergence of the estimates of Hubble's constant from a variety of totally independent approaches. The challenge is to reduce the uncertainties even further – that is no small undertaking.

5.2. The Supernova Cosmology Project

The determination of cosmological parameters by the traditional procedures has been dominated by the use of the Type 1a supernovae as standard candles. Perlmutter and his colleagues have announced revised estimates of combinations of the cosmological parameters Ω_0 and Ω_Λ from a sample of 40 Type 1a supernovae discovered in the Supernova Cosmology Project, all of them with redshifts $z > 0.35$ and extending to redshifts of 0.9. Many of the supernovae lie in the redshift interval $0.4 \leq z \leq 0.6$. The quoted estimates of the cosmological parameters are:

- If $\Omega_0 + \Omega_\Lambda = 1$, $\Omega_0 = 0.25 \pm 0.06(\text{stat}) \pm 0.4(\text{syst})$.
- If $\Omega_\Lambda = 0$, $\Omega_0 = -0.4 \pm 0.1(\text{stat}) \pm 0.6(\text{syst})$.

I am sure that the detailed analysis of these data will be a major topic of debate at this symposium. A glance at the redshift-apparent magnitude relation for these supernovae indicates immediately that there is a remarkably narrow dispersion about the mean relation even at large redshifts. It will be recalled that this narrow dispersion is only found once allowance is made for the correlation between the duration of the supernova outburst and the absolute magnitude of the supernova at maximum. It is of the first importance that the astrophysical origin of this correlation is understood in some detail and also that the physics of the processes which lead to Type 1a supernovae are established convincingly. There is the real prospect of extending these studies to very much large samples of supernovae by extending the current campaigns and also extending them to much larger redshifts with the Next Generation Space Telescope.

5.3. The Angular Power-spectrum of Fluctuations in the Cosmic Microwave Background Radiation

The other area which has rightly hit the headlines over the last few months has been the results of balloon experiments to determine with precision the angular power-spectrum of fluctuations in the Cosmic Microwave Background Radiation. The first generations of experiments have now been superseded by the new generation of high precision experiments. The new experiments are of such improved precision that it is not unfair to combine only the results of the COBE DRM experiment with those of the Boomerang and Maxima experiments, a procedure carried out by Jaffe *et al.* (2000). I am sure their combined power-spectrum will appear many times in this meeting. It is a wonderful achievement and an indication of what has to be done next. Combining all three data sets, Jaffe *et al.* (2000) quote the following estimates:

$$\begin{aligned}\Omega_0 + \Omega_\Lambda &= 1.11 \pm 0.07 \\ \Omega_B h^2 &= 0.032_{-0.008}^{+0.005} \\ n &= 1.01_{-0.07}^{+0.09}\end{aligned}$$

As discussed above, these numbers disguise a number of assumptions concerning the selection of the model for the evolution of the perturbation spectrum, but there is no doubt that these numbers represent a major advance. It is also correct to caution that they are *model dependent* and much higher statistical precision is necessary to exclude some of the more extreme possibilities with confidence. The importance of future space missions such as MAP and the PLANCK Surveyor cannot be overstated.

It is striking that these results suggest that we live in a Universe which is geometrically flat and that the spectral index of the power-spectrum is close to the preferred Harrison-Zeldovich value, $n = 1$. The estimate of the mean baryon density is somewhat greater than that found from primordial nucleosynthesis arguments. The origin of the high value can be traced to the small amplitude of the second maximum in observed angular power-spectrum.

5.4. Other Estimates of these Density Parameters

The estimates in Sections 5.2 and 5.3 should be compared with those derived from other approaches. Examples include:

- The best estimates of $\Omega_B h^2$ from primordial nucleosynthesis suggest values of $\Omega_B h^2 = 0.019 \pm 0.002$, somewhat less than the value found from the power-spectrum of the fluctuations in the Cosmic Microwave Background Radiation, although only at the 2σ level.
- The value of Ω_0 can be found from the masses of clusters of galaxies, Bahcall (1997) quoting values of Ω_0 of about 0.2 to 0.3. There remains the problem that larger values are inferred from reconstructions of the local velocity field (see, for example, the pleasant discussion by Dekel, Burstein and White (1997)). Part of the problem lies in knowing what the appropriate values of the biasing factor b should be and whether or not it varies from one part of the Universe to another.
- Limits to the value of Ω_Λ can be found from the statistics of large redshift gravitational lenses. If the value of Ω_Λ is large, this has the effect of increasing the volume elements at large redshifts and so larger numbers of gravitational lenses would be expected. The analysis of the data is highly non-trivial. I find Kochanek's analysis of 1996 an impressive piece of work. He quotes an upper limit to the value of Ω_Λ of 0.65 to account for the observed numbers of gravitational lenses. This result should be compared with those quoted in Section 5.3. There is a marginal discrepancy between the best-fitting values.

5.5. The Age of the Universe

- As reported at the 1997 Kyoto symposium, the upward revision of the local distance scale from Hipparcos observations has increased the luminosity of the stars and suggested ages for the globular clusters in the region of $(11-12) \times 10^9$ years, although Bolte (1997) favours values closer to 14×10^{10} years.
- According to Schramm's analysis of the abundances of radioactive nuclei, nucleocosmochronology has suggested values of the same order for the age of our Galaxy (Schramm 1997).
- Expansion ages for the best fit models are probably consistent with these data, but ...

6. Reflections

It is no exaggeration to state that the whole nature of the discipline of estimating the values of cosmological parameters has changed dramatically over the matter of a decade. The figures quoted above show that we are entering a wholly new realm of parameter estimation. In my view, it is crucial that the understanding of the astrophysics go hand-in-hand with the improvement in the ability to estimate cosmological parameters. Without convincing astrophysical underpinning, there will always be scope for changing the astrophysics and so the best estimates of the parameters. I would apply this concern to the estimation of parameters using the power-spectrum of the Cosmic Microwave Background Radiation as well. It is not really good enough to say that it all comes out of inflationary *models* of

the early Universe – I find arguments derived from the power-spectrum of the largest scale structures we observe at the present day more convincing evidence.

Despite all the worries one might have about how secure the various estimates are, particularly when one tries to understand the real uncertainties in the observations and the methods used to extract them, the apparent convergence of independent estimates of the cosmological parameters is distinctly encouraging. However, the devil is in the detail and it will require a very careful and objective estimate of how the data are treated to convince the sceptic that we really are on the right lines. This must be one of the main concerns of this Symposium, if, as I believe, we really are entering the epoch of precision cosmology.

A whole new range of challenges is just on the horizon. It is becoming more and more feasible to extend precision observations to large redshifts at which the Universe of galaxies was very different from their appearance at the present day. I fully expect that, with the new types of facilities which are planned for the future, we will obtain a much more secure astrophysical understanding of the processes by which galaxies and active galaxies came about. In turn, this understanding can be applied to the determination of cosmological parameters at large redshifts and, in the end, we would expect it all to hang together. It would be remarkable if we did not encounter a number of major surprises along the way, which will take the whole discipline off in new and unexpected directions.

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