VII. PROPERTIES OF NEUTRON STARS

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NEUTRON STAR PROPERTIES FROM OBSERVATIONS OF PULSARS AND PULSING X-RAY SOURCES

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This paper reviews the information on neutron stars that has been obtained from radio and X-ray observations, with emphasis on recent results from timing studies. Included are summaries of the current theoretical picture of neutron stars, and the most recent evidence concerning stellar masses, radii, inertial moments, and magnetic fields. Also included are discussions of large frequency jumps ("macroglitches"), small-scale timing irregularities ("microglitches"), the characterization of the latter phenomenon in terms of random noise processes, and the use of both as probes of the internal structure and dynamics of neutron stars. Finally, some of the most promising directions for future work are briefly noted.

1. INTRODUCTION

Most of our knowledge of neutron stars has come from studies of pulsars and pulsing X-ray sources, and especially from pulse timing observations. It was, after all, the regularity of pulsars that first attracted attention (Hewish et al. 1968). Soon after their discovery, timing measurements by Richards and Comella (1969) revealed that the Crab pulsar is slowing down, a result that was crucial in establishing that pulsars are rotating neutron stars (Gold 1969). The detection of a large jump in the frequency of the Vela pulsar (Radhakrishnan and Manchester 1969, Reichley and Downs 1969) provided the first evidence of complex internal structure, while the pioneering work of Boynton et al. (1972) showed that the small timing irregularities ("restless behavior") observed in the Crab pulsar could be described as noise. The discovery of pulsing X-ray sources (Schreier et al. 1972) and the convincing interpretation of these objects as a second class of rotating neutron stars (Pringle and Rees 1972, Davidson and Ostriker 1973, Lamb et al. 1973) opened the way for the study of neutron stars using X-ray techniques.

The present review surveys our current observational knowledge of neutron stars, with emphasis on recent results from timing studies.

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After summarizing the present theoretical picture of neutron stars in Section 2, I briefly review the evidence on stellar masses and radii in Section 3. In Section 4 I discuss the available information on inertial moments and magnetic fields. The evidence provided by large frequency jumps is described in Section 5, while in Section 6 I discuss the characterization of timing irregularities as noise, and the use of timing noise as a probe of neutron star interiors. Reviews of early developments and of other topics may be found in the articles by Ruderman (1972), Pines, Shaham, and Ruderman (1974), Lamb (1975, 1977, 1979a), and Pines (1980), and in the monograph by Manchester and Taylor (1977).

2. THEORETICAL PICTURE

In the discussion that follows it will be helpful to have in mind current theoretical ideas on neutron star structure and the response of such a star to changes in its rotation rate. Theoretical work on neutron stars has been recently reviewed by Baym and Pethick (1975, 1979), and the expected structure is shown in Figure 1 (the quoted dimensions are those of a model based on the tensor-interaction equation of state of Pandharipande and Smith; see Pandharipande et al. 1976). A neutron star may be conveniently divided into five regions: (1) the surface, where the properties of the matter can be strongly affected by the temperature and the magnetic field. (2) The outer crust, consisting of a solid lattice of nuclei embedded in a sea of degenerate, relativistic electrons. (3) The inner crust, consisting of that part interior to the neutron drip point. In this region neutrons exist outside the nuclei, so that one has a solid lattice immersed in a sea of electrons and neutrons. (4) The neutron liquid, consisting largely of neutrons

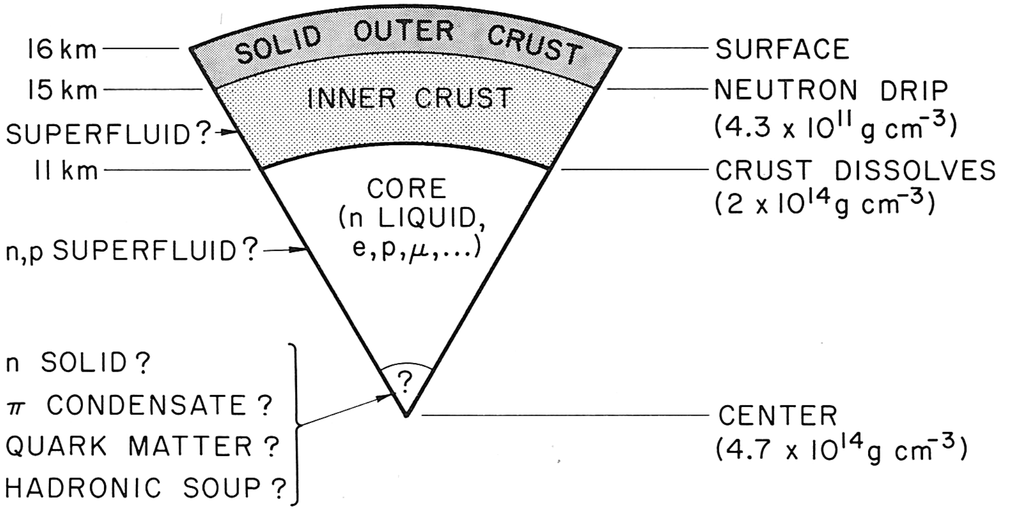

Fig. 1. Cross section of a 1.3 $M_\odot$ neutron star illustrating the five regions discussed in the text

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but with some electrons and protons, and a few muons. (5) Possibly a distinct core. Such a core could consist of condensed pions, or a solid neutron lattice. Alternatively, the composition of the interior may simply change gradually to a quark or hyperonic soup as the density increases.

The response of such a star to changes in the rotation rate of the crust is expected to be very complex (see Lamb 1977). From the dynamical point of view it has at least five distinct components: the crust, the neutron fluid, the proton fluid, the electron fluid, and, in heavier stars, possibly a solid core. These components posses a variety of normal modes that can be excited by rotational disturbances, including inertial and hydromagnetic-inertial waves in the electron and proton fluids, Tkachenko oscillations of the neutrons (if they are superfluid), wobble of the crust, and wobble of the solid core (if present). There are also a variety of different relaxation times, including the time required for relative motions of the various components to damp, and the time scale for changes in the crust due to cracking, crumbling, creep, and plastic flow. One of the key challenges facing observers is to devise methods of probing the dynamical behavior of neutron stars.

3. MASSES AND RADII

The determination of neutron star masses is discussed in detail by Kelley and Rappaport (1981), and I will therefore only summarize the results here. The mass of a neutron star can be measured directly only if it is in a binary system. Thus, all but one of the current estimates come from the study of binary X-ray sources. The seven systems that have provided mass estimates are listed in Table I, which also gives the period, P_{orb}, and the projected semi-major axis, a_n sin i, of the neutron star orbit, and the allowed range of the neutron star mass, M_n.

TABLE I. MEASURED NEUTRON STAR MASSES

System	P_{orb}(d)	a_n sin i (lt-s)	$M_n(M_\odot)$
4U0115-73 (SMC X-1)[a]	3.892	53.46±0.03	0.7-1.5
4U0900-40 (Vela X-1)[a]	8.965	113.0±0.8	1.6-2.5
4U1118-60 (Cen X-3)[a]	2.087	39.792±0.005	0.4-1.7
4U1538-52[a]	3.730	55.2±3.7	0.8-3.4
4U1626-67[b]	2.88×10^{-2}	$\lesssim$0.04	0.5-4.7
4U1653+35 (Her X-1)[c]	1.700	13.206±0.006	0.6-2.0
PSR 1913+16[d]	3.23×10^{-1}	2.3424±0.0007	1.24-1.54

[a]Kelley and Rappaport (1981). [b]Middleditch et al. (1980).
[c]Middleditch and Nelson (1976), Bahcall and Chester (1977).
[d]Taylor et al. (1979).

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In four systems (SMC X-1, Vela X-1, Cen X-3, and 1538-52) the mass determination is based on the orbital period and semimajor axis derived from pulse timing, the radial velocity of the companion star derived from measurements of the Doppler shift of its spectral lines, and the eclipse duration, together with some assumptions about the fraction of the critical lobe filled by the companion star and its rate of rotation. So far it has not been possible to determine the radial velocities of the companion stars in Her X-1 and 1626-67 spectroscopically, but orbital solutions have nevertheless been obtained by careful study of the optical pulses from these two systems. The very close binary system PSR 1913+16 is a unique case. Assuming that general relativity is the correct theory of gravitation and that both stars behave like point masses, the pulsar mass can be determined from pulse timing alone, with the result given.

The masses listed in Table I are consistent with the possibility that all neutron stars have masses in the range 1.2 - 1.6 $M_\odot$, as suggested by current theories of stellar collapse. The maximum stable neutron star mass depends on the equation of state of matter at high densities and the behavior of gravitational forces. Assuming that general relativity is correct and that the errors quoted in Table I reflect the true uncertainties in the masses, the 1.6 $M_\odot$ lower limit on the mass of Vela X-1 is inconsistent with several of the softer equations of state that have been proposed, including those of Leung and Wang (1971), which give maximum masses $\lesssim 0.5\ M_\odot$, Hagedorn's asymptotic equation of state and that of an ideal neutron gas, both of which give a maximum mass $\sim 0.7\ M_\odot$ (see Ruffini 1974), and the equations of state of Pandharipande (for hyperons), Arponen, and Canuto and Chitre, which give maximum masses in the range 1.36 - 1.46 $M_\odot$ (see Arnett and Bowers 1977, and references therein). The mass estimates in Table I are consistent with the somewhat stiffer equations of state of Bethe and Johnson, Moszkowski, Walecka, Pandharipande and Smith, and Bowers, Gleeson, and Pedigo (again see Arnett and Bowers 1977).

New evidence on the masses and radii of neutron stars and the equation of state at high densities is being provided by recent X- and γ-ray observations. The discovery of $\sim$ 420 keV lines in the spectra of several γ-ray bursts will make possible the determination of the surface gravitational redshift and hence M/R, if these are due to redshifted 511 keV e^+e^- annihilation radiation from the stellar surface (see for example Mazets et al. 1979). Comparison of the observed surface luminosities of cooling neutron stars with detailed models is providing new information about the equation of state at high densities (see Van Riper and Lamb 1980, and references therein). Measurements of the spectra of X-ray bursts may furnish estimates of neutron star radii if they are produced by thermonuclear flashes in neutron stars (van Paradijs 1978; Swank, Eardley, and Serlemitsos 1980), but such estimates may be seriously confused if only part of the stellar surface is involved, if the X-ray photosphere is not at the stellar surface, or if the X-rays interact appreciably with matter above the surface.

4. INERTIAL MOMENTS AND MAGNETIC FIELDS

The inertial moments of neutron stars are estimated from observations of period changes in pulsars and pulsing X-ray sources, and therefore usually require knowledge of some combination of the star's mass M, radius R, and dipole magnetic moment μ. An exception is the lower bound, $I \gtrsim 1.5 \times 10^{44}$g cm^2, on the inertial moment of the Crab pulsar, which follows if one assumes that the total luminosity of the Crab Nebula is supplied by the rotational energy lost by the pulsar (for a discussion, see Baym and Pethick 1975). This result is entirely consistent with stellar models, which yield $I = 2 \times 10^{44} - 3 \times 10^{45}$ g cm^2, depending on the mass and equation of state (see Arnett and Bowers 1977).

Direct determinations of surface magnetic field strengths are possible by measuring the effect of cyclotron scattering on the waveforms and spectra of pulsing X-ray stars (Elsner and Lamb 1976). Trümper et al. (1978) discovered a feature in the hard X-ray spectrum of Her X-1, which if due to cyclotron scattering at $\lesssim$ 42 keV (as suggested by Lamb 1977) implies a surface field $B_s \simeq 4 \times 10^{12}$ G (or if emission at ~ 58 keV, $B_s \simeq 6 \times 10^{12}$ G). A similar feature has been reported in the spectrum of 4U 0115+63 (Wheaton et al. 1979), which if due to cyclotron scattering at ~ 25 keV implies $B_s \sim 2 \times 10^{12}$ G.

Measurement of the period P and period derivative $\dot{P}$ of a pulsar provides an estimate of the quantity $\mu I^{-1/2}$, which is proportional to $(P\dot{P})^{1/2}$ for the simplest models of the braking torque. The observed values of P and $\dot{P}$ imply $\mu(I/10^{45}\text{ g cm}^2)^{-1/2} \simeq 2 \times 10^{28} - 2 \times 10^{31}$ G cm^3, with the typical value being ~ 10^{30} G cm^3. These are only rough estimates, however, since there are both observational and theoretical reasons for expecting significant departures from the simplest models of pulsar braking (see Manchester and Taylor 1977).

More promising are measurements of the accretion luminosities and secular spin-up rates of disk-fed pulsing X-ray stars (there are too many unknowns if the star is wind-fed). This method can be widely applied, since most pulsing X-ray sources are disk-fed (Elsner et al. 1980). For a disk-fed star of given M and μ, $\dot{P}$ is a function of $PL^{3/7}$, independent of the details of the model (Ghosh et al. 1980). Figure 2 shows plots of $\dot{P}$ against $PL^{3/7}$, with the data from nine pulsing X-ray stars superposed on the theoretical spin-up curves given by the disk accretion model of Ghosh and Lamb (1979a, 1979b) for $\mu_{30} = 0.48$ and $M = 0.5$, 1.3, and 1.9 $M_\odot$ (Fig. 2a), and for $\mu_{30} = 10^{-2}$, 10^{-1}, 1, 10, and 10^2 assuming $M = 1.3\ M_\odot$ (Fig. 2b). Here $\mu_{30} = \mu/10^{30}$ G cm^3. The choice $\mu_{30} = 0.48$ and $M = 1.3\ M_\odot$ is consistent with the data for all stars except Vela X-1, which may be wind-fed (see Lamb 1977). However,

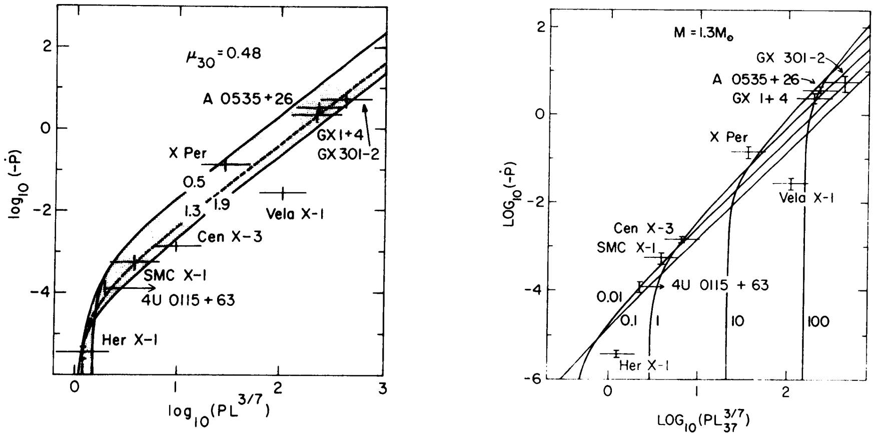

Fig. 2. Data from nine pulsing X-ray sources plotted against theoretical spin-up curves for disk accretion (from Ghosh and Lamb 1979b and Ghosh et al. 1980; see also Mason 1977, Rappaport and Joss 1977)

$M = 1.3\ M_\odot$ with μ_{30} ranging from 0.5 to 80 for different sources is also consistent with the data. Indeed, the short-term behavior of A 0535+26 suggests that this star has $\mu_{30} = 8.6$ (Elsner et al. 1980). These ambiguities can be resolved by measuring P, $\dot{P}$, and L as the luminosity of the star varies (Lamb 1979b). Such measurements determine the quantities $S_1\mu^{2/7}$ and $S_2\mu^{6/7}$, where S_1 ($\equiv R^{6/7}M^{-3/7}I^{-1}$) and S_2 ($\equiv R^{-3/7}M^{-2/7}$) are stellar structure parameters which, given the equation of state, depend only on M (Ghosh and Lamb 1979b). Thus, if M can be separately determined, both μ and the equation of state can be constrained (Lamb and Zylstra 1980).

From the point of view of magnetic field determinations, the most interesting star is perhaps Her X-1, which is known to be disk-fed, is a fast rotator, and has an accurately measured period, period derivative, and luminosity (see Lamb 1977). The model of disk accretion developed by Ghosh and Lamb (1979a, 1979b) gives $\mu_{30} = 0.47$ and a fastness $\omega_s = 0.35$ for the 1.3 $M_\odot$ star of Figure 1, implying a surface dipole magnetic field $B_d = 1.2 \times 10^{11}$ G. Even if ω_s is as large as 1.0 (its maximum permissible value) and R is as small as 10 km (the smallest value given by currently acceptable equations of state), the implied value of B_d is just 9.4×10^{11} G, substantially smaller than the total surface field strength $B_s \cong 4 \times 10^{12} - 6 \times 10^{12}$ G inferred from the cyclotron interpretation of the 40 - 60 keV spectral feature. This suggests that the surface field strength is largely in higher multipole moments (Lamb 1978). That the surface magnetic fields of some X-ray stars and most pulsars are complex has been argued on different grounds by Elsner and Lamb (1976) and by numerous pulsar theorists (see Arons 1980).

The presence of such a strong surface magnetic field in Her X-1 and its estimated age (~5 x 10^8 y, van den Heuvel 1977), place a severe constraint on the electrical resistivity of the crust (Ewart et al. 1975, Flowers and Itoh 1976). The likely presence of quadrupole or higher moments, which tend to decay faster than the dipole moment, makes this constraint even more severe. This evidence argues against the universality of dipole field decay in ~10^6 y, a hypothesis that has been widely invoked to explain the observed P-$\dot{P}$ distribution of pulsars (see Manchester and Taylor 1977, p. 164). This discrepancy could be resolved if (1) the long field decay time in Her X-1 is relatively unique, (2) either the age or the magnetic field of Her X-1 are greatly overestimated, or (3) pulsars turn off as the result of alignment of the dipole axis with the spin axis or by some other mechanism, rather than as a result of magnetic field decay (note that torques can cause alignment only if the dipole and figure axes of the star are already aligned, otherwise rearrangement of the crust must occur; see Lamb et al. 1975).

5. MACROGLITCHES

The size and rate of sudden, large changes in the frequencies of pulsars and pulsing X-ray sources ("macroglitches") provide clues about the physical processes responsible. In addition, the frequency behavior following a macroglitch provides information about the internal structure and dynamical properties of the star. Macroglitches have now been detected in four pulsars: one each in PSR 1325-43 (Manchester 1981) and PSR 1641-45 (Manchester et al. 1978), two in the Crab pulsar (see Boynton et al. 1972, Lohsen 1975), and four in the Vela pulsar (see Downs 1981). Although other macroglitches have been reported, Groth (1975) and Cordes and Helfand (1980) have argued convincingly that these are more likely just local accumulations of random small events. Rapid, large changes in the frequencies of the pulsing X-ray sources Cen X-3 and Vela X-1 have been observed (see Fabbiano and Schreier 1977, Becker et al. 1978), but at least in Cen X-3, the change was not as sudden as the pulsar macroglitches. Observations and interpretations of macroglitches and theoretical studies of the response of the star have been extensively reviewed elsewhere (see Ruderman 1972, Pines et al. 1974, Groth 1975, Lamb 1977, 1979a, and Pines 1980), and only a brief summary of the current status will be given here.

Macroglitches are usually characterized by three parameters: the relative size $(\Delta\Omega/\Omega)_o$ of the initial frequency jump, the fraction Q of the initial jump that eventually decays away, and the time scale τ of the decay. For the Crab and Vela glitches, these parameters have been determined with some precision. Successive glitches in the same star appear similar, but not identical. In the Crab pulsar, $(\Delta\Omega/\Omega)_o \sim 10^{-8}$, $Q \simeq 0.95$, and $\tau \sim 4$-15 d, while in the Vela pulsar, $(\Delta\Omega/\Omega)_o \sim 10^{-6}$, $Q \sim 0.2$, and $\tau \sim 450$ d.

Pulsar macroglitches have been attributed to a wide variety of causes. Among the explanations considered most promising at present are crustquakes (Ruderman 1969, Baym et al. 1969, Baym and Pines 1971), corequakes (Pines et al. 1972, but see Baym et al. 1976), and crust-breaking by vortex pinning (Ruderman 1976). None of these models is entirely satisfactory. The Vela glitches are too large to be accounted for by crustquakes, while the Crab glitches have too large a Q if the mass of the star is, as expected, $\gtrsim 1\ M_{\odot}$. Corequakes can in principle cause glitches as large as those seen in the Vela pulsar, but may produce too much heat, while the change in the crust moment of inertia in a crust-breaking event, and hence the size of the resulting glitch, is uncertain.

Recently, Greenstein (1979b) has pointed out that sudden heating of the neutron superfluid could cause angular momentum to be transfered from the faster core to the slower crust, creating a glitch. Among the heating mechanisms he has proposed, accretion, magnetospheric instabilities, and electrical currents all heat the crust and are likely to heat the interior too slowly and inefficiently; crustquakes and vortex unpinning will themselves cause a change in the rotation of the crust in addition to that caused by the associated heating. Pines et al. (1980) have developed a model in which ~10% of the pinned vortices in the inner crust break free once every few years. They argue that such events can produce glitches as large as those seen in the Vela pulsar and PSR 1641-45 while releasing only $\sim 10^{39}$ ergs in heat.

Regardless of the cause of a macroglitch, the behavior of the pulse frequency following the event furnishes information about the internal structure and dynamics of the star. In the starquake models, the post-glitch relaxation time τ is the coupling time between the crust and neutron superfluid, while in the thermal-pulse and vortex-unpinning models it reflects cooling of the interior and spin-down of the crust by the external torque, and hence only provides a bound on the crust-superfluid coupling time. In the crust-breaking model of Ruderman, τ is the time scale for plastic flow of the crust, and is unrelated to the crust-superfluid coupling time.

Greenstein (1979a, 1979b) has argued that a sudden small heating of the interior of a cold pulsar will lead to runaway frictional heating of the interior, causing a massive transfer of angular momentum from the core to the crust, and a speed-up of the pulse frequency by a factor of two or more ("thermal/timing instability"). However, Lamb (1980) has argued that if the star is hot when formed and temperature perturbations occur more often than once per cooling time, this instability is unlikely to have a dramatic effect. The reason is that for the crust-superfluid coupling and cooling mechanisms considered by Greenstein, such perturbations repeatedly damp the rotation of the core during the era when the instability is still quenched by dynamical effects. Thus, the effect of such perturbations is to feed rotational energy from the core into the crust and into heat as the star ages, without sudden large changes in the rotation rate of the crust.

6. MICROGLITCHES

At least four pulsing X-ray stars (Her X-1, Cen X-3, Vela X-1, and X Per) and more than a dozen pulsars (see Helfand et al. 1980) show detectable small-scale variations in pulse frequency. The pioneering work of Boynton et al. (1972, see also Groth 1975 and Cordes 1980) showed that in the Crab pulsar, these variations can be described by a series of frequent, small events ("microglitches") which can be represented mathematically as a random noise process. This same description has since been shown to be consistent with the data from ten other pulsars (see Cordes and Helfand 1980). Previously, Lamb et al. (1974, 1976, 1978a, 1978b) had argued on theoretical grounds that the frequency variations seen in some pulsing X-ray stars are due to excitation of the crust by broad-band torque noise, and showed that the data then available were consistent with this conjecture. As in the case of macroglitches, the rate and size of microglitches furnish information about the processes that cause them, while the response of the star reveals its internal structure.

The description in terms of random noise processes allows this complicated phenomenon to be characterized by a small number of parameters. For example, the two types of events shown in Figure 3 can be represented mathematically by the two expressions

$$\delta\Omega_c(t) = \sum_i \delta\phi_i \ \delta(t-t_i)$$

and

$$\delta\Omega_c(t) = \sum_i \delta\Omega_i \ \theta(t-t_i) \quad ,$$

where $\delta(t)$ and $\theta(t)$ are the delta and unit step functions, and $\delta\phi_i = \delta\Omega_i \delta t_i$ in terms of the size $\delta\Omega_i$ and duration δt_i of the i^{th} event. Provided that there are a large number of events in the observing interval T ($RT \gg 1$, where R is the mean event rate) and none of the events is resolved ($\delta t \ll \delta T$, where δT is the sampling interval of the observations), the only observable quantity besides the type of noise is its strength ($R\langle\delta\phi^2\rangle$ and $R\langle\delta\Omega^2\rangle$ for the two noise processes above; here $\langle\delta\phi^2\rangle$ and $\langle\delta\Omega^2\rangle$ are the second moments of $\delta\phi_i$ and $\delta\Omega_i$).

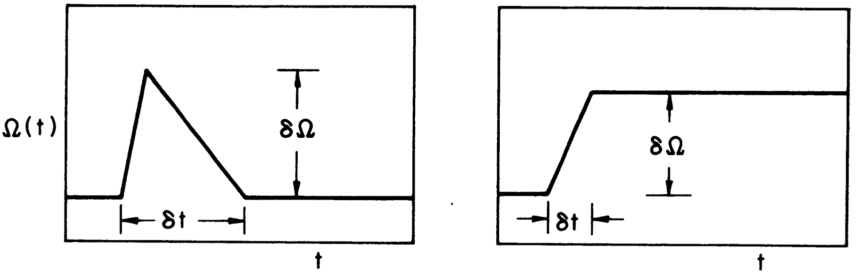

Fig. 3. Two examples of possible small-scale timing irregularities. Variations (a) and (b) can be modeled by white noise in Ω and $\dot{\Omega}$, respectively (see Lamb, Pines and Shaham 1978a).

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Table II lists a variety of simple noise processes, physical processes that might produce these types of timing noise in pulsars and pulsing X-ray sources, the resulting power density P_ϕ of pulse phase fluctuations as a function of angular frequency ω, the spectrum designation, and the expected behavior of the rms phase residual $\Delta\phi$ as a function of the length of the observing interval T. In the nomenclature adopted here, a noise process is identified by specifying the variable in which the process is white noise. This contrasts with the older nomenclature (peculiar to pulsar work) in which white noise in Ω, $\dot{\Omega}$, and $\ddot{\Omega}$ is referred to as "phase noise (PN)", "frequency noise (FN)", and "slowing-down noise (SN)". The older nomenclature can be confusing, since <u>all</u> of these processes produce noise in the pulse phase, frequency, etc. Moreover, the old nomenclature cannot be easily generalized to the larger number of noise processes needed here. Finally, the nomenclature used here emphasizes the variable in which the process is stationary (white noise), which plays a special role in analyzing data.

Information on the physical processes that cause timing noise is furnished by the noise spectrum and strength. Thus, Lamb et al. (1978a) have argued that the strength of the timing noise observed in Cen X-3 and Her X-1 are consistent with estimates of the possible strength of accretion torque fluctuations or fluctuations in the torque on the crust caused by vortex unpinning. They have also discussed the noise spectra to be expected from these processes. Information on the causes of timing noise and on variations in stellar properties can be obtained by comparing the noise strengths and rms phase residuals of a collection

TABLE II. CHARACTERISTICS OF TIMING NOISE PROCESSES

Noise Type	Possible Origin	$P_\phi(\omega)$	Spectrum	$\Delta\phi(T)$
White noise in ϕ	Measurement errors, pulse shape changes	$\propto\omega^0$	White	$\propto T^0$
White noise in Ω (PN)	Quakes in heavy stars, temperature fluctuations	$\propto\omega^{-2}$	First-order red	$\propto T^{-1/2}$
White noise in $\dot{\Omega}$ (FN)	Quakes in light stars, temperature fluctuations, vortex unpinning, torque fluctuations	$\propto\omega^{-4}$	Second-order red	$\propto T^{3/2}$
White noise in $\ddot{\Omega}$ (SN)	Torque steps	$\propto\omega^{-6}$	Third-order red	$\propto T^{5/2}$

TABLE III. TIMING NOISE SCALING LAWS

Physical Process	Noise Strength	$\Delta\phi$
White torque noise	$\propto\dot{\Omega}^2$	$\propto\dot{\Omega}T^{3/2}$
Red torque noise	$\propto\dot{\Omega}^2$	$\propto\dot{\Omega}T^{5/2}$
Vortex unpinning noise[a]	$\propto\Omega\dot{\Omega}$	$\propto\Omega^{1/2}\dot{\Omega}^{1/2}T^{3/2}$
Small crustquakes	$\propto\Omega^3\dot{\Omega}$	$\propto\Omega^{3/2}\dot{\Omega}^{1/2}T^{3/2}$

[a]According to the model proposed by Lamb et al. (1978a).

of neutron stars. Table III shows how the timing noise produced by various physical processes would scale from star to star if all had similar properties. Lamb (1979a) has argued that the timing noise in pulsing X-ray sources is due to white torque noise which scales as shown. More recently, Cordes and Greenstein (1980) have used the results of Cordes and Helfand (1980) on the spectrum and strength of the timing noise in a collection of eleven pulsars to discuss critically a variety of physical models of the origins of pulsar timing noise.

Regardless of its cause, excitation of the stellar crust by a broad-band noise process can be used to systematically probe the interior of the star. This possibility was pointed out by Lamb (1977) and has been developed in detail by Lamb et al. (1978a, 1978b). The reason is that the noise "signal" applied to the crust is, in effect, "filtered" by the crust-superfluid system to produce the "output"

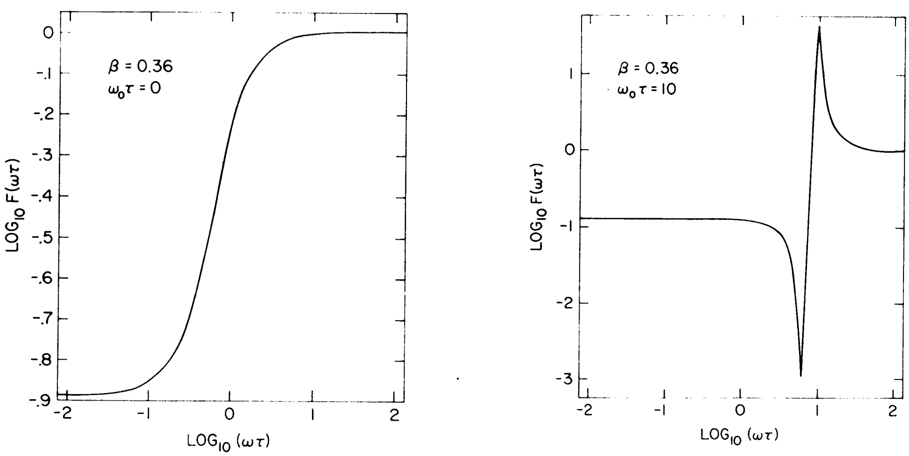

Fig. 4. Neutron star power transfer functions for (a) the two-component model and (b) a generalized two-component model with a resonant mode

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represented by pulse phase fluctuations. This is expressed mathematically by the relation $P_{obs}(\omega) = F(\omega)\, P_{ex}(\omega)$ between the power density of the exciting noise process P_{ex} and the observed power density P_{obs}. Here F is the power transfer function of the neutron star, which depends on its internal dynamical properties. The form of the power transfer function is immediately apparent if one works with the variable in which the exciting noise is white, since in this case $P_{ex}(\omega)$ = const. and hence $P_{obs}(\omega) \propto F(\omega)$. Figure 4a shows the power transfer function for a two-component star with $\beta \equiv I_c/I = 0.36$, where I_c and I are the inertial moments of the crust and the whole star, respectively. The power transfer at $\omega << \tau^{-1}$ is a factor β^2 smaller than at $\omega >> \tau^{-1}$ because perturbations of the crust rotation at the lower frequency must drive the whole star, whereas those at the higher frequency drive only the crust. The quantity β^2 is a sensitive function of the star's mass and the equation of state. Figure 4b shows the power transfer function for a more complicated star with an internal resonant mode at the frequency $\omega_o = 10\tau^{-1}$.

An alternative approach to the power spectrum method is to study the behavior of the rms phase residual $\Delta\phi$ as a function of the length T of the observing interval. Although mathematically equivalent for equally-spaced noise-free data, this approach has many disadvantages when handling real data (see for example Blackman and Tukey 1958).

The power density spectrum of actual data is usually composite, even if that intrinsic to the source is a simple power-law. This is illustrated in Figure 5a, which shows schematically the effect on a power-law noise signal (denoted N) of (1) fitting a polynomial of degree n to the measured pulse phase (which removes power at $\omega \lesssim \omega_n$), and (2) measurement error (which produces the white noise in pulse phase denoted ME). Figure 5b shows the same spectrum after prewhitening. The precision with which the power density spectrum of the noise signal can be determined is governed by the precision ΔP of the local power density

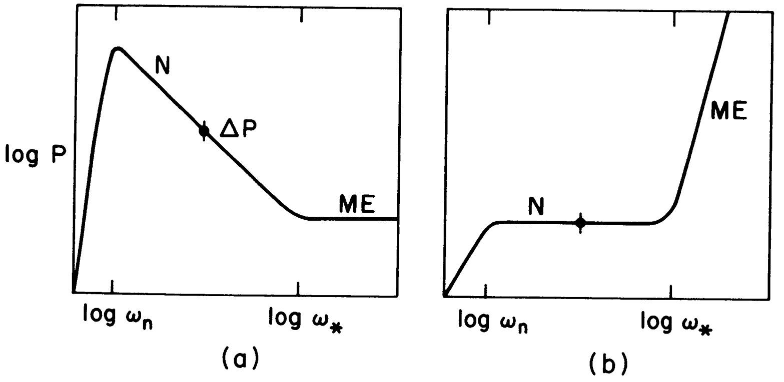

Fig. 5. Schematic illustration of observed power density spectrum (a) and prewhitened spectrum (b)

estimates and the frequency ω_* above which the power density produced by measurement error exceeds that of the noise signal. In dealing with the extremely steep spectra characteristic of timing noise, great care must be taken to insure that the local power density estimates are not contaminated by leakage of power from distant parts of the spectrum. Further details are given by Lamb (1979a) and Boynton and Deeter (1979, 1980).

Boynton and Deeter (1979, 1980) have determined the power density spectrum of pulse phase fluctuations in the Crab pulsar, using the Princeton optical timing data, and in Her X-1, using the Uhuru X-ray data. The results are shown in Figure 6, where the power density in the variable $\nu = \Omega/2\pi$ is plotted against frequency $f = \omega/2\pi$. In each case the data points to the left of the dashed line show the timing noise intrinsic to the star, while the dashed line indicates the calculated spectrum of the white phase noise introduced by the measurement errors. The latter has been made "blue" by the process used to whiten the intrinsic noise spectrum. Both intrinsic noise spectra are relatively flat over more than two decades in frequency, indicating that a description in terms of a simple noise process (in this case, white noise in $\dot{\Omega}$) is appropriate, and that the neutron star response is approximately that of a rigid body (F = const.). Comparison with the power transfer function for a two-component star excludes τ's in the range 4-14 d, for the Crab pulsar, and 1-9 d, for Her X-1, with 99% confidence, assuming $Q \cong 1 - \beta$ is as large as 0.7 (see Boynton 1981).

In the case of the Crab pulsar, these results conflict with the parameters derived from analysis of the two macroglitches (see also Groth 1975), implying that the internal dynamics of this neutron star are more complicated than can be described by the two-component model. This represents a major advance in the continuing effort to confront

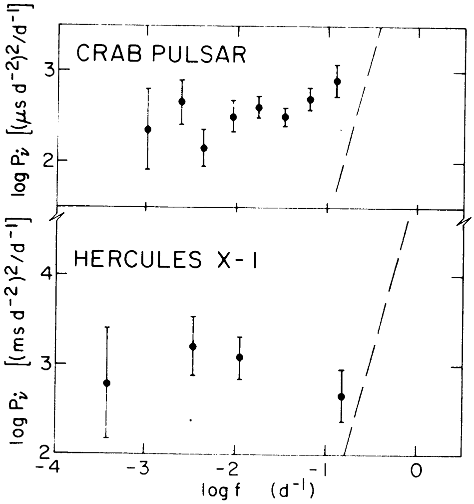

Fig. 6. Prewhitened power density spectra observed in the Crab pulsar and Her X-1 (from Boynton 1981)

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theories of neutron star dynamics with observation, and supports theoretical efforts to develop more realistic models. One possibility is that the response of the star depends on the size and rate of events, which are very different for macroglitches and microglitches. A macroglitch could, for example, heat a portion of the neutron superfluid, thereby reducing τ from a "normal" value >> 40 d to 4-15 d. Alternatively, macroglitches may release a second component which is not released by the much smaller microglitches. This would be the case if neutron superfluid vortices in the inner crust remain pinned except when unpinned by a macroglitch. If in addition the inertial moment of the unpinned superfluid is at least comparable to that of the crust component, the behavior could be consistent with that observed.

Cordes and Helfand (1980) have adopted the alternative approach of studying $\Delta\phi(T)$ for eleven pulsars, but were only able to estimate $\Delta\phi$ for two values of T. This is equivalent to determining the power density at two points in the spectrum. Given the complexity of the response expected even for simple neutron star models, results based on so few points must be treated very cautiously. Two pulsars are consistent with white noise in Ω, four with white noise in $\dot{\Omega}$, and two with white noise in $\ddot{\Omega}$. The timing residuals of three others are ambiguous with this data set.

7. CONCLUDING REMARKS

We have seen that observations of pulsars and pulsing X-ray sources have furnished a substantial amount of information on the properties of neutron stars, including estimates of the masses of seven probable neutron stars, the magnetic moments of eight pulsing X-ray stars and tens of pulsars, and the surface magnetic fields of two pulsing X-ray stars. Measurements of macroglitches in four pulsars constrain models of their internal structure, while smaller scale timing irregularities have been detected in more than a dozen pulsars and at least four pulsing X-ray stars. Studies of the latter have provided important information on the causes of timing noise. Regardless of the cause, such irregularities can be used to probe the internal structure of neutron stars, as has been done in the Crab pulsar and Her X-1.

The prospects for future progress are bright. New mass estimates may come from the two most recently discovered binary pulsars (Taylor 1981) and from additional X-ray stars. X-ray stars are the most promising candidates for studies of neutron star inertial moments and magnetic fields, and for studies of internal dynamical time scales of two years or less. Such studies are already in progress using data from the HEAO-1 satellite (Lamb 1979b) and fresh opportunities will be provided by the launch of the X-ray Timing Explorer in the mid-1980's. Radio observations are the most promising for studies of internal dynamical time scales longer than two years. Tests of noise strengths, spectra, and scaling laws between sources will provide new constraints on the causes of timing noise and the variation in neutron star properties from source to source.

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