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ON CONCIRCULAR TRANSFORMATIONS IN RIEMANNIAN SPACES

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Abstract

This paper introduces a tensor that contains the Riemannian curvature tensor and the conformal curvature tensor as special examples in the Riemannian space (M^n, g), and by using this tensor we define C'-semi-symmetric space. In this paper, we have the following main result: if there is a non-trivial concircular transformation between two C'-semi-symmetric spaces, then both spaces are of quasi-constant curvature.

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1. Introduction and preliminaries

It is well known that if a curvature tensor in Riemannian space (M^n, g) satisfies $R^h{}_{ijk,lm} = R^h{}_{ijk,ml}$, then (M^n, g) is said to be S-manifold or semi-symmetric space (here the comma "," followed by a Latin index denotes covariant derivative with respect to g). If the conformal curvature tensor

$$C^h{}_{ijk} = R^h{}_{ijk} + \frac{1}{n-2}\left(\delta^h{}_j R_{ik} - \delta^h{}_k R_{ij} + g_{ik} R^h{}_j - g_{ij} R^h{}_k\right) + \frac{R}{(n-1)(n-2)}\left(\delta^h{}_k g_{ij} - \delta^h{}_j g_{ik}\right)$$

of (M^n, g) satisfies $C^h{}_{ijk,lm} = C^h{}_{ijk,ml}$, then (M^n, g) is said to be a *conformally semi-symmetric space*. In order to treat a semi-symmetric space and conformally semi-symmetric space simultaneously, we introduce the following tensor in the

Riemannian space (M^n, g):

$$(1.1)\qquad C^{h\prime}{}_{ijk} = R^h{}_{ijk} + a\left(\delta^h{}_j R_{ik} - \delta^h{}_k R_{ij} + g_{ik} R^h{}_j - g_{ij} R^h{}_k\right) + bR\left(\delta^h{}_k g_{ij} - \delta^h{}_j g_{ik}\right)$$

where a and b are constants ($a \neq -1$). It is obvious that if $a = b = 0$, then $C^{h\prime}{}_{ijk} = R^h{}_{ijk}$; if $a = 0$, $b = -1/n(n-1)$, then $C^{h\prime}{}_{ijk} = Z^h{}_{ijk}$ is the concircular curvature tensor; if $a = 1/(n-2)$, $b = 0$, then $C^{h\prime}{}_{ijk} = Z^{h\prime}{}_{ijk} = R^h{}_{ijk} + 1/(n-2)(\delta^h{}_j R_{ik} - \delta^h{}_k R_{ij} + g_{ik} R^h{}_j - g_{ij} R^h{}_k)$ is the conharmonic curvature tensor; if $a = 1/(n-2)$, $b = 1/(n-1)(n-2)$, then $C^{h\prime}{}_{ijk} = C^h{}_{ijk}$ is the conformal curvature tensor.

It is easy to verify that the tensors $C^{h\prime}{}_{ijk}$ and $C'_{hijk} \equiv g_{hl} C^{l\prime}{}_{ijk}$ satisfy the following identities

$$(1.2)\qquad C^{h\prime}{}_{ijk} = -C^{h\prime}{}_{ikj},$$

$$(1.3)\qquad C^{a\prime}{}_{ajk} = 0,$$

$$(1.4)\qquad C^{h\prime}{}_{ijk} + C^{h\prime}{}_{jki} + C^{h\prime}{}_{kij} = 0,$$

$$(1.5)\qquad C'_{hijk} = C'_{jkhi} = -C'_{ihjk} = -C'_{hikj},$$

$$(1.6)\qquad C'_{ij} \equiv C^{a\prime}{}_{ija} = \left(1-(n-2)a\right)R_{ij} + \left(b(n-1)-a\right)Rg_{ij}.$$

If tensor (1.1) satisfies

$$(1.7)\qquad C^{h\prime}{}_{ijk,lm} = C^{h\prime}{}_{ijk,ml}$$

then we say that (M^n, g) is a *C'-semi-symmetric space*. If the Ricci tensor R_{ij} satisfies

$$(1.8)\qquad R_{ij,lm} = R_{ij,ml}$$

then (M^n, g) is called a *Ricci semi-symmetric space*.

In 1940 to 1942 K. Yano [1] introduced the concept of a concircular transformation between two Riemannian spaces (M^n, g) and $(M^n, \bar{g})$. A concircular transformation between two Riemannian spaces (M^n, g) and $(M^n, \bar{g})$ is by definition a conformal transformation of (M^n, g) to $(M^n, \bar{g})$ which carries geodesic circles in (M^n, g) to geodesic circles in $(M^n, \bar{g})$. K. Yano showed that a conformal transformation $\bar{g}_{ij} = e^{2\rho} g_{ij}$ is a concircular transformation if and only if the equation $\rho_{,ij} - \rho_{,i}\rho_{,j} = \varphi g_{ij}$ holds. Using the change $1/\sigma = e^\rho$, it is easy to verify that a conformal transformation $\bar{g}_{ij} = \sigma^{-2} g_{ij}$ is concircular if and only if the equation $\sigma_{,ij} = \psi g_{ij}$ holds for a certain function ψ. We shall use this simple form. It is obvious that if $\sigma =$ constant, then the concircular transformation

$$(1.9)\qquad \bar{g}_{ij} = \sigma^{-2} g_{ij}, \qquad \sigma_{,ij} = \psi g_{ij}$$

are the homothety or trivial transformation. In this paper, we only study non-trivial transformations.

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It is easy to verify that, under the concircular transformation (1.9), the Christoffel symbols, the Riemannian curvature tensors, the Ricci tensors, the scalar curvature and tensor (1.1) of $(M^n, \bar{g})$ and (M^n, g) are related as follows:

$$\left\{ \overline{\begin{matrix} h \\ ij \end{matrix}} \right\} = \left\{ \begin{matrix} h \\ ij \end{matrix} \right\} - \frac{1}{\sigma}\left(\delta^h{}_j \sigma_{,j} + \delta^h{}_i \sigma_{,j} - g_{ij} \sigma_{,}{}^h \right), \tag{1.10}$$

$$\bar{R}^h{}_{ijk} = R^h{}_{ijk} + \alpha\left(\delta^h{}_j g_{ik} - \delta^h{}_k g_{ij} \right), \tag{1.11}$$

$$\bar{R}_{ij} = R_{ij} - (n-1)\alpha g_{ij}, \tag{1.12}$$

$$\bar{R}^i_j = \sigma^2\left(R^i_j - (n-1)\alpha \delta^i{}_j \right), \tag{1.13}$$

$$\bar{R} = \sigma^2(R - n(n-1)\alpha), \tag{1.14}$$

$$\bar{C}^{h\prime}{}_{ijk} = C^{h\prime}{}_{ijk} + \alpha\beta\left(\delta^h{}_j g_{ik} - \delta^h{}_k g_{ij} \right) \tag{1.15}$$

where $\sigma_{,}{}^h = g^{kh}\sigma_{,k}$

$$\alpha = \frac{2\psi}{\sigma} + \frac{1}{\sigma^2}\Delta_1 \sigma, \tag{1.16}$$

$$\Delta_1 \sigma = g^{ab}\alpha_{,a}\sigma_{,b},$$

$$\beta = 1 - 2a(n-1) + b(n-1)n. \tag{1.17}$$

We know that when $n > 3$ a space of quasi-constant curvature is a Riemannian space whose curvature tensor satisfies

$$R^h{}_{ijk} = p\left(\delta^h{}_k g_{ij} - \delta^h{}_j g_{ij} \right) + q\left(\left(\delta^h{}_k v_j - \delta^h{}_j v_k \right) v_i + \left(v_k g_{ij} - v_j g_{ik} \right) v^h \right) \tag{1.18}$$

where p and q are scalar functions and v_i is a unit covariant vector field. The vector field v^i is called the generator of the space ([4] and [5]).

The purpose of this paper is to study the non-trivial concircular transformations of C'-semi-symmetric Riemannian spaces. In Section 2 we study concircular transformations of a C'-semi-symmetric space to a Riemannian space; in Section 3 we study concircular transformations between two C'-semi-symmetric spaces. In this paper we always assume that $n > 3$, the metrics are positive definite and the indices $h, i, j, k, l, m, \ldots$ run over the range $1, 2, \ldots, n$.

2. Concircular transformations of a C'-semi-symmetric space to a Riemannian space

It is obvious that a semi-symmetric space is C'-semi-symmetric. Conversely, a C'-semi-symmetric space is semi-symmetric if it is Ricci semi-symmetric.

LEMMA 1. *If there is a concircular transformation of a space (M^n, g) of quasi-constant curvature to a Riemannian space $(M^n, \bar{g})$, then $(M^n, \bar{g})$ is also of quasi-constant curvature.*

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PROOF. Substituting (1.18) into (1.11) and from $\bar{g}_{ij} = \sigma^{-2} g_{ij}$ we get

$$\bar{R}^h{}_{ijk} = (p - \alpha)\sigma^2(\delta^h{}_k \bar{g}_{ij} - \delta^h{}_j \bar{g}_{ik}) + q\sigma^2\big((\delta^h{}_k \bar{v}_j - \delta^h{}_j \bar{v}_k)\bar{v}_i + (\bar{v}_k \bar{g}_{ij} - \bar{v}_j \bar{g}_{ik})\bar{v}^h\big)$$

where $\bar{v}_j = \sigma^{-1} v_j$ is a unit covariant vector field under the metric $\bar{g}$. Consequently $(M^n, \bar{g})$ is also of quasi-constant curvature.

LEMMA 2. *If a Riemannian space (M^n, g) admits a concircular transformation* (1.9), *then there is a scalar function K such that the following equations hold*:

$$K(\sigma_{,k} g_{ik} - \sigma_{,j} g_{ik}) = \sigma_{,a} R^a{}_{ijk}, \tag{2.1}$$

$$(n-1)K\sigma_{,k} = \sigma_{,a} R^a{}_k. \tag{2.2}$$

PROOF. If a Riemannian space (M^n, g) admits a concircular transformation (1.9), then we have

$$\sigma_{,ij} = \psi g_{ij}. \tag{2.3}$$

Covariant differentiation of (2.3) with respect to g_{ij} and Ricci's identity give us

$$\psi_{,k} g_{ij} - \psi_{,j} g_{ik} = \sigma_{,a} R^a{}_{ijk}. \tag{2.4}$$

Transvecting (2.4) with $\sigma_{,}{}^i$, we obtain

$$\psi_{,k}\sigma_{,j} - \psi_{,j}\sigma_{,k} = 0.$$

Consequently there exists a function K such that

$$\psi_{,k} = K\sigma_{,k}. \tag{2.5}$$

Substituting (2.5) into (2.4), we get (2.1). Again contracting (2.1) with g^{ij} we get (2.2).

Now we study non-trivial concircular transformations of a C'-semi-symmetric space (M^n, g) to a Riemannian space $(M^n, \bar{g})$. Twice covariant differentiation of (2.1) with respect to g_{ij}, and (2.3) and (2.5) give us that

$$\begin{aligned} K_{,lm}(\sigma_{,k} g_{ij} - \sigma_{,j} g_{ik}) + K_{,l}\psi(g_{km} g_{ij} - g_{jm} g_{ik}) \\ + \sigma_{,m} K^2(g_{kl} g_{ij} - g_{jl} g_{ik}) + \psi K_{,m}(g_{kl} g_{ij} - g_{jl} g_{ik}) \\ = \psi_{,m} R_{lijk} + \psi R_{lijk,m} + \psi R_{mijk,l} + \sigma_{,a} R^a{}_{ijk,lm}. \end{aligned} \tag{2.6}$$

Interchanging the place of the indices l and m in (2.6), and subtracting equation (2.6) from the obtained relation, we get

$$\begin{aligned} K\sigma_{,l}\big(R_{mijk} - K(g_{km} g_{ij} - g_{jm} g_{ik})\big) - K\sigma_{,m}\big(R_{lijk} - K(g_{kl} g_{ij} - g_{jl} g_{ik})\big) \\ = \sigma_{,a}\big(R^a{}_{ijk,lm} - R^a{}_{ijk,ml}\big). \end{aligned} \tag{2.7}$$

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On the other hand, from (1.1) we obtain easily that

$$C^{h\prime}{}_{ijk,lm} - C^{h\prime}{}_{ijk,ml}$$
$$= R^h{}_{ijk,lm} - R^h{}_{ijk,ml} + a\big(\delta^h{}_j(R_{ik,lm} - R_{ik,ml}) - \delta^h{}_k(R_{ij,lm} - R_{ij,ml})$$
$$+ g_{ij}\big(R^h{}_{j,lm} - R^h{}_{j,ml}\big) - g_{ij}\big(R^h{}_{k,lm} - R^h{}_{k,ml}\big)\big).$$

Assume that (M^n, g) is a C'-semi-symmetric space. Consequently the above mentioned equation becomes

(2.8)
$$R^h{}_{ijk,lm} - R^h{}_{ijk,ml} = a\big(\delta^h{}_k(R_{ij,lm} - R_{ij,ml}) - \delta^h{}_j(R_{ik,lm} - R_{ik,ml})$$
$$+ g_{ij}\big(R^h{}_{k,lm} - R^h{}_{k,ml}\big) - g_{ik}\big(R^h{}_{j,lm} - R^h{}_{j,ml}\big)\big).$$

Substituting (2.8) into (2.7) and using the Ricci identity, we get

(2.9)
$$K\sigma_{,l}\big(R_{mijk} - K(g_{km}g_{ij} - g_{jm}g_{ik})\big) - K\sigma_{,m}\big(R_{lijk} - K(g_{kl}g_{ij} - g_{jm}g_{ik})\big)$$
$$= a\big(\sigma_{,k}\big(R_{aj}R^a{}_{ilm} + R_{ia}R^a{}_{jlm}\big) - \sigma_{,j}\big(R_{ak}R^a{}_{ilm} + R_{ia}R^a{}_{klm}\big)$$
$$+ g_{ij}\sigma_{,a}\big(R^a{}_bR^b{}_{klm} - R^b{}_kR^a{}_{blm}\big) - g_{ik}\sigma_{,a}\big(R^a{}_bR^b{}_{jlm} - R^b{}_jR^a{}_{blm}\big)\big).$$

From (2.1) and (2.2), equation (2.9) becomes

$$(2.10)\quad K\sigma_{,l}\big(R_{mijk} + dK(g_{mk}g_{ij} - g_{ik}g_{jm}) - a(g_{ij}R_{mk} - g_{ik}R_{mj})\big)$$
$$- K\sigma_{,m}\big(R_{lijk} + dK(g_{kl}g_{ij} - g_{ik}g_{jl}) - a(g_{ij}R_{lk} - g_{ik}R_{lj})\big)$$
$$= a\sigma_{,k}\big(R^a{}_jR_{lmai} + R^a{}_iR_{lmaj}\big) - a\sigma_{,j}\big(R^a{}_kR_{lmai} + R^a{}_iR_{lmak}\big)$$

where

$$(2.11)\qquad d = a(n-1) - 1.$$

Transvecting (2.10) with $\sigma^l_{,}$, we obtain $K = 0$ or

$$(2.12)\quad \Delta_1\sigma\big(R_{mijk} + dK(g_{mk}g_{ij} - g_{ik}g_{jm}) - a(g_{ij}R_{mk} - g_{ik}R_{mj})\big)$$
$$= a\big\{\sigma_{,k}\sigma_{,i}\big(R_{mj} - (n-1)Kg_{mj}\big) - \sigma_{,j}\sigma_{,i}\big(R_{mk} - (n-1)Kg_{mk}\big)\big\}.$$

If $K = 0$, then from (2.5) we have $\psi =$ constant. If (2.12) holds, contracting (2.12) with g^{mk}, we get

(2.13)
$$\Delta_1\sigma(1 + a)R_{ij} = \Delta_1\sigma(aR - (n-1)dK)g_{ij} + a(n(n-1)K - R)\sigma_{,i}\sigma_{,j}.$$

We put

$$(2.14)\qquad v_i = \sigma_{,i}/\sqrt{\Delta_1\sigma}\,.$$

Then it is obvious that v_i is a unit vector field under the metric g. Substituting (2.14) into (2.13), we get

$$(2.15)\quad R_{ij} = \frac{1}{1+a}(aR - (n-1)dK)g_{ij} + \frac{a}{1+a}(n(n-1)K - R)v_iv_j.$$

Substituting (2.14) and (2.15) into (2.12), we finally obtain

$$R_{mijk} = p(g_{mk}g_{ij} - g_{mj}g_{ik}) + q\big(v_i(g_{mk}v_j - g_{mj}v_k) + (v_k g_{ij} - v_j g_{ik})v_m\big)$$

where

$$p = \frac{1}{1+a}\big(a^2R - (an+1)(an-a-1)K\big), \qquad q = \frac{a^2}{1+a}\big(n(n-1)K - R\big).$$

Therefore (M^n, g) is of quasi-constant curvature, and from Lemma 1 $(M^n, \bar{g})$ is also of quasi-constant curvature. Thus we have

THEOREM 1. *If there is a non-trivial concircular transformation* (1.9) *of a C'-semi-symmetric space to a Riemannian space, then both spaces are of quasi-constant curvature or ψ = constant.*

In particular, different values of a and b must be considered, and then from Theorem 1 we have

THEOREM 2. *If there is a non-trivial concircular transformation* (1.9) *of a semi-symmetric space to a Riemannian space, then both spaces are of constant curvature or ψ = constant.*

THEOREM 3. *If there is a non-trivial concircular transformation* (1.9) *of a conformally semi-symmetric space to a Riemannian space, then both spaces are of quasi-constant curvature or ψ = constant.*

3. Concircular transformations between two C'-semi-symmetric spaces

Now we further investigate the case ψ = constant. In this case (2.1), (2.2) become respectively

(3.1) $$\sigma_{,a}R^a{}_{ijk} = 0,$$

(3.2) $$\sigma_{,a}R^a{}_k = 0.$$

Again assume that $(M^n, \bar{g})$ is also C'-semi-symmetric, namely that

(3.3) $$\bar{C}^{h\prime}{}_{ijk|lm} = C^{h\prime}{}_{ijk|ml}$$

where "|" denotes covariant differentiation with respect to $\bar{g}_{ik}$. Applying the Ricci identity to (3.3), we have

(3.4) $$\bar{C}^{h\prime}{}_{ajk}\bar{R}^a{}_{ilm} + \bar{C}^{h\prime}{}_{iak}\bar{R}^a{}_{jlm} + \bar{C}^{h\prime}{}_{ija}\bar{R}^a{}_{klm} - \bar{C}^{a\prime}{}_{ijk}\bar{R}^h{}_{alm} = 0.$$

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Substituting (1.11) and (1.16) into (3.4), we have

$$C^{h\prime}{}_{ajk}R^{a}{}_{ilm}+C^{h\prime}{}_{iak}R^{a}{}_{jlm}+C^{h\prime}{}_{ija}R^{a}{}_{klm}-C^{a\prime}{}_{ijk}R^{h}{}_{alm}$$
$$+\alpha\big(g_{im}C^{h\prime}{}_{ljk}-g_{il}C^{h\prime}{}_{mjk}+g_{jm}C^{h\prime}{}_{ilk}-g_{jl}C^{h\prime}{}_{imk}$$
$$+g_{km}C^{h\prime}{}_{ijl}-g_{kl}C^{h\prime}{}_{ijm}-\delta^{h}{}_{l}C'{}_{mijk}+\delta^{h}{}_{m}C'_{lijk}\big)=0. \tag{3.5}$$

Since (M^n, g) is a C'-semi-symmetric space, from (3.5) we have $\alpha = 0$ or

$$g_{im}C^{h\prime}{}_{ljk}-g_{il}C^{h\prime}{}_{mjk}+g_{jm}C^{h\prime}{}_{ilk}-g_{jl}C^{h\prime}{}_{imk}$$
$$+g_{km}C^{h\prime}{}_{ijl}-g_{lk}C^{h\prime}{}_{ijm}-\delta^{h}{}_{l}C'_{mijk}+\delta^{h}{}_{m}C'_{lijk}=0. \tag{3.6}$$

If $\alpha = 0$, then in consequence of (1.14), we have

$$2\psi\sigma+\Delta_1\sigma=0 \qquad (\psi=\text{constant}). \tag{3.7}$$

Differentiation (3.7), we get

$$2\psi\sigma_{,k}=0. \tag{3.8}$$

Since the transformation is non-trivial, equation (3.8) does not hold, and therefore $\alpha \neq 0$. Next we investigate the case where (3.6) holds. It will be contracted for h and l, and from (1.3), (1.4) and (1.5), we obtain

$$g_{km}C'{}_{ij}-g_{jm}C'{}_{ik}-(n-1)C'{}_{mijk}=0. \tag{3.9}$$

Substituting (1.6) in (3.9), we get

$$(n-1)C'{}_{mijk}+(1-(n-2)a)(g_{jm}R_{ik}-g_{km}R_{ij})$$
$$+(b(n-1)-a)R(g_{mj}g_{ik}-g_{km}g_{ij})=0. \tag{3.10}$$

Transvecting (3.10) with $\sigma_{,}^{k}$, and considering (1.1), (3.1) and (3.2), we obtain

$$(1+a)\sigma_{,m}R_{ij}=(n-1)a\sigma_{,i}R_{jm}+aR(\sigma_{,m}g_{ji}-\sigma_{,i}g_{jm}). \tag{3.11}$$

Again transvecting (3.11) with $\sigma_{,}^{i}$, and considering (2.14), we get

$$aR_{jm}=\frac{a}{n-1}R(g_{jm}-v_jv_m). \tag{3.12}$$

On the other hand, transvecting (3.11) with $\sigma_{,}^{m}$, and considering (2.14), we have

$$R_{ij}=\frac{a}{1+a}R(g_{ij}-v_iv_j).$$

Again transvecting the above equation with g^{ij}, we find

$$\frac{a}{1+a}R=\frac{R}{n-1}. \tag{3.13}$$

Substituting (3.12) and (1.1) into (3.10), and considering (3.13), we finally obtain

$$R_{mijk}=\frac{a}{n-1}R(g_{mk}g_{ij}-g_{mj}g_{ik})$$
$$-\frac{a}{n-1}R\big((g_{km}v_iv_j-g_{jm}v_iv_j)+(g_{ij}v_mv_k-g_{ik}v_mv_j)\big).$$

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Therefore (M^n, g) is of quasi-constant curvature, and from Lemma 1 $(M^n, \bar{g})$ is also of quasi-constant curvature. Thus we have

THEOREM 4. *If there is a non-trivial concircular transformation* (1.9), *where* $\psi =$ *constant, between two* C'*-semi-symmetric spaces, then both spaces are of quasi-constant curvature.*

In particular, we have

THEOREM 5. *If there is a non-trivial concircular transformation* (1.9), *where* $\psi =$ *constant, between two semi-symmetric spaces, then* (M^n, g) *is locally Euclidean and* $(M^n, \bar{g})$ *is of constant curvature.*

THEOREM 6. *If there is a non-trivial concircular transformation* (1.9), *where* $\psi =$ *constant, between two conformally semi-symmetric spaces, then both spaces are of quasi-constant curvature.*

From Theorems 1 and 4, we have the following theorem.

THEOREM 7. *If there is a non-trivial concircular transformation between* C'*-semi-symmetric spaces, then both spaces are of quasi-constant curvature.*

REMARK. Applying the method of this paper to the study of concircular transformations of Ricci semi-symmetric spaces we may get the following conclusion: if there is a non-trivial concircular transformation between Ricci semi-symmetric spaces, then both spaces are Einstein spaces.

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