THE CAPTURE OF HALLEY-TYPE AND JUPITER-FAMILY COMETS FROM THE NEAR-PARABOLIC FLUX

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Abstract. We have considered the transfer of comets from the near-parabolic flux to short-period orbits under the perturbations of Jupiter, Saturn, Uranus, and Neptune for 5 Gyr. We have developed a combined analytical and numerical scheme which includes all essential features of the dynamical evolution. Secular resonances are amongst the most important factors causing large changes in perihelion distances. We have studied the evolution of about 10^5 randomly oriented near-parabolic orbits with initial inclinations i uniformly distributed in $\cos i$ and perihelia q distributed in all the planetary region. The main contribution to Halley-type comets comes from $q < 2$ AU where the probability of the capture is 0.02. The number of Halley-type objects arising from the observed near-parabolic cometary flux of all inclinations and absolute magnitudes brighter than $H_{10} = 7$, is hundreds of times larger than the number of known Halley-type comets. In contrast with Halley-type comets, the majority of observable Jupiter-family comets originate from orbits with $q > 10$ AU. The flux of comets in high-eccentricity orbits may be the dominant source of the observed Jupiter family.

1. Introduction

There have been many attempts to calculate the probabilities of the dynamical transfer from near-parabolic to short-period orbits (Everhart, 1972; Stagg and Bailey, 1989; Quinn *et al.*, 1990; Wetherill, 1991; Fernandez and

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Gallardo, 1994). All these considerations are based on using simplified schemes due to extremely large CPU time that the numerical integration of the equations of motion takes. It is very difficult to estimate uncertainties introduced by these simplifications. But, it was shown (Bailey and Emel'yanenko, 1996) that all statistical considerations ignoring secular perturbations cannot accurately describe the long-term dynamical evolution of short-period comets. Even numerical integrations based on circular planetary orbits do not take into account such a powerful mechanism as secular resonances which can lead to the change of orbits from the outer planetary region down to the near-Earth region. This suggests that short-period comets may originate from a source of orbits with perihelia in the outer planetary region. Therefore, there is a need to estimate the probability of the dynamical transfer from the near-parabolic flux to short-period orbits for all perihelion distances in the planetary region.

2. Model and Methods

We consider the process of transformations from nearly parabolic orbits to short-period orbits under the perturbations of Jupiter, Saturn, Uranus, and Neptune. We have investigated separately five ranges: $0 < q < 4\,\mathrm{AU}$, $4 < q < 6\,\mathrm{AU}$, $6 < q < 10.5\,\mathrm{AU}$, $10.5 < q < 18\,\mathrm{AU}$, $18 < q < 31\,\mathrm{AU}$ where initial perihelion distances q are distributed uniformly. Initial inclinations i are distributed uniformly in $\cos i$. Initial semi-major axes a are distributed uniformly in the interval $(2\times10^4, 3\times10^4\,\mathrm{AU})$. Initial mean longitudes of the planets are distributed uniformly in the range $(0, 2\pi)$. Our investigations cover the interval $\sim 5\times10^9\,\mathrm{yr}$. In this paper, we regard an orbit as Halley-type if the perihelion distance $q < 1.5\,\mathrm{AU}$ and the period of revolution $20 < P < 200\,\mathrm{yr}$, and as Jupiter-family type if $q < 1.5\,\mathrm{AU}$ and $P < 20\,\mathrm{yr}$.

We combine two methods of calculating planetary perturbations. For the near-parabolic motion, we use the method of mappings (Emel'yanenko, 1992) describing analytically the changes of semi-major axis at each perihelion passage. This approximation is not valid for the description of close encounters with planets and for orbits with perihelia located near the planetary orbits. Therefore, in this work, we start to use the numerical integration if the orbital parameters satisfy one of the following conditions: $e < 0.95$, $a < 200\,\mathrm{AU}$, $4.6 < q < 6.0\,\mathrm{AU}$, $8.5 < q < 11.0\,\mathrm{AU}$, $17.0 < q < 22.0\,\mathrm{AU}$, $26.6 < q < 34.6\,\mathrm{AU}$, $i < 10°$, $i > 170°$, $4.7 < r_\Omega < 5.7\,\mathrm{AU}$, $9.0 < r_\Omega < 10.0\,\mathrm{AU}$, $18.7 < r_\Omega < 19.7\,\mathrm{AU}$, $29.6 < r_\Omega < 30.6\,\mathrm{AU}$, where e is the eccentricity, r_Ω is the heliocentric distance in the ascending or descending node. We use Everhart's method (Everhart, 1974) for the numerical integration of the equations of motion in heliocentric rectangular coordinates. The coordinates of the planets are calculated on the basis of the theory of secular perturbations (Brower and van Woerkom, 1950; Sharaf and Budnikova, 1967). Outside the heliocentric distance 40 AU, we transfer to the barycentric coordinate system and consider the motion of objects as unperturbed in this system.

3. The Probabilities of Capture and the Number of Halley-Type and Jupiter-Family Comets

Table 1 shows the relative frequencies $\overline{p}_c^{HT}$ and $\overline{p}_c^{JF}$ of the transfer from near-parabolic orbits into Halley-type and Jupiter-family orbits by gravitational perturbations of four outer planets in our calculations. S is the initial number of objects in each range of perihelion distances.

TABLE 1. The relative frequences of the capture from near-parabolic orbits into short-period orbits.

q [AU]	$\overline{p}_c^{HT}$	$\overline{p}_c^{JF}$	S
0–4	0.0128	0.0002	20 000
4–6	0.0013	0.0008	20 000
6–10.5	0.0003	0.0003	20 000
10.5 –18	0.0002	0.0002	17 500
18–31	0	0.0003	7600

The maximum probability of the transfer from the near-parabolic flux into Halley-type comets takes place in the region $0 < q < 4$ AU with the bulk coming from $q < 2$ AU where $\overline{p}_c = 0.0195$. This probability is comparatively small for the zone $4 < q < 6$ AU suggested by Everhart (1972) for the origin of Jupiter-family comets. By contrast, the majority of Jupiter-family comets come from the region of perihelion distances which lies outside Everhart's zone even if we assume the same flux of "new" comets for all perihelion distances.

Different estimates of the observed flux ν of "new" comets spread from 0.2 to 0.8 per year and AU for comets with absolute magnitudes brighter than $H_{10} = 7$. We have done a special consideration of the evolution of 10 000 near-parabolic comets with $0 < q < 4$ AU which has shown that the mean time which captured objects spend in the Halley-type region $\overline{L}_{HT} \approx 3 \times 10^5$ yrs. Assuming these parameters and the estimated values of the capture probability from Table 1 for near-parabolic comets with $0 < q < 4$ AU we obtain the steady-state number of Halley-type comets $N_{HT} = 3.1 \times 10^3 - 1.2 \times 10^4$.

The minimum equilibrium number of Halley-type objects dynamically produced by capture from the near-parabolic flux with $H_{10} < 7$ is hundreds of times as large as the observed one, and this does not take into account the additional contribution from $q > 4\,\mathrm{AU}$ and the possible action of nongravitational forces. Up to now, only 20 comets with $20 < P < 200$ and $q < 1.5\,\mathrm{AU}$ (Marsden and Williams, 1995) have been discovered. This discrepancy is, of course, caused largely by the limitation on the interval of time, during which comets can be observable. Our investigations have shown that we should suggest the physical lifetime less than 200 revolutions in order to explain the observed number of Halley-type comets (Emel'yanenko and Bailey, 1996). In this case the question of physical evolution of numerous "extinct" comets in Halley-type orbits is extremely important in the consideration of the near-Earth population of small bodies.

In contrast with the Halley-type comets, the steady-state number of Jupiter-family comets N_{JF} captured from $0 < q < 4$ is substantially less than the observed one: $N_{JF} = 1.6 - 6.4$ if we assume that the mean lifetime for Jupiter-family comets $L_{JF} = 10^4$ years (Rickman, 1992). According to the catalogue (Marsden and Williams, 1995), 53 Jupiter-family comets with $q < 1.5\,\mathrm{AU}$ have been discovered. This shows that the near-parabolic flux in the observable region is not the main source of Jupiter-family comets.

The estimates of the cometary distribution at large heliocentric distances are uncertain. Let us assume that the perihelion distances of "new" comets are distributed according to the formula suggested in the paper by Zheng *et al.* (1996):

$$\frac{\nu(q)}{\nu(1)} = 1 + 0.014q^{1.82}, \quad q < 13\,\mathrm{AU}; \qquad \frac{\nu(q)}{\nu(1)} = 5, \quad 13 < q < 30\,\mathrm{AU}. \quad (1)$$

Taking again for all q $\overline{L}_{HT} = 3 \times 10^5$, $L_{JF} = 10^4$, and $\nu(1) = 0.2 - 0.8$, we obtain $N_{HT} = 3.7 \times 10^3 - 1.5 \times 10^4$, and $N_{JF} = 61 - 243$ for comets with the initial values of $H_{10} < 7$. The steady-state number of Halley-type comets is hardly dependent on the contribution of comets with large perihelion distances. But, the number of Jupiter-family comets changes drastically if we consider the near-parabolic flux at large distances. This number should be even larger if we take into account the prediction (Bailey, 1986) that the near-parabolic flux with large perihelion distances contains many comets from the inner part of the Oort cloud for which the probability of the capture into short-period comets is larger. We have found for 8200 objects with $10.5 < q < 18\,\mathrm{AU}$, $a \sim 3 \times 10^3\,\mathrm{AU}$ that $\overline{p}_c^{JF} = 0.0006$. So there is no difficulty in explaining 53 observed comets with $P < 20\,\mathrm{yr}$ and $q < 1.5\,\mathrm{AU}$.

4. The Distribution of Jupiter-Family Comet Inclinations

The explanation of the observed distribution of inclinations for Jupiter-family comets by the capture from the isotropic near-parabolic flux is a more serious problem (Duncan *et al.*, 1988; Bailey, 1992). Table 2 shows the observed distribution and two modelled distributions for perihelion distances of near-parabolic comets distributed according to the formula (1) and uniformly. Each column contains data about the steady-state numbers of comets and the relative frequences w for 30-degree intervals of inclinations at the above-mentioned assumptions about $\nu(1)$ and L_{JF}. Though almost all comets captured into Jupiter-family orbits have prograde orbits in our model, the discrepancy between the observed distributions and the modelled ones is large. The capture from the isotropic near-parabolic flux produces almost equal numbers of comets in the ranges $0 - 30°$ and $30° - 60°$. This result is weakly dependent on the assumption about the distribution of perihelion distances of "new" comets. The observed distribution demonstrates that the ratio of the number of Jupiter-family comets in the range $0 - 30°$ to that in the range $30° - 60°$ is about 7.7. Therefore, we should conclude that the isotropic flux produces directly only a small number of Jupiter-family comets inside the observed set with $q < 1.5\,\mathrm{AU}$. This implies that either the adopted value of $L_{JF} = 10^4$ years or the increase of the isotropic flux according to the formula (1) is too large. The main part of the observed Jupiter-family population should originate from a flatter source. Since there are no difficulties in producing the required number of Jupiter-family comets from high-eccentricity orbits, the model of a cometary disc (Fernandez and Ip, 1991) looks, at least, not worse than the popular idea of the Kuiper belt. Splitting of comets on their way from large perihelion distances to the Jupiter family can play an important role as well. We would like to stress in this connection that the mean number of

TABLE 2. Distribution of inclinations.

Inclination [deg]	Observed		Distribution Formula(1)		Uniform	
	N_{JF}	w	N_{JF}	w	N_{JF}	w
0–30	46	0.868	29–116	0.477	8–31	0.449
30–60	6	0.113	27–109	0.449	6–25	0.362
60–90	1	0.019	2–9	0.037	1–6	0.087
90–120	0	0	0–1	0.004	0–1	0.014
120–150	0	0	2–7	0.029	1–5	0.072
150–180	0	0	0–1	0.004	0–1	0.014

revolutions which comets spend in the planetary region before the capture to the Jupiter-family orbits with the inclinations in the range 0 – 30° is twice as large as that in the range 30 – 60° (for the model (1) these values are 2.5×10^5 and 1.1×10^5 revolutions, respectively). This implies larger possibilities of splitting in low-inclination orbits.

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