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Roche Instability in Ejecting Stellar Systems

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ABSTRACT

We consider a small stellar system ejected from the interior of a bigger one. The two systems are represented by unequal Plummer models and energy changes in them are derived analytically under the impulse approximation. These are used to obtain the conditions for the Roche instability of the systems. For systems greatly differing in dimension,the density ratio of the two systems is a useful parameter for specifying the Roche limit for the disruption of the smaller system and the mass ratio is a meaningful parameter for specifying the Roche limit for the disruption of the bigger system. Density ratios and mass ratios at Roche limit are given for both systems for various scale-length ratios of the two systems.

INTRODUCTION

Of much interest in recent times has been the idea of ejection of matter from galaxies in various forms - gas (Gerola and Carnevali 1983, Fukunaga-Nakamuraand Toss 1989), radio jets (Yokosawa and Inoue 1985), quasars (Hutchings 1983, Narlikar and Das 1986, Burbidge and Hewitt 1989),black holes (Kapoor 1985, Mikkola and Valtonen 1990) and stellar systems (Ambartsumian 1965, Arp 1972, 1986).

Saslaw, Valtonen and Asrseth (1974) discuss the gravitational slingshot mechanism for the ejection of an object from the centre of a galaxy, when three or more massive objects near the centre interact strongly.

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In the 1950's Ambartsumian suggested that galaxies ejected material from which new galaxies formed. Arp (1986) has drawn attention to observations which indicate that in the Virgo cluster of galaxies, the galaxy M 84 had originated as a protogalaxy within the bigger galaxy M 87 and had been ejected out along the line of the jet. X-ray observations show that M 84 is moving almost exactly along the same line from M 87. In the Stephen's Quintet, one of the five galaxies has a very different red shift from those of the rest and could have been ejected from the group. Our galaxy and M 31 were probably ejected from the same initial protogalaxy. Thus the ejection of a smaller system for a bigger one is of much astronomical interest.

In this paper we consider the ejection of a smaller stellar system by a bigger stellar system and determine the increase in energy of the constituent stars in each system on account of tidal forces. Initially both the systems are assumed to be in virial equilibrium. Using the Jacobi' criterion of stability, namely, that a system is unstable if its total energy is positive, we deduce the conditions under which each of the two systems becomes unstable.

STELLAR VELOCITY PERTURBATIONS

Let M_1 and M_2 be the masses of the bigger system (ejector) and the smaller system (ejected) respectively. We consider both systems to be spherically symmetric configurations of density distribution represented by Plummer model, namely,

$$\rho_i(r) = \frac{3M_i\alpha_i^2}{4\pi} \frac{1}{(r^2+\alpha_i^2)^{5/2}}, \quad i = 1,2 \tag{1}$$

where

$$\alpha_i = \left(\frac{3M_i}{4\pi(\rho_i)_i} \right)^{1/3} \tag{2}$$

ρ_i being the central density. Let $\alpha_{12} = \frac{\alpha_1}{\alpha_2}$ be the scale length ratio and let $\alpha_1 \geq \alpha_2$.

Consider the tidal effects of M_1 and M_2. Let the centre of M_2 be the origin of the coordinate system. Let $\vec{r}'$ and $\vec{r}$ denote the position of the centre of M_1 and of a representative star in M_2 respectively and let $\vec{r}'' = \vec{r}' - \vec{r}$. The motion of the star in M_2 is given by

$$\ddot{r} = - \nabla\Phi_2 + \nabla'' \Phi_1 - \frac{1}{M_2} \nabla' W \tag{3}$$

and Φ and W represent the potential and the interaction potential energy given by

$$\Phi_i(r) = - \frac{GM_i}{(\alpha_i^2 + r^2)^{1/2}} , \quad i = 1,2 \tag{4}$$

$$W(r) = - \frac{GM_1 M_2}{(\alpha^2 + r^2)^{1/2}} \tag{5}$$

α is a complicated function of α_1 and α_2. We shall make the approximation $\alpha^2 = \alpha_1^2 + \alpha_2^2$ as in Aarseth and Fall (1980) in carrying out subsequent integration. The first term on the right hand side in equation (3) represents the gravitational attraction of M_2 on the star while the remaining two terms together give the tidal acceleration $\vec{f}_T$ due to the ejector M_1.

Let M_2 be ejected from the centre of M_1 along the z-axis with velocity $V \geq V_{par}$ where V_{par} is parabolic velocity of escape of the two systems. We obtain the change in the velocity of the star by integrating $\vec{f}_T$ over time using the impulse approximation, i.e. neglecting the motion of the star during the ejection. The impulse approximation gives reasonable estimates for the energy changes in stellary systems moving with relative speeds equal to or greater than the parabolic speed (Alladin and Narasimhan 1982).

Following the analysis as in Zafarullah, Narasimhan and Sastry (1983), we get the following expressions for the velocity changes of the star perpendicular to and parallel to the direction of ejection:

$$\Delta V_{\perp} = [(\Delta V_x)^2 + (\Delta V_y)^2]^{1/2} = \frac{GM_1 \tilde{w}}{V(\tilde{w}^2 + \alpha_1^2)} \tag{6}$$

$$\Delta V_{11} = \Delta V_z = \frac{GM_1}{V}\left[\frac{1}{(\tilde{w}^2 + \alpha_1^2)^{1/2}} - \frac{1}{(\alpha_1^2 + \alpha_2^2)^{1/2}} \right] \tag{7}$$

where $\tilde{w} = (x^2 + y^2)^{1/2}$.

ENERGY CHANGES IN EJECTION

The increment in the kinetic energy ΔT_2 due to the tidaleffects M_1 which in the impulse approximation is the same as increment in the total energy ΔU_2 is obtained from

$$\Delta U_2 = \int_0^\infty [\{\Delta V_\perp(\tilde{w})\}^2 + \{\Delta V_{11}(\tilde{w})\}^2] dM_2 \tag{8}$$

We follow the analysis given in Ahmed (1979) and Binney and Tremaine (1987) for integrating over the mass. The integration is carried out analytically and obtain

$$\Delta U_2 \equiv (\Delta U_\perp)_2 + (\Delta U_{11})_2$$

$$= \frac{G^2 M_1^2 M_2 \; B(\alpha_{12})}{2V^2 \alpha_1^2} + \frac{G^2 M_1^2 M_2}{2V^2 \alpha_1^2} [C(\alpha_{12}) + D(\alpha_{12}) + E(\alpha_{12})]$$

$$\equiv \frac{G^2 M_1^2 M_2}{2V^2 \alpha_1^2} H(\alpha_{12}) \tag{9}$$

where $B(\alpha_{12}) = \dfrac{2\alpha_{12}^2}{\alpha_{12}^2 - 1} \left[\dfrac{\alpha_{12}^2 + 1}{\alpha_{12}^2 - 1} \ln \alpha_{12} - 1 \right]$

$$C(\alpha_{12}) = \frac{\alpha_{12}^2}{\alpha_{12}^2 - 1} \left[1 - \frac{2}{\alpha_{12}^2 - 1} \ln \alpha_{12} \right]$$

$$D(\alpha_{12}) = \frac{\alpha_{12}^2}{\alpha_{12}^2 + 1}$$

$$-E(\alpha_{12}) = \frac{2\alpha_{12}^2}{\alpha_{12}^2 - 1} \frac{1}{(\alpha_{12}^2 + 1)^{1/2}} \left[\frac{\ln\{\alpha_{12} + (\alpha_{12}^2 - 1)^{1/2}\}}{\alpha_{12}^2 - 1} - \alpha_{12} \right]$$

Table I gives $B(\alpha_{12})$ and $H(\alpha_{12})$.

Table I: The Functions $B(\alpha_{12})$ and $H(\alpha_{12})$

α_{12}	$B(\alpha_{12})$	$H(\alpha_{12})$
1.0	1.67(-1)	2.24(-1)
1.5	1.56(-1)	2.04(-1)
2.0	1.38(-1)	1.77(-1)
3.0	1.05(-1)	1.30(-1)
4.0	8.12(-2)	9.80(-2)
5.0	6.45(-2)	7.64(-2)
10	2.75(-2)	3.10(-2)
20	1.01(-2)	1.10(-2)
50	2.33(-3)	2.48(-3)
100	7.21(-4)	7.60(-4)
500	4.17(-5)	4.36(-5)
1000	1.16(-5)	1.20(-5)

$(n) = 10^n$

The self-gravitational potential energy of M_2 is given by

$$\Omega = -\frac{3\pi}{32}\frac{GM_2^2}{\alpha_2} = 2U_2 \text{ by vivial theorem} \tag{10}$$

whence the fractional increase in the binding energy of M_2 is given by

$$\mu_2 \equiv \frac{\Delta U_2}{|U_2|} = \frac{32}{3\pi}\frac{GM_1^2\alpha_2}{M_2V^2\alpha_1^2} H(\alpha_{12}) \tag{11}$$

Following the analysis in Ahmed (1979) and Alladin and Narasimhan (1986), equation (11) can be written in the form

$$\mu_2 = \frac{16}{3\pi}\frac{1}{M_{21}(1+M_{21})}\frac{\alpha_{21}\ H(\alpha_{12})}{A(\alpha_{12})}\left(\frac{V_{par}}{V}\right)^2 \tag{12}$$

where $M_{21} = \frac{M_2}{M_1}$, $\alpha_{21} = \frac{\alpha_2}{\alpha_1}$; V_{par} is the parabolic speed of escape of the two systems of ejection. The function $A(\alpha_{12})$ is given in Ahmed (1979).

Similarly we obtain for the change in the binding energy of the ejector M_1:

$$\mu_1 \equiv \frac{\Delta U_1}{|U_1|} = \frac{32}{3\pi}\frac{GM_2^2\alpha_1}{M_1V^2\alpha_2^2} H(\alpha_{12}) \tag{13}$$

$$= \frac{16}{3\pi}\frac{1}{M_{12}(1+M_{12})}\frac{\alpha_{12}\ H(\alpha_{12})}{A(\alpha_{12})}\left(\frac{V_{par}}{V}\right)^2 \tag{14}$$

ROCHE LIMITS IN EJECTION

According to Jacobi's criterion of stability, a stellar system is unstable if its total energy is positive. This implies that $\mu \equiv \frac{\Delta U}{|U|} > 1$. However, the numerical experiments by Miller (1986) indicate that a satellite system is in danger of disintegration if $\mu > 2$. When μ lies between 1 and 2 many stars remain bound. We shall adopt the criterion $\mu = 1$ for obtaining the Roche limits in ejecting systems.

Table II gives the mass ratios and density ratios at the Roche limits for various values of α_{12}. $\mu_1 = 1$ gives the limit for the disruption of the bigger system and $\mu_2 = 1$ gigives the corresponding limit for the smaller system. For a given α_{12}, the bigger system is unstable for the values of $M_{12} = \frac{M_1}{M_2}$ or equivalently $\rho_{12} = \rho_1/\rho_2$ smaller than the tabular value and the smaller system is unstable for values of M_{12}

Table II: Mass ratios and density ratios at Roche limit when $V = V_{par}$

	Roche Limit for the Disruption of			
	Bigger System		Smaller System	
α_{12}	M_{12}	ρ_{12}	M_{12}	ρ_{12}
1.0	0.45	4.50(-1)	2.22	2.22
1.5	0.67	1.99(-1)	3.83	1.13
2.0	0.84	1.05(-1)	6.03	7.60(-1)
3.0	1.10	4.74(-2)	1.25(+1)	4.64(-1)
4.0	1.29	2.02(-2)	2.27(+1)	3.53(-1)
5.0	1.44	1.15(-2)	3.66(+1)	2.13(-1)
10	1.88	1.88(-3)	1.87(+2)	1.85(-1)
20	2.29	2.86(-4)	1.07(+2)	1.34(-1)
50	2.79	2.23(-5)	1.19(+4)	9.52(-1)
100	3.13	3.13(-6)	7.76(+4)	7.76(-2)
500	3.83	3.06(-7)	6.78(+6)	5.42(-2)
1000	4.04	4.04(-9)	4.92(+7)	4.92(-2)

$(n) = 10^n$

or ρ_{12} greater than the tabular value. It may be noted from the table that for the bigger system the range in M_{12} is considerably smaller than the range in ρ_{12} while for the smaller system the reverse holds. Hence for specifying the instability of the bigger system, the mass ratio is a useful parameter; while for specifying the instability of the smaller system, the density ratio is a meaningful parameter.

The dependence of M_{12} on α_{21} when $V = V_{par}$ and $\mu_1 = 1$ can be expressed as

$$M_{12} = (2.6)(0.13)^{\alpha_{21}} \tag{15}$$

Using the Chi-square Test, we find the fit to be good both at 5% and 1% levels of significance. Further as $\alpha_{12} \to \infty$, $M_{12} \to 2.6 \sim 3$. We therefore, infer that a smaller system will generally disrupt a bigger system at $V = V_{par}$ if its mass is greater than one-third the mass of the bigger.

The dependence of $(\rho_i)_{12}$ on M_{21} when $V = V_{par}$ and $\mu_2 = 1$ is of the form

$$(\rho_i)_{12} = 0.16(10)^{3M_{21}} \tag{16}$$

The fit is again found to be good. As $M_{12} \to \infty$, $(\rho_i)_{12} \to 0.16 \sim \frac{1}{7}$. This indicates that the bigger system will generally make the smaller unstable if its density is greater than one-seventh of the density of the smaller at $V = V_{par}$.

A similar analysis yields the values 1/2 and 1/3 for the disintegration of the bigger and smaller systems respectively if Miller's (1986) criterion $\mu = 2$ is used.

COMPARISON WITH COLLIDING STELLAR SYSTEMS

Energy transfer in head-on collisions of galaxies were studied analytically by Toomre (1977) and Ahmed (1979). Narasimhan and Alladin (1986) derived the conditions for Roche instability for head-on collisions of stellar systems. A comparison of the results for the colliding systems with those obtained here for ejecting systems indicates that the fractional increase in the binding energy of a system in the case of a collision is about 3 times that in ejection. The corresponding Roche limits in the case of a collision are 1/6 and 1/25.

In the case of a collision $\Delta V_{11} = 0$. In ejection ΔV_{11} is small but is not zero.

CONCLUSION

The analytical treatment of energy changes under the impulse approximation leads to the following results: (i) The energy change in an ejection is about 1/3 of that in a complete collision.(ii) For stellar systems differing widely in dimensions the disruption of the smaller system in an ejection is conveniently specified by the density ratio of the two systems and the disruption of the larger by the mass ratio. (iii) When the velocity of ejection is equal to the parabolic velocity of escape of the pair, the smaller system is generally unstable if its density is less than about 7 times the density of the bigger and the bigger is unstable if its mass is less than about 3 times the mass of the smaller.

It is desirable to test the accuracy of the predictions of the analytical formulae presented here by numerical experiments.

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