12 Supersymmetric grand unification

In this brief chapter we discuss one of the most compelling pieces of circumstantial evidence in favor of supersymmetry: the unification of coupling constants. Earlier, we introduced grand unification without supersymmetry. In this chapter we consider how supersymmetry modifies that story.

12.1 A supersymmetric grand unified model

Just as in theories without supersymmetry, the simplest group into which one can unify the gauge group of the Standard Model is $SU(5)$. The quark and lepton superfields of a single generation again fit naturally into a $\bar{5}$ and a 10.

To break $SU(5)$ down to $SU(3) \times SU(2) \times U(1)$, we can again consider a 24-dimensional representation of the Higgs field Σ. If we wish supersymmetry to be unbroken at high energies, the superpotential for this field should not lead to supersymmetry breaking. The simplest renormalizable superpotential is

$$W(\Sigma) = m \,\mathrm{Tr}\, \Sigma^2 + \frac{\lambda}{3} \,\mathrm{Tr}\, \Sigma^3. \tag{12.1}$$

Treating this as a globally supersymmetric theory (i.e. ignoring supergravity corrections), the equations

$$\frac{\partial W}{\partial \Sigma} = 0 \tag{12.2}$$

are conveniently studied by introducing a Lagrange multiplier to enforce $\mathrm{Tr}\,\Sigma = 0$. The resulting equations have three solutions:

$$\Sigma = 0, \quad \Sigma = \frac{m}{\lambda}\,\mathrm{diag}(1,1,1,-4), \quad \Sigma = \frac{m}{\lambda}\,\mathrm{diag}(2,2,2,-3,-3). \tag{12.3}$$

These solutions either leave $SU(5)$ unbroken or break $SU(5)$ down to $SU(4) \times U(1)$ or the Standard Model group. Each solution is isolated; you can check that there are no massless fields from Σ in any of these states. At the classical level they are degenerate.

If we include supergravity corrections, however, these states are split in energy. Provided that the unification scale m is substantially below the Planck scale, these corrections can be treated perturbatively. In order to make the cosmological constant vanish in the

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$SU(3) \times SU(2) \times U(1)$ (in brief, $(3, 2, 1)$) vacuum, it is necessary to include a constant in the superpotential such that, in this vacuum, the expectation value of the superpotential is zero. As a result the other two states have negative energy (as we will see in the chapter on gravitation, they correspond to solutions in which space–time is not Minkowski but *anti-de Sitter*).

We will leave working out the details of these computations to the exercises and turn to other features of this model. It is necessary to include Higgs fields to break $SU(2) \times U(1)$ down to $U(1)$. The simplest choice for the Higgs is the 5-dimensional representation. As in the MSSM, it is actually necessary to introduce two sets of fields so as to avoid anomalies: a 5 and $\bar{5}$ are the minimal choice. We denote these fields by H and $\bar{H}$.

Once again it is important that the color triplet Higgs fields in these multiplets be massive in the $(3, 2, 1)$ vacuum. The most general renormalizable superpotential that couples the Higgs to the adjoint is

$$m_H H\bar{H} + y\bar{H}\Sigma H. \tag{12.4}$$

By carefully adjusting y (or m) we can arrange that the Higgs doublet is massless. As a result the triplet is automatically massive, with a mass of order m_H. Of course, this represents an extreme fine tuning. We will see that the unification scale is about 10^{16} GeV, so this is a tuning of one part in 10^{13} or so. But it is curious that this tuning only needs be done classically. Because the superpotential is not renormalized, radiative corrections do not lead to large masses for the doublets.

12.2 Coupling constant unification

The calculation of coupling constant unification in supersymmetric theories is quite similar to that in non-supersymmetric theories. We assume that the threshold for the supersymmetric particles is somewhere around 1 TeV. So, up to that scale, we run the renormalization group equations just as in the Standard Model. Above that scale there are new contributions from the superpartners of ordinary particles. The leading terms in the beta functions are as follows:

$$SU(3), b_0 = 3; \quad SU(2), b_0 = -1; \quad U(1), b_0 = -\frac{33}{5}. \tag{12.5}$$

One can be more thorough, including two-loop corrections and threshold effects. The result of such an analysis are shown in Fig. 12.1. One has:

$$M_{\rm gut} = 1.2 \times 10^{16}\text{GeV}, \quad \alpha_{\rm gut} \approx \frac{1}{25}. \tag{12.6}$$

The agreement in the figure is striking. One can view this as a successful prediction of α_s (see below Eq. (3.100)), given the values of the $SU(2)$ and $U(1)$ couplings.

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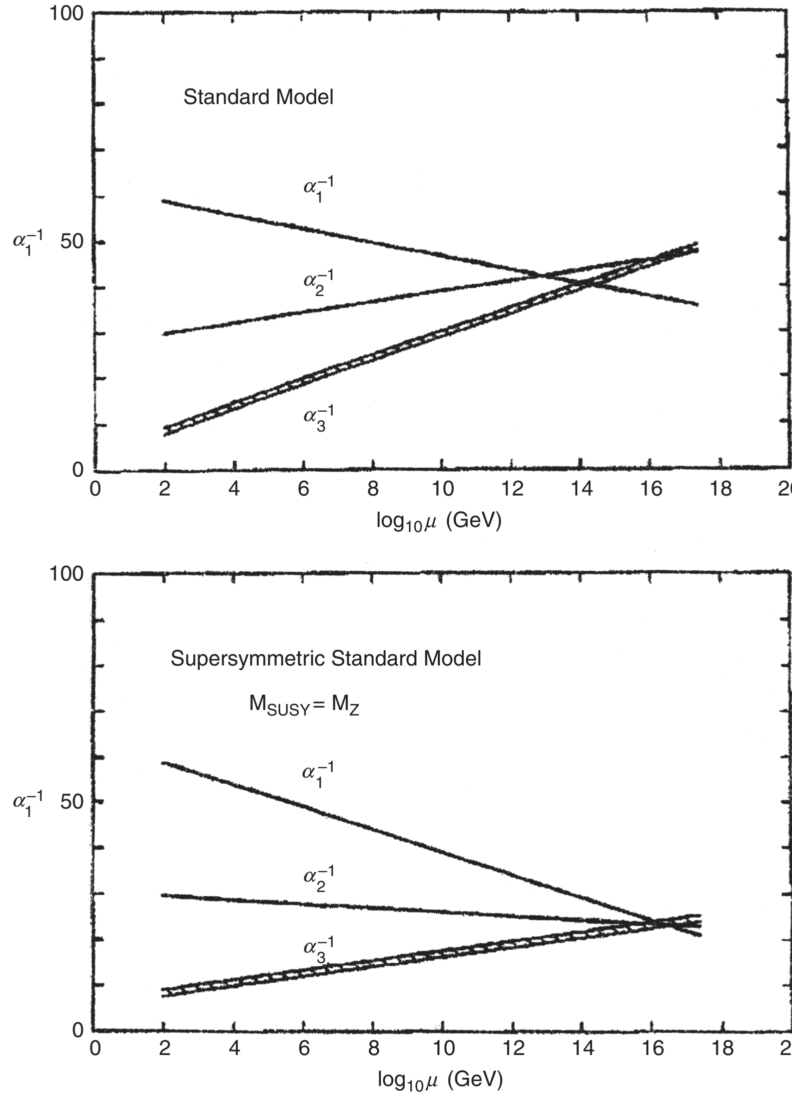

Fig. 12.1 In the Standard Model the couplings do not unify at a point. In the MSSM they do, provided that the threshold for new particle production is at about 1 TeV. Reprinted with permission from P. Langacker and N. Polonsky, Uncertainties in coupling constant unification, *Phys. Rev. D*, **47**, 4028, 1993. Copyright (1993) by the American Physical Society.

12.3 Dimension-five operators and proton decay

We have seen that, in supersymmetric theories, there are dangerous dimension-four operators. These can be forbidden by a simple Z_2 symmetry, i.e. R-parity. But there are also operators of dimension five which can potentially lead to proton decay rates far larger

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than the experimental limits. The MSSM possesses B- and L-violating dimension-five operators which are permitted by all symmetries. For example, R-parity does not forbid such operators as

$$\mathcal{O}_5^a = \frac{1}{M}\int d^2\theta\, \bar{u}\bar{u}\bar{d}e^+, \quad \mathcal{O}_5^b = \frac{1}{M}\int d^2\theta\, QQQL. \tag{12.7}$$

These are still potentially very dangerous. When one integrates out the squarks and gauginos they will lead to dimension-six B- and L-violating operators in the Standard Model with coefficients (optimistically) of order

$$\frac{\alpha}{4\pi}\frac{1}{Mm_{\text{susy}}}. \tag{12.8}$$

Comparing with the usual minimal $SU(5)$ prediction, and supposing that $M \sim 10^{16}$ GeV, one sees that a suppression of order 10^9 or so is needed.

Fortunately, such a suppression is quite plausible, at least in the framework of supersymmetric GUTs. In a simple $SU(5)$ model, for example, the operators in Eq. (12.7) will be generated by exchange of the color triplet partners of ordinary Higgs fields, and thus one obtains two factors of Yukawa couplings. Also, in order that the operators be $SU(3)$ invariant the color indices must be completely antisymmetrized, so more than one generation must be involved. This suggests that suppression by factors of order the CKM angles is plausible. So we can readily imagine a suppression by factors 10^{-9}–10^{-11}. Proton decay can be used to restrict – and does severely restrict – the parameter space of particular models. The simplest $SU(5)$ model, with TeV-scale squarks and gauginos and the simplest Higgs structure, can be ruled out, for example. But what is quite striking is that we are automatically in the right range to be compatible with experimental constraints, and perhaps even to see something. It is not obvious that things would work out like this.

So far we have phrased this discussion in terms of baryon-violating physics at M_{gut}. But, whatever the underlying theory at M_{p} may be, there is no reason to think that it should preserve baryon number. So one expects that already at scales just below M_{p} these dimension-five terms are present. If their coefficients were simply of order $1/M_{\text{p}}$, the proton decay rate would be enormous, five orders of magnitude or more faster than the current bounds. In any such theory one must also explain the smallness of the Yukawa couplings. One popular approach is to postulate approximate symmetries. Such symmetries could well suppress the dangerous operators at the Planck scale. One might expect that there would be further suppression in any successful underlying theory. After all, the rate from Higgs exchange in GUTs is very small because the Yukawa couplings are small. We do not really know why the Yukawa couplings are small, but it is natural to suspect that this is a consequence of (approximate) symmetries. These same symmetries, if present, would also suppress dimension-five operators from Planck-scale sources, presumably by a comparable amount.

Finally, we mentioned earlier that one can contemplate symmetries that would suppress dimension-four operators beyond a Z_2 R-parity. Such symmetries, as we will see, are common in string theory. One can write down R-symmetries which forbid not only all

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the dangerous dimension-four operators but some or all the dimension-five operators as well. In this case, proton decay could be unobservable in feasible experiments.

Suggested reading

A good introduction to supersymmetric GUTs is provided in Witten (1981). The reviews and texts which we have mentioned on supersymmetry and grand unification all provide good coverage of the topic. The Particle Data Group website has an excellent survey, including up-to-date unification calculations and constraints on dimension-five operators. Murayama and Pierce (2002) discussed the constraints on minimal $SU(5)$ unification from dimension-five operators.

Exercises

(1) Work through the details of the simplest $SU(5)$ supersymmetric grand unified model. Solve the equations

$$\frac{\partial W}{\partial \Sigma} = 0.$$

Couple the system to supergravity, and determine the value of the constant in the superpotential required to cancel the cosmological constant in the $(3, 2, 1)$ minimum. Determine the resulting value of the vacuum energy in the $SU(5)$ symmetric minimum.

(2) In the simplest $SU(5)$ model, include a 5 and a $\bar{5}$ representation of Higgs fields. Write down the most general renormalizable superpotential for these fields and the 24-dimensional representation, Σ. Find the condition on the parameters of the superpotential such that there is a single light doublet. Using the fact that only the Kahler potential is renormalized, show that this tuning of parameters at tree level ensures that the doublet remains massless to all orders of perturbation theory. Now consider the couplings of quarks and leptons required to generate masses for the fermions. Show that exchanges of 5 and $\bar{5}$ Higgs lead to baryon- and lepton-number-violating dimension-five couplings.

(3) Show how various B-violating four-fermion operators are generated by squark and slepton exchange, starting with the general set of B- and L-violating terms in the superpotential.