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Some Peculiaritiers of the Relativistic Electron-Positron Plasma Dynamics in the Pulsar Magnetosphere

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Abstract. Some aspects of relativistic electron-positron plasma dynamics in the pulsar magnetosphere are discussed. Namely it is shown that if we take into account the influence of centrifugal force, this gives rise to the new dynamical effect of plasma particle radial braking in the pulsar magnetosphere. Also the possibility of aperiodic instability development in the pulsar magnetosphere is demonstrated. The mechanism for the toroidal magnetic field generation is proposed. Besides, the possible changes of the pulsar magnetosphere structure caused by above mentioned processes is considered.

1. Introduction

We will discuss the case when the pulsar rotation axis and magnetic axis are perpendicular, for example the case of Crab Pulsar, and treat only the region above the polar cap, close to the pulsar surface. It will be also assumed that the pulsar magnetic field lines are the radial straight lines located in the plane perpendicular to the pulsar rotation axis, because we discuss the physical processes in the magnetospheric layer the thickness of which is much less than the curvature radius of the magnetic field lines.

In the pulsar magnetosphere, especially close to the star surface, the energy of the pulsar magnetic field is much larger than the energy of the pulsar magnetosphere relativistic electron-poistron plasma. Pulsar magnetic field lines are frozen in the magnetospheric plasma. The magnetic field lines are also frozen in pulsar, because the matter inside it is in the superconductive state. So, the "solid body type rotation" - corotation of pulsar, its magnetic field and the magnetospheric plasma takes place. In such case centrifugal force must play the important role in the dynamics of the magnetospheric plasma.

2. Main consideration

It is convenient to study the dynamics of the pulsar magnetosphere relativistic electron-positron plasma in two frames - in the reference frame of rotating magnetic field line, which is the noninertial frame and in the rest inertial frame. Let us begin the discussion from the noninertial rotating frame. The metric of this frame has such a form:

$$dS^2 = -\left(1-\Omega^2 r^2\right) dt^2 + dr^2, \tag{1}$$

where Ω is the pulsar rotation frequency. According to the Einstein principle of equivalence, we can not tell gravitation from noninertiality. Because of this for the investigation of the puslar magnetosphere plasma behaviour in the non-inertial rotating frame the "3+1" formalism can be used. According to this formalism, the equation of the motion for the pulsar magnetosphere relativistic electron-positron plasma in the metric (1) has the form (Chedia et al. 1996):

$$\frac{1}{\alpha}\frac{\partial \vec{p}}{\partial t} + \left(\vec{V}\vec{\nabla}\right)\vec{p} = -\gamma m \frac{\vec{\nabla}\alpha}{\alpha} + e\left(\vec{E} + \left[\vec{V}\vec{B}\right]\right). \tag{2}$$

In this equation $\vec{p}$ is the pulsar magnetosphere plasma particle momentum. Here and below we use the "geometric units" - $c = G = 1$. As for the first term in the right hand side of the equation (2), it is the centrifugal force acting on the plasma particles. $\alpha = \sqrt{1-\Omega^2 r^2}$ is the "lapse function" and γ - it is the plasma particle Lorentz-factor.

If we take into account the freezing-in condition

$$\vec{E} + \left[\vec{V}\vec{B}\right] = 0, \tag{3}$$

then in the right hand side of the equation of the motion there will be only the centrifugal force. One can easily find that in this case the radial acceleration of the pulsar magnetoshere relativistic electron-positron plasma particles can be expressed in this way:

$$\frac{d^2 r}{dt^2} = \frac{\Omega^2 r}{1-\Omega^2 r^2}\left(1-\Omega^2 r^2 - 2{V_r}^2\right). \tag{4}$$

Here V_r is the plasma particle radial velocity. From this equation it is evident that if the condition

$$V_r > \frac{1}{\sqrt{2}}, \tag{5}$$

is fulfilled the radial acceleration of the plasma particles changes its sign and bocomes negative, i.e. it is directed to the rotation axis and not outwards from it.

So, we can see that the centrifugal force really plays the important role in the pulsar magnetosphere plasma dynamics, namely if the condition (5) is fulfilled, and it is well fulfilled in the pulsar magnetosphere, the radial braking of the plasma takes place. The reason of this is that during the motion with relativistic velocities plasma particle mass increases and when the condition (5) is fulfilled their inertia is such a large that the plasma radial braking begins.

During the investigation of the pulsar magnetosphere relativistic electron-positron plasma behaviour in the noninertial rotating frame centrifigal force naturally appeares in the consideration and plays important role in its dynamics. But if we will discuss the dynamics of the magnetospheric plasma in the rest inertial frame which is described by the Minkowskian metric the centrifugal foce does not appear in the equation of the motion. We have only the Lorentz force in the right hand side of the equation and no centrifugal force. According to our opinion the reason of this is that the magnetic field is inhomogeneous in the rest inertial frame - $\vec{B} = B\cdot(\vec{r}/r)$. Really, if we take into account the inhomogeneity of the magnetic field in the equation of the plasma motion which is obtained in the framework of the drift approximation, we can easily mark out the centrifugal force from the Lorentz force in the evident form (see in detail Nanobashvili 1997).

Then one can easily find that in the rest inertial frame the expression for the plasma particle radial acceleration has exactly the same form as in the previous case, when the plasma dynamics has been studied in the noninertial rotating frame. So, during the investigation of the pulsar magnetosphere plasma dynamics in the rest inertial frame, like the case when the plasma dynamics has been studied in the noninertial rotating frame, if the condition (5) is fulfilled, the plasma particle radial braking begins in the pulsar magnetosphere. Of course this will cause the perturbation of the pulsar magnetosphere relativistic electron-positron plasma and it is very interesting to study the stability of this plasma. We will discuss the stability of the plasma with respect to the radial potential perturbations. For this purpose one needs the equation of the motion, continuity equation and Poisson equation:

$$\left(\frac{\partial}{\partial t} + \left(\vec{V}\vec{\nabla}\right)\right)\vec{p} = \gamma m\Omega^2\vec{r} + e\left(\vec{E} + \left[\vec{V}\vec{B}\right]\right), \tag{6}$$

$$\frac{\partial n}{\partial t} + div\left(n\vec{V}\right) = 0, \tag{7}$$

$$div\vec{E} = 4\pi q. \tag{8}$$

Here n is the plasma particle density and q is the electric charge.

Of course, we will not go into the details of the calculations and will present the despersion relation for the law frequency ($\omega \ll \omega_p$, where $\omega_p = \sqrt{\frac{4\pi n_0 e^2}{m}}$ is the plasma frequency) perturbations (see for details Kahniashvili, Machabeli, & Nanobashvili 1997):

$$\omega^2 = \frac{{\omega_p}^2}{2\gamma_0} - \frac{{k_r}^2{V_{0i}}^2}{2}. \tag{9}$$

The time dependence of the perturbed electric field has the form:

$$E_1 \sim e^{-i\omega t}. \tag{10}$$

So, we can see that when this condition is fulfilled

$${k_r}^2{V_{0i}}^2 > \frac{{\omega_p}^2}{\gamma_0}, \tag{11}$$

and it is well fulfilled in the pulsar magnetosphere, the aperiodic instability development takes place in it, which means that the exponentially growing radial electric field is generated in the pulsar magnetosphere.

Now about the toroidal magnetic field generation in the pulsar magnetosphere. During the plasma braking both - electrons and positrons are braked in the same manner. The appearance of the increasing radial electric field will cause the additional braking of the particles of one sign and the decreasing of braking of the particles of another sign (this depends on the direction of the generated electric field). So, the plasma particles with different sign of charge will move radially by nonzero relative velocities with respect to each other. This will cause the generation of the increasing radial electric current and corresponding toroidal magnetic field in the pulsar magnetosphere.

Now about the influence of all above mentioned processes on the pulsar magnetosphere structure. After the appearance of the toroidal magnetic field in the pulsar magnetosphere, the pulsar magnetic field structure will become spiral. Plasma particles follow the magnetic field lines and the corotation will be disturbed in the pulsar magnetosphere. Of course this is the simplified scenario, but I beleive that it will contribute to the solution of this problem - namely how the corotation is disturbed in the pulsar magnetosphere inside the light cylinder and how the pulsar magnetic field and magnetospheric plasma cross the light cylinder.

3. Conclusion

Generally, in most cases I mean, the dynamics of the magnetospheric plasma and pulsar magnetic field is studied in the region near the pulsar surface, where the corotation takes place or outside the light cylinder, where the corotation is disturbed, and in fact we have no corotation. It's very difficult to say what happens in the area between these two regions and how the magnetospheric plasma and pulsar magnetic field cross the light cylinder. I think the above described mechanism will contirbute to the solution of this problem. We must mention that we still know very few about the magnetopsheric plasma dynamics in this region. To our opinion in this region the differential rotation of plasma takes place and the magnetic field has the spiral structure. So, we think that still a lot of work has to be done in this field.

References

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