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THE EFFECT OF MULTIPLICITY OF STELLAR ENCOUNTERS AND THE CONVERGENCE OF COLLISION INTEGRAL IN THE STELLAR SYSTEM

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ABSTRACT

The multiplicity factor of stellar encounters in the infinite homogeneous layer with the constant thickness H has been studied. It is seen that simultaneous effect of Chaotically distributed distant field stars leads to the convergence of the collision integral for the great distances in the plane disk and in the layer of the constant finite thickness.

Many problems of stellar dynamics have the essential difficulty - the divergence of the collision integral. This convergence leads to the impossibility to find some expressions and values under the account of the distant gravitational interactions of the stellar system members. One of the example of this convergence is the increasing of the probability of the stellar encounter of the test star with the field star, the velocity variation after the encounter being Δv^2.

Let's denote v - the test start velocity, $\overline{v_1^2}$ - the mean square of the field star velocity, $\beta = v^2/\overline{v_1^2}$, $g = \Delta v^2/\overline{v_1^2}$. Then probability density (the transition function) $\phi(\beta,g)$ increases as g^{-3} for $g \to 0$, i.e. for faint interaction, in 3-dimensional stellar system. This result was obtained by Agekyan (1959) and Henon (1959), Agekyan found the transition probability for homogeneous stellar medium with the spherical Maxwellian velocity distribution of the field stars. And then he introduced the screening factor for the account of the simultaneous interaction of the test star by the field stars

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which are at the distance R from the test star,

$$\lambda(R) = \Delta\bar{F}/\Delta F_1 \qquad (1)$$

where $\Delta\bar{F}$ is the mean force from these field stars, ΔF_1-the arithmetic sum of the force moduls from these stars (on the unit mass). That factor takes into account the geometrical summing of the gravitational forces from all these stars instead of the arithmetical summing, as it was done when the double encounter probability, $\phi(\beta,g)dg$, was found (Agekyn, 1961).

Agekyan (1961) found the coefficient of the multiplicity (1) for 3-dimensional medium as the function of $\bar{N}$-the mean number of the field stars up to distance R from the test star,

$$\lambda(\bar{N}) = \frac{4}{\pi} \int_0^\infty \frac{x-\sin x}{x^3} e^{-\frac{2\sqrt{2\pi}}{5}\bar{N}x^{3/2}} dx \qquad (2)$$

This function increases as g^2 when $g \to 0$, as $\lambda\phi \sim g^{-1}$. This means that we can find the first order and higher moments of the velocity variation $\int_{-\infty}^{+\infty} g^k \lambda\phi \, dg$ $(k > 0)$, but can't find the probability of any encounter.

Petrovskaya, Chumak and Chumak (1984) found the double encounter probability density $\phi(\beta,g)$, for the plane (2-dimensional) medium. In that medium the effect of the distant encounters is smaller than in the 3-dimensional system, so $\phi \sim g^{-2}$ when $g \to 0$.

To find the multiplicity factor for the plane case we must consider the field stars in the ring of small thickness $h(\ll R)$. The test star is in the centre of the ring contained $\bar{N} = \pi R^2 D$. The arithmetic sum of the forces module from these stars is

$$\Delta F_1 = 2\pi G m D h/R \qquad (3)$$

G is the gravitational constant, D the number of stars in the unit of volume. For the mean force from these field stars we have

$$\Delta\bar{F} = \int F[W(F) - \rho(N) W_N(F)] d\vec{F} \qquad (4)$$

where $W(F)$ is the distribution function of the force from the whole stellar field, $W_N(F)$ is the force moduls distribution function from the whole stellar field without stars of the

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ring region (R, R+h), $p(N) = \frac{N^{-N} e^{-\bar{N}}}{N!}$. The force distribution functions in (4) may be found by Holtsmark method (Chandrasekhar, 1947) using Fourier transformation of that functions

$$W(\vec{F}) = \frac{1}{4\pi^2} \int \exp(-i\vec{\rho}\vec{F}) A(\vec{\rho}) d\vec{\rho},$$

$$W_N(\vec{F}) = \frac{1}{4\pi^2} \int \exp(-i\vec{\rho}\vec{F}) A_N(\vec{\rho}) d\vec{\rho} \tag{5}$$

and

$$\psi = Gm\vec{r}/r^3,$$

$$A(\vec{\rho}) = \lim_{N\to\infty} \{ \frac{D}{N} \int_{i=0}^{R} \exp(i\vec{\rho}\vec{\psi}) d\vec{r} \}^N, \tag{6}$$

$$A_N(\vec{\rho}) = [\frac{D}{N} \int_{i=0}^{\infty} \exp(i\vec{\rho}\vec{\psi}) d\vec{r}]^N \exp -D \int_{r=R+h}^{\infty} (1-e^{i\vec{\rho}\vec{\psi}}) d\vec{r}.$$

From (1), (3)-(6) we find

$$\lambda(\bar{N}) = \frac{1-\bar{N}^2}{\sqrt{\bar{N}^2+1}} + \bar{N} - \frac{2\bar{N}}{\sqrt{\bar{N}^2+1}\,(\bar{N}+\sqrt{\bar{N}^2+1})} + \bar{N} \ln$$

$$+ \bar{N} \ln \frac{2\bar{N}}{\bar{N}+\sqrt{\bar{N}^2+1}}, \tag{7}$$

where $\bar{N}$ is the mean number of stars which are nearer than the star of the field from which the approach is considered. For $\bar{N} \to \infty$ we have from (7) $\lambda \sim \bar{N}^{-1}$.

Using the approximate expression for the square velocity variation

$$v^2 = 2v\bar{v}_1\, Gm/(pw^2),$$

we find $\lambda \sim g^2$ when $N \to \infty$, or $g \to 0$. The product $\lambda\phi$ is finite when $g \to 0$ in that case.

The analogous task was decided for the more real system: the homogeneous layer of finite thickness H. The double encounter probability function found for this case by Chumak and Chumak (1988) is varying as g^{-2} when $g \to 0$. We consider now the multiplicity factor of encounters in the infinite homogeneous layer with the constant thickness H.

Let's consider the stars of equal masses m which are R to R+h from the test star (h << R). There are two cases (Fig. 1):

1) $R < H/2$. Then the arithmetic sum of the force modulus from these stars is

$$\Delta_1 F = 4\pi G m D h \qquad (8)$$

2) $R > H/2$. Then we add up the force modulus from the part of spherical layer.

$$\Delta_1 F = 2\pi G m D h \, H/R \qquad (9)$$

For $\overline{\Delta F}$ in (1) we have (4)-(6). In the case $R < H/2$ the volume $R < r < R+h$ is the spherical layer with thickness h, in the case $R > H/2$ - the part of spherical layer (Fig. 1). Therefore we find two different expression for the multiplicity coefficient

$$\lambda(\bar{N}) = \frac{4}{\pi} \int_0^\infty \frac{x - \sin x}{x^3} e^{-\frac{2\sqrt{2\pi}}{5} \bar{N} x^{3/2}} \left(1 + \frac{7}{24} \bar{N}^{4/3} N_o^{1/3} x^2\right) dx \qquad (10)$$

$R < H/2, \quad \bar{N} < N_o$;

and

$$\lambda(\bar{N}) = \frac{4}{\pi} \int_0^\infty \frac{x - \sin x}{x^3} e^{-\frac{2\sqrt{2\pi}}{5} N_o \left(\frac{2}{3}\frac{\bar{N}}{N_o} + \frac{1}{3}\right)^{3/2} x^{3/2}} \times \left[1 + \frac{N_o}{4}\left(\frac{2}{3}\frac{\bar{N}}{N_o} + \frac{1}{3}\right)^2 x^2\right] dx, \qquad (11)$$

$R > H/2, \quad \bar{N} > N_o$.

where $N_o = -\frac{4}{3} D (H/2)^3$ is the number of stars in the sphere with radius H/2. These expressions are different a little when $R = H/2$, $N = N_o$, because we kept no more than the lowest power of R/H in the first case, and the lowest power H/R in the second case. The difference after integrating is small ($\approx$ 4%).

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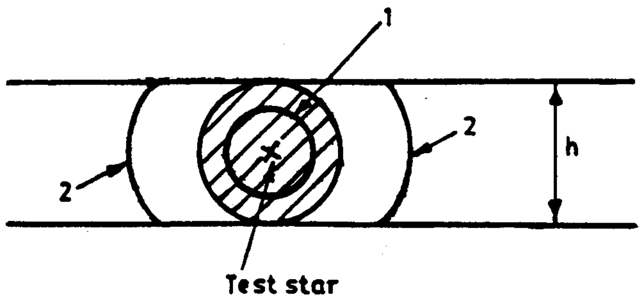

Fig. 1

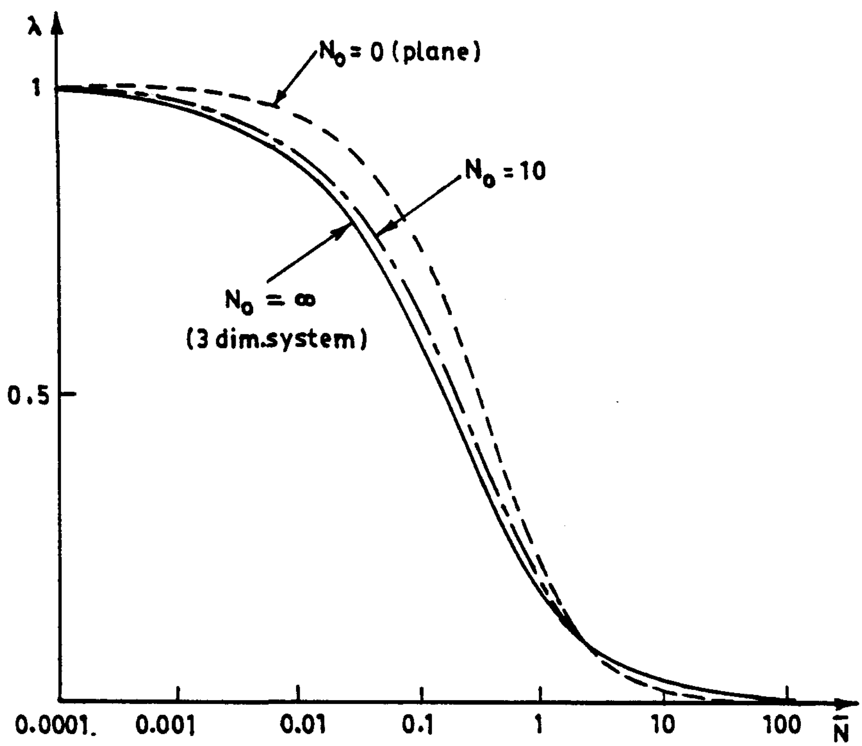

Fig. 2

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If $H \to \infty$, $N_o \to \infty$ (10) changes to the expression (2) for the 3-dimensional medium.

One can see from (11) that $\lambda(\bar{N}) \sim \bar{N}^{-1}$ when $\bar{N} \to \infty$. As it was shown for the plane system, $N \sim g^{-2}$, $\lambda \to g^2$ when $g \to 0$. Therefore simultaneous effect of chaotically distributed distant field stars leads to the convergence of the collision integral for the great distances in the plane disk and in the layer of the constant finite thickness because in such systems the transition probability is a finite value for the infinitestimal velocity variations.

In Fig. 2 we show the function $\lambda(\bar{N})$ for different N_o. Two limit cases are: 3-dimensional infinite medium, $N_o = \infty$, and the plane system, $N_o = 0$. The value $N_o = 10^8$ corresponds to our Galaxy.

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