DISTRIBUTION - FREE TESTS OF SUBHYPOTHESES

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This work investigates the problem of devising a permutation test of
the subhypothesis $H_0 : \beta_2 = 0$ in the linear model

(1) $$y = X_1\beta_1 + X_2\beta_2 + \varepsilon \ldots .$$

Whilst permutation tests have already been proposed for special cases
of (1) - such as for the Randomized Block Design by Pitman [3], and for the
case of testing the full hypothesis (where X_1 is a column of 1's) by
Box and Watson [1], no permutation test has previously been put forward for
the general model.

To construct a test for the general case, recursive residuals (Brown,
Durbin and Evans [2]) are used. It is argued that under $H_0 : \beta_2 = 0$,
sets of recursive residuals obtained from different orderings of the data
are exchangeable.

From this point two permutation tests are devised and their properties
compared. Of the two tests, the author states a preference for the first
on the basis that it is compatible with already established permutation
tests; when the first test is applied to specific cases of (1), such as
those given above, the test statistic and its null distribution reduce
identically to those of the already established permutation tests.
Approximate permutation moments of the test statistic are derived, leading

to information on the robustness of the classical F test for the general model.

References

[1] G.E.P. Box and G.S. Watson, "Robustness to non-normality of regression tests", *Biometrika* **49** (1962), 93-106.

[2] R.L. Brown, J. Durbin and J.M. Evans, "Techniques for testing the constancy of regression relationships over time (with discussion)", *J. Roy. Statist. Soc. Ser. B* **37** (1975), 149-192.

[3] E.J.G. Pitman, "Significance tests which may be applied to samples from any populations. III. The analysis of variance test", *Biometrika* **29** (1937), 322-335.

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