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CORRIGENDUM

Statistical mechanics of the Euler equations without vortex stretching – CORRIGENDUM

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In recent work (Wu & Bos 2021) we focused on an incompressible turbulent flow governed by a modified version of the Navier–Stokes equations. The essential difference with respect to the full Navier–Stokes equations is that the curl of the modified version does not contain the vortex-stretching term and writes

$$\frac{\partial \boldsymbol{\omega}}{\partial t} + \boldsymbol{u} \boldsymbol{\cdot} \nabla \boldsymbol{\omega} = -\nabla \mathcal{P}_\omega + \nu \Delta \boldsymbol{\omega} + \nabla \times \boldsymbol{f}, \tag{0.1}$$

with $\boldsymbol{\omega} = \nabla \times \boldsymbol{u}$ the vorticity, $\boldsymbol{u}$ the velocity, ν the kinematic viscosity and $\boldsymbol{f}$ a solenoidal force term. The first term on the right-hand side $\nabla \mathcal{P}_\omega$ represents a pressure gradient which ensures the vorticity field to remain divergence free, $\nabla \boldsymbol{\cdot} \boldsymbol{\omega} = 0$. Its value is determined by the Poisson-equation,

$$\Delta \mathcal{P}_\omega = -\nabla \boldsymbol{\cdot} (\boldsymbol{u} \boldsymbol{\cdot} \nabla \boldsymbol{\omega}). \tag{0.2}$$

In the original manuscript, this pressure term was omitted in the vorticity equation, and incompressibility was only imposed on the level of the velocity equation. The thereby defined set of equations was not consistent.

Denoting $\mathcal{F}[x] = \hat{x}$ the Fourier transform and $\boldsymbol{k}$ the wave-vector, we defined the velocity equation (ignoring forcing and viscous stress) as

$$\frac{\partial \hat{\boldsymbol{u}}}{\partial t} + \mathrm{i}k^{-2}\boldsymbol{k} \times \mathcal{F}[\boldsymbol{u} \boldsymbol{\cdot} \nabla \boldsymbol{\omega}] = 0. \tag{0.3}$$

It is this equation which was solved numerically in our previous investigations. Taking the curl of this equation, we find that

$$\frac{\partial \hat{\boldsymbol{\omega}}}{\partial t} - k^{-2}\boldsymbol{k} \times (\boldsymbol{k} \times \mathcal{F}[\boldsymbol{u} \boldsymbol{\cdot} \nabla \boldsymbol{\omega}]) = 0, \tag{0.4}$$

or, working out the double cross-product and applying the inverse Fourier transform, $\mathcal{F}[x]^{-1}$

$$\frac{\partial \boldsymbol{\omega}}{\partial t} + \boldsymbol{u} \boldsymbol{\cdot} \nabla \boldsymbol{\omega} = \mathcal{F}^{-1}\left[\frac{\boldsymbol{k}}{k^2}\boldsymbol{k} \boldsymbol{\cdot} \mathcal{F}[\boldsymbol{u} \boldsymbol{\cdot} \nabla \boldsymbol{\omega}]\right]. \tag{0.5}$$

The right-hand side equals the pressure term $-\nabla \mathcal{P}_\omega$ and was therefore present in the numerically investigated system, but was missing in several equations in our manuscript. The omission of the pressure term should be corrected in equations (1.1), (3.1), (3.10) and

(3.11) and the last two sentences of § 3.1. 'Furthermore … enstrophy', should be removed. The rest of the text and results remain unaltered.

This mistake does therefore not change the numerical results, which were all obtained from the modified Navier–Stokes equation for $\boldsymbol{u}$ (0.3), which was correctly formulated in the articles. Furthermore, the presence of this term does not influence the conservation of enstrophy and helicity. The only implication is that the enstrophy is conserved as a whole but the different components $\langle \omega_x^2 \rangle$, $\langle \omega_y^2 \rangle$, $\langle \omega_z^2 \rangle$ are not conserved individually by the nonlinear interaction. The lesson here is that one cannot throw away half the nonlinear term in the vorticity equation without adding an additional mechanism to maintain incompressibility.

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Reference

WU, T. & BOS, W.J.T. 2021 Statistical mechanics of the Euler equations without vortex stretching. *J. Fluid Mech.* **929**.