ROTATIONAL ANGULAR MOMENTA OF CLOSE BINARY SYSTEM COMPONENTS

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Abstract. The ratio between rotational angular momentum, J_{rot}, and orbital angular momentum, J_{orb}, in close binary systems and its variation with mass ratio is studied. The tables and the graphs give this variation for detached systems, contact systems, semidetached systems and for systems containing a supernova-remnant component and a contact component. For this study some statistical relations of close binary stars were used.

The ratio $J_{\text{rot}}/J_{\text{orb}}$ is sensitive to the variation of the mass ratio q. If q differs much from unity and if the concentration of the stellar matter is moderate (polytropic index $n \sim 3$), the neglect of rotational angular momentum, J_{rot}, is not justified.

1. Introduction

The rotation of the star, characterized by its angular velocity, plays an important role in the study of the stellar stability and stellar structure (Ledoux, 1965). The study of the rotational angular velocity variations in time can give evidence on some fundamental aspects of stellar evolution. As is well known, the problem of rotation is a central one in the theory of the formation and evolution of our solar system, as Laplace pointed out first. In the modern cosmogonic theories, the rotation is also in the foreground, the solar system being, it seems, a result of the interaction between the rotation and the magnetic field of the primary nebula.

In the study of the close binary systems, the rotation was taken into account by different authors (Darwin, 1897; Kruszewski, 1963; Huang, 1966). The role of the orbital angular momentum, J_{orb}, in the evolution of the close binary systems is well known (Huang, 1966; Paczyński, 1966; Svechnikov, 1969; Plavec, 1973). If we denote by M_1, M_2 the mass of primary and secondary component respectively and by a the semi-major axis of the relative orbit of eccentricity e and period P, then the (total) orbital angular momentum, J_{orb}, has the expression (Huang, 1966)

$$J_{\text{orb}} = \frac{M_1 M_2}{M_1 + M_2} \cdot \frac{2\pi a^2 (1-e^2)^{1/2}}{P}. \tag{1}$$

For the study of the close binary system evolution, different authors (Paczyński, 1966; Svechnikov, 1969) take into account only this orbital angular momentum, neglecting the rotational angular momentum of the system, J_{rot}. This is equal to the sum of the rotational angular momenta of the components. In order to justify the neglect of J_{rot}, some authors cite sometimes a paper of Smak (1964), which shows that for close binary systems of W Ursae Majoris type, $J_{\text{rot}} < 10^{-2} J_{\text{orb}}$ and it is expected that for such systems J_{rot} to be maximum.

In this paper we shall study in more detail the ratio $J_{\text{rot}}/J_{\text{orb}}$. There exist three factors which lead to the conclusion that $J_{\text{rot}}/J_{\text{orb}}$ can surpass Smak's limit:

(i) The large variation of $J_{\text{rot}}/J_{\text{orb}}$ with mass ratio $q = M_2/M_1$. For $q \sim 1$, $J_{\text{rot}}/J_{\text{orb}}$ becomes minimum.

P. Eggleton et al. (eds.), Structure and Evolution of Close Binary Systems, 351–363.

(ii) The non-synchronism between rotation and revolution for some close binary systems (for example: U Cep, RZ Sct).

(iii) The presence of rings or disks around the components of some close binary systems.

Therefore, in certain cases, the neglect of the rotational angular momentum J_{rot} is not justified. In the present paper we shall concentrate our attention mainly on the variation of the ratio J_{rot}/J_{orb} with the variation of q.

2. Expression of Ratio J_{rot}/J_{orb}

We shall consider that each component executes a rigid rotation around an axis which is perpendicular on the orbital plane. Let I_1, I_2 be the moments of inertia, and ω_1, ω_2 the rotational angular velocities of the two components. Then

$$J_{rot} = I_1 \omega_1 + I_2 \omega_2 \tag{2}$$

Let R_i $(i = 1, 2)$ be the radii of the components and $k_i R_i$ their gyration radii, where the non-dimensional gyration radii, k_i, are defined by the relation

$$k_i^2 \cdot M_i R_i^2 = \int_{V_i} \rho d^2 \, \mathrm{d}\tau; \quad i = 1, 2, \tag{3}$$

where ρ is the density in a point of the star, situated at the distance d from rotation axis, V_i is the volume of the star, while $\mathrm{d}\tau$ is the elementary volume. Let $\tilde{\omega}_i = \omega_i/\omega_k$, where $\omega_k \equiv \omega_{orb} = 2\pi/P$ is (mean) angular velocity in the Keplerian orbit.

The total angular momentum $J \equiv J_{tot} = J_{orb} + J_{rot}$ will have the expression

$$J = \frac{2\pi}{P} \left\{ \frac{M_1 M_2}{M_1 + M_2} a^2 (1 - e^2)^{1/2} + k_1^2 R_1^2 M_1 \tilde{\omega}_1 + k_2^2 R_2^2 M_2 \tilde{\omega}_2 \right\} \tag{4}$$

Further, we shall introduce the mass ratio $q = M_2/M_1$ and the fractional radii of the components, r_i, by the relation $R_i = r_i a$ $(i = 1, 2)$. Using the third Kepler law

$$\frac{G(M_1 + M_2)}{a^3} = \frac{4\pi^2}{P^2}, \tag{5}$$

where G is gravitational constant, and observing that

$$M_1 = (M_1 + M_2) \frac{1}{1+q}, \quad M_2 = (M_1 + M_2) \frac{q}{1+q} \tag{6}$$

expression (4) becomes

$$J = G^{1/2} (M_1 + M_2)^{3/2} a^{1/2} \frac{q}{(1+q)^2} \left\{ (1 - e^2)^{1/2} + \left(1 + \frac{1}{q}\right) k_1^2 r_1^2 \tilde{\omega}_1 + (1 + q) k_2^2 r_2^2 \tilde{\omega}_2 \right\} \tag{7}$$

The ratio between the rotational angular momentum and orbital angular momentum will have the expression

$$F = \frac{J_{\text{rot}}}{J_{\text{orb}}} = (1 - e^2)^{-1/2} \left\{ \left(1 + \frac{1}{q}\right) k_1^2 r_1^2 \tilde{\omega}_1 + (1 + q) k_2^2 r_2^2 \tilde{\omega}_2 \right\} \tag{8}$$

This ratio does not depend on the orbital period, the sum of mass of the components, or their separation. It depends only on the mass ratio, the internal structure of the components, their fractional radii, their relative angular velocities and the shape of orbit. As a function of q, the ratio F reaches the minimum for

$$q^* = \frac{k_1 r_1}{k_2 r_2} \left(\frac{\tilde{\omega}_1}{\tilde{\omega}_2}\right)^{1/2} \tag{9}$$

In the case of synchronism ($\tilde{\omega}_1 = \tilde{\omega}_2 = 1$), for systems having components with comparable dimensions ($r_1 \simeq r_2$) and similar internal structure ($k_1 \simeq k_2$) (as is the case of W UMa systems), it turns out that $q^* \sim 1$; that is the ratio F reaches the minimum for a value of q of the order of unity. The corresponding minimal value of F is

$$F_{\min} = (1-e^2)^{-\frac{1}{2}} (k_1 r_1 \tilde{\omega}_1^{1/2} + k_2 r_2 \tilde{\omega}_2^{1/2})^2. \tag{10}$$

In order to evaluate the ratio F for a given close binary system, we need the non-dimensional gyration radii of the components.

3. Non-dimensional Gyration Radii

We shall neglect the tidal and rotational distortions of the components. Considering that these possess a spherical symmetry, relation (3) can be written thus, with the omission of the index i,

$$k^2 = \frac{8\pi}{3} \frac{1}{MR^2} \int_0^R \rho(r) r^4 \, \mathrm{d}r. \tag{11}$$

3.1. POLYTROPIC MODELS

By use of the classical notations (Chandrasekhar, 1939), formula (11) becomes

$$k^2(n) = \frac{8\pi}{3} \cdot \frac{1}{MR^2} \rho_c(n) \alpha^5(n) \int_0^{\xi_1} \xi^4 \theta_n^n(\xi) \, \mathrm{d}\xi. \tag{12}$$

Introducing the known expression of constant α and using the relation between ρ_c and $\bar{\rho}$, the differential equation of Lane-Emden, as well as the mass-radius relation for polytropic models, Equation (12) can be written thus (Motz, 1952)

$$k^2(n) = \tfrac{2}{3} \left\{ 1 - \frac{6H(n)}{\xi_1^4 \left(-\dfrac{\mathrm{d}\theta_n}{\mathrm{d}\xi}\right)_{\xi=\xi_1}} \right\} \tag{13}$$

where

$$H(n) = \int_0^{\xi_1} \xi^2 \theta_n(\xi)\, d\xi \tag{14}$$

Motz computed the values of $k^2(n)$ for some typical values of polytropic index n, namely $n = 1$; 1.5; 2; 2.5; 3; 3.5; 4. Observing that $k^2(0) = 0.4$ and $k^2(5) = 0$, we obtain Figure 1, which gives the variation of the non-dimensional gyration radius k as function of polytropic index n.

3.2. OTHER MODELS

For other spherical models, the non-dimensional gyration radius k can be directly computed with formula (11), if we know the distribution of the density ρ along the radius r. From Motz's (1952) computations, it results that in Figure 1, the points corresponding to different models lay around the curve of polytropic models. Therefore this curve can be approximately used as an interpolation curve in order to obtain the non-dimensional gyration radius of the star from the known effective polytropic index of the corresponding model.

3.3. USE OF APSIDAL MOTION CONSTANT

For the close binary systems with known apsidal line motion, we can obtain from the observations the value of (mean) apsidal motion constant, $K_{2\,\mathrm{aps}}$, of the system. Table I gives the values of the apsidal motion constant $K_{2\,\mathrm{aps}}$ (Russell, 1939; Motz, 1952) and the values of the non-dimensional gyration radius k as functions of polytropic index n.

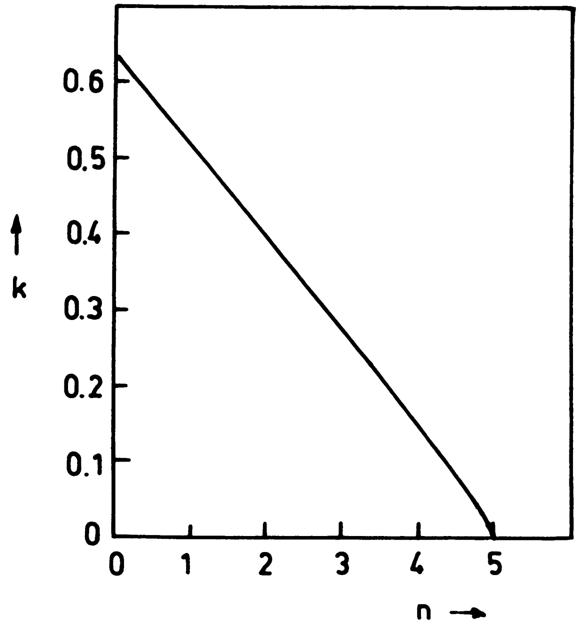

Fig. 1. Variation of the non-dimensional gyration radius k with polytropic index n.

TABLE I

Values of gyration radius and of apsidal motion constant

n	k^2	Gyration radius k	Apsidal motion constant $K_{2\,\text{aps}}$	$K^{1/4}_{2\,\text{aps}}$
0	0.4	0.6325	0.75	0.9306
1	0.26138	0.5113	0.25992	0.7140
1.5	0.20502	0.4528	0.14460	0.6167
2	0.15704	0.3963	0.07410	0.5217
2.5	0.11203	0.3347	–	–
3	0.07583	0.2754	0.01440	0.3464
3.5	0.04558	0.2135	0.00470	0.2618
4	0.02358	0.1536	0.00134	0.1913
5	0	0	0	0

Having the value $K_{2\,\text{aps}}$ from the observations, we can obtain the effective polytropic index, and from this we can obtain the non-dimensional gyration radius. Figure 2 gives the variation of the non-dimensional gyration radius k as function of parameter $K^{1/4}_{2\,\text{aps}}$. It allows the direct determination of the non-dimensional gyration radius k from the observed apsidal motion constant.

4. Statistical Evaluation of ratio F

It is difficult to obtain, with sufficient accuracy, the non-dimensional gyration radii of the components for a real close binary system, because we do not know exactly their internal structure. Therefore we shall proceed to a statistical evaluation of the ratio F.

For the rotational angular velocities of the close binary system components, Kopal (1965) gives the following statistical relation

$$\tilde{\omega}_i^2 \equiv \left(\frac{\omega_i}{\omega_k}\right)^2 = \frac{1+e}{(1-e)^3}\,. \tag{15}$$

From the relations (8) and (15) it follows that

$$F = \frac{(1+e)^{1/2}}{(1-e)^2} f(q) \tag{16}$$

where

$$f(q) = \left(1+\frac{1}{q}\right)k_1^2 r_1^2 + (1+q)\,k_2^2 r_2^2\,. \tag{17}$$

The factor depending on e in the expression (16) has little variation, because the close binary systems have orbits with small eccentricities; the majority of the orbits are circular or quasicircular. From the catalogue of Kopal and Shapley (1956) it is clear that, generally, $e \lesssim 0.2$. Therefore we can consider that the factor depending on e in (16) is generally between 1 and 2. This means that study of the variation of F is reduced to study of the

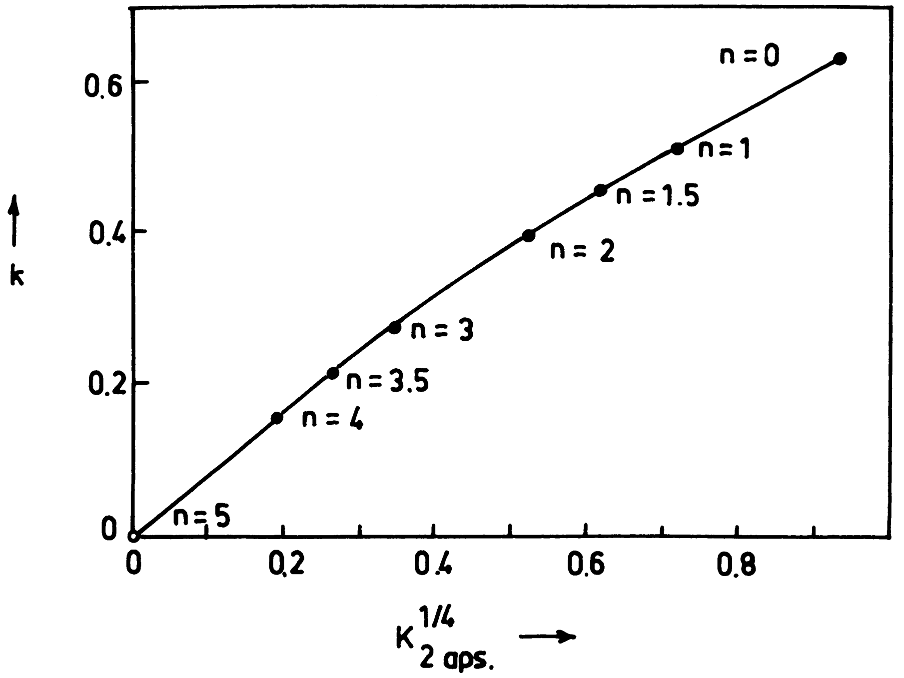

Fig. 2. Variation of the non-dimensional gyration radius k with the parameter $K_{2\,\mathrm{aps}}^{1/4}$ (where $K_{2\,\mathrm{aps}}$ is the apsidal motion constant).

variation of the function $f(q)$. In order to evaluate the function $f(q)$ we shall consider separately the different types of close binary systems.

4.1. DETACHED SYSTEMS

In this case we consider some statistical delimitations for the fractional radii of the components. Detached system components are generally stars of the main sequence, which satisfy the mass-radius relation. We shall use the mass-radius relation given by Svechnikov (1969), which can be written approximately thus

$$R \simeq 1.05 M^{3/4} \tag{18}$$

The relation between the semimajor axis of the relative orbit, a, and the sum of the masses of the close binary system components, $M_1 + M_2$, is given by Svechnikov in a diagram, from which we can see that the majority of the detached systems lie between the straight lines

$$\begin{aligned} \log a &= 0.35 + 0.75 \log (M_1 + M_2) \\ \log a &= 0.95 + 0.75 \log (M_1 + M_2) \end{aligned} \tag{19}$$

that is

$$2.24 (M_1 + M_2)^{3/4} \lesssim a \lesssim 8.91 (M_1 + M_2)^{3/4} \tag{20}$$

From the relations (6), (18) and (20) it results that

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$$\frac{0.12}{(1+q)^{3/4}} \lesssim r_1 \lesssim \frac{0.47}{(1+q)^{3/4}} \tag{21}$$

$$\frac{0.12\,q^{3/4}}{(1+q)^{3/4}} \lesssim r_2 \lesssim \frac{0.47\,q^{3/4}}{(1+q)^{3/4}}. \tag{22}$$

Considering that the two components have a similar internal structure, we can put, approximately, $k_1^2 \simeq k_2^2 = k^2$. From relations (17), (21) and (22) we obtain

$$\text{(L)} \quad \frac{0.014(1+q^{5/2})}{q(1+q)^{1/2}} k^2 \lesssim f(q) \lesssim \frac{0.22(1+q^{5/2})}{q(1+q)^{1/2}} k^2 \quad \text{(U)} \tag{23}$$

where by (L) is noted the lower limit and by (U) the upper limit for $f(q)$. One observes that in this case $f(q) = f(1/q)$, that is in a logarithmic scale we have symmetry with respect to the straight line $\log q = 0$. Therefore here it has no importance which component is named primary and which secondary.

In Table II, the numerical values for the lower and upper limits of the function $\log f(q)$ are given, for $\log q \in [-2, 2]$ and taking the values of k^2 corresponding to the polytropic indexes 1.5; 3 and 4. The results obtained are presented graphically in Figure 3.

Let us consider that $J_{\rm rot}$ is negligible if $F = J_{\rm rot}/J_{\rm orb} \lesssim 10\%$ and is not negligible if $J_{\rm rot}/J_{\rm orb} > 10\%$. From the observational data it results that, generally, q is between 0.1 and 10; that is $\log q$ is between -1 and 1. On the other hand, from the study of the apsidal line motion, it results that the effective polytropic index of the close binary stars is between 3 and 4 (Kopal, 1965; Mathis, 1967). So, one observes from Figure 3 that for the detached systems, in the great majority of the cases, $J_{\rm rot}$ is negligible. But there can exist detached systems for which $q \sim 0.1$, the dimensions of the components would correspond to the limit (U), and the concentration of the matter would be moderate ($n = 3$). Then $\log f(q) \simeq -0.8$ and $J_{\rm rot}/J_{\rm orb} \simeq 16\%$. This means that the momentum $J_{\rm rot}$ is not negligible.

4.2. CONTACT SYSTEMS

In this case both components are in contact with the Roche critical equipotential surface. For mean radius of the Roche lobe, we can use the approximate formulae of Paczyński (1966, 1971), that is

$$r_1 = 0.38 - 0.2 \log q, \quad r_2 = 0.38 + 0.2 \log q \tag{24}$$

TABLE II

Values of $\log f(q)$ as function of $\log q$ for detached systems

$\log f(q)$ \ $\log q$		0	±0.3	±0.6	±0.9	±1.0	±1.3	±2.0
for $n = 1.5$	(L)	−2.40	−2.26	−1.96	−1.66	−1.57	−1.25	−0.54
	(U)	−1.19	−1.06	−0.78	−0.47	−0.37	−0.06	0.65
for $n = 3$	(L)	−2.82	−2.70	−2.40	−2.10	−2.00	−1.68	−0.98
	(U)	−1.62	−1.50	−1.21	−0.90	−0.80	−0.49	0.22
for $n = 4$	(L)	−3.35	−3.22	−2.92	−2.60	−2.52	−2.22	−1.48
	(U)	−2.16	−2.00	−1.72	−1.41	−1.30	−1.00	−0.29

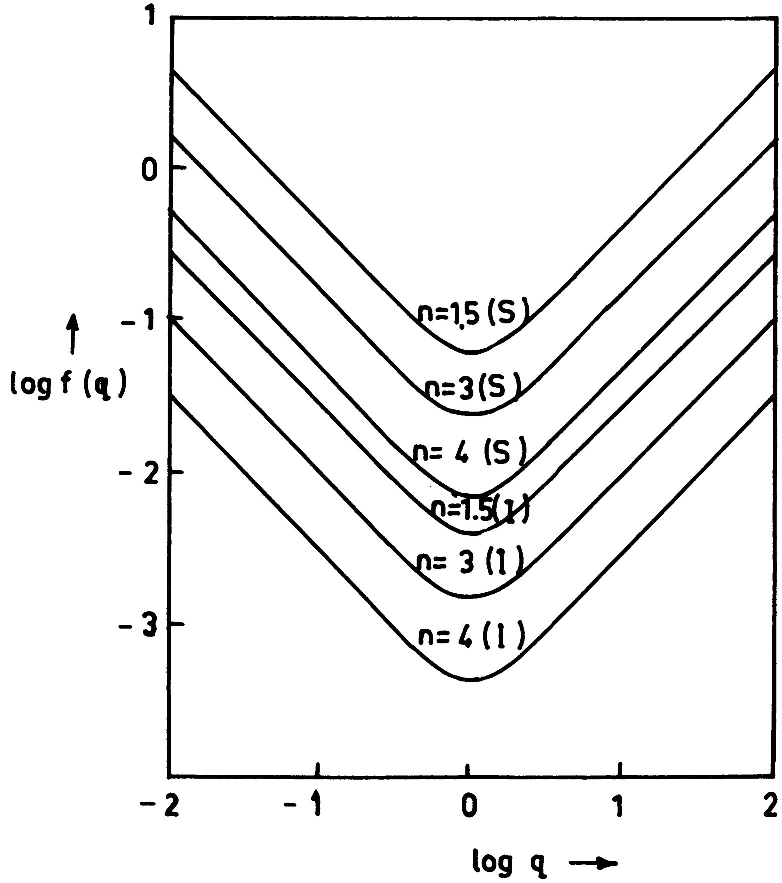

Fig. 3. Variation of log $f(q)$ with log q for detached systems. S and I denote upper and lower limits.

These expressions give an error less than 1% for $0.2 < q < 10$. For our qualitative conclusions we can extrapolate these formulae within the limits given by the conditions $r_1 \geqslant 0$, $r_2 \geqslant 0$; therefore we shall consider $-1.9 \leqslant \log q \leqslant 1.9$.

Taking again $k_1^2 \simeq k_2^2 = k^2$, expression (17) becomes

$$f(q) = \left\{ \left(1 + \frac{1}{q}\right) (0.38 - 0.2 \log q)^2 + (1 + q)(0.38 + 0.2 \log q)^2 \right\} k^2 \quad (25)$$

In this case we have also $f(q) = f(1/q)$. In the Table III, the numerical values of the function $\log f(q)$ are given for $\log q \epsilon [-1.9, 1.9]$ and taking the values of k^2 corresponding to the polytropic indexes 1.5; 3 and 4. The results obtained are presented graphically in Figure 4. One observes that for $q \sim 0.1$, (or $q \sim 10$), and $n \sim 3$ the angular momentum J_{rot} is not negligible.

4.3. SEMIDETACHED SYSTEMS

This is an intermediate case with respect to the preceding two. The different positions of the close binary system components with respect to the Roche critical equipotential sur-

TABLE III

Values of log $f(q)$ as function of log q for contact systems

log q / log $f(q)$	0	±0.3	±0.6	±0.9	±1.0	±1.3	±1.9
n = 1.5	−0.93	−0.82	−0.56	−0.23	−0.12	0.25	0.98
n = 3	−1.36	−1.25	−1.00	−0.66	−0.55	−0.18	0.55
n = 4	−1.85	−1.76	−1.50	−1.17	−1.06	−0.69	0.04

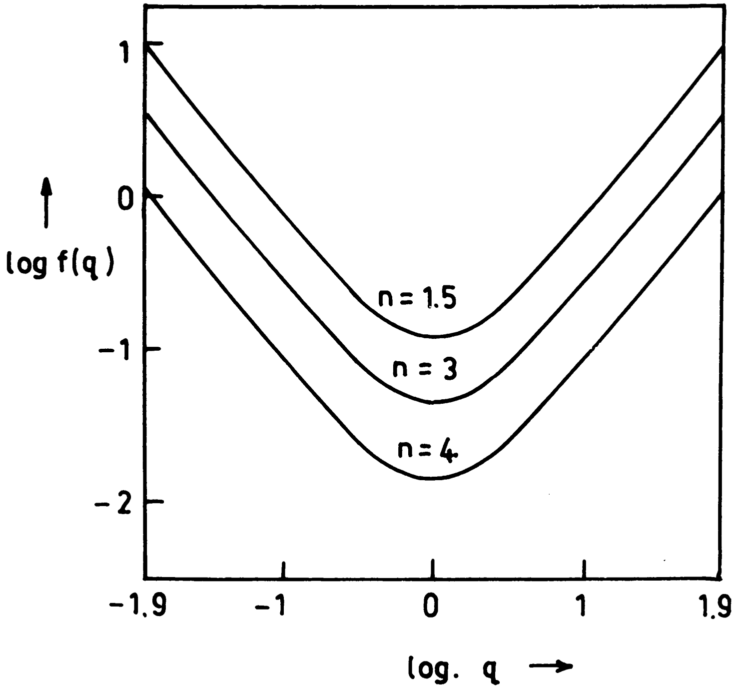

Fig. 4. Variation of log $f(q)$ with log q for semidetached systems.

face (Roche limit) will carry to the loss of the symmetry of log $f(q)$ as function of log q. On the other hand the internal structure of the components can differ much, but we shall neglect this last fact.

Now we shall define the mass ratio thus: $q = M_{\text{cont}}/M_{\text{det}}$, where M_{cont} is the mass of the component in contact with the Roche critical equipotential surface and M_{det} is the mass of the component detached from the same surface. The fractional radii of the components are

$$\left.\begin{aligned} r_{\text{cont}} &= 0.38 + 0.2 \log q \\ \frac{0.12}{(1+q)^{3/4}} &\leqslant r_{\text{det}} \leqslant \frac{0.47}{(1+q)^{3/4}}\,. \end{aligned}\right\} \qquad (26)$$

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Taking again $k_1^2 \simeq k_2^2 \simeq k^2$, from (17) we obtain

$$\text{(L)} \quad \left[(1+q)(0.38+0.2\log q)^2 + \frac{0.014}{q(1+q)^{1/2}} \right] k^2 \leqslant f(q) \leqslant$$
$$\leqslant \left[(1+q)(0.38+0.2\log q)^2 + \frac{0.22}{q(1+q)^{1/2}} \right] k^2 \quad \text{(U)} \qquad (27)$$

where by (L) and (U) the lower and upper limits of the function $f(q)$ are noted. In the Tables IV and V, the numerical values of these limits are given, for $\log q \epsilon [-1.9, 1.9]$ and taking the values of k^2 corresponding to the polytropic indexes 1.5; 3 and 4. The results obtained are presented graphically in Figure 5.

For $-1.9 \leqslant \log q \leqslant 0$ we have the case when the secondary component (the less massive one) is in contact (Table IV). If q is sufficiently small, the first terms from the square brackets, in (27), are negligible, therefore for $\log q \rightarrow -1.9$ Figure 5 is similar to Figure 3 (the case of detached systems).

For $0 \leqslant \log q \leqslant 1.9$ we have the case when the primary component (the more massive one) is in contact (Table V). If q is sufficiently large, the first terms from square brackets, in (27), dominate, therefore for $\log q \rightarrow 1.9$ Figure 5 is similar to Figure 4 (the case of contact systems).

TABLE IV

Values of $\log f(q)$ as function of $\log q$ for semidetached systems (secondary component in contact)

log f(q) \ log q		0	−0.3	−0.6	−0.9	−1.0	−1.3	−1.9
n = 1.5	(L)	−1.22	−1.44	−1.55	−1.51	−1.46	−1.25	−0.65
	(U)	−1.04	−0.98	−0.75	−0.46	−0.36	−0.06	0.55
n = 3	(L)	−1.64	−1.89	−2.00	−1.99	−1.96	−1.68	−1.08
	(U)	−1.47	−1.41	−1.18	−0.89	−0.80	−0.49	0.12
n = 4	(L)	−2.16	−2.40	−2.52	−2.50	−2.46	−2.22	−1.59
	(U)	−2.00	−1.92	−1.68	−1.40	−1.30	−1.00	−0.39

TABLE V

Values of $\log f(q)$ as function of $\log q$ for semidetached systems (primary component in contact)

log f(q) \ log q		0	0.3	0.6	0.9	1.0	1.3	1.9
n = 1.5	(L)	−1.22	−0.92	−0.59	−0.24	−0.12	0.25	0.98
	(U)	−1.04	−0.88	−0.58	−0.24	−0.12	0.25	0.98
n = 3	(L)	−1.64	−1.36	−1.02	−0.67	−0.55	−0.19	0.55
	(U)	−1.47	−1.31	−1.01	−0.67	−0.55	−0.19	0.55
n = 4	(L)	−2.16	−1.85	−1.52	−1.17	−1.06	−0.69	0.04
	(U)	−2.00	−1.82	−1.52	−1.17	−1.06	−0.69	0.04

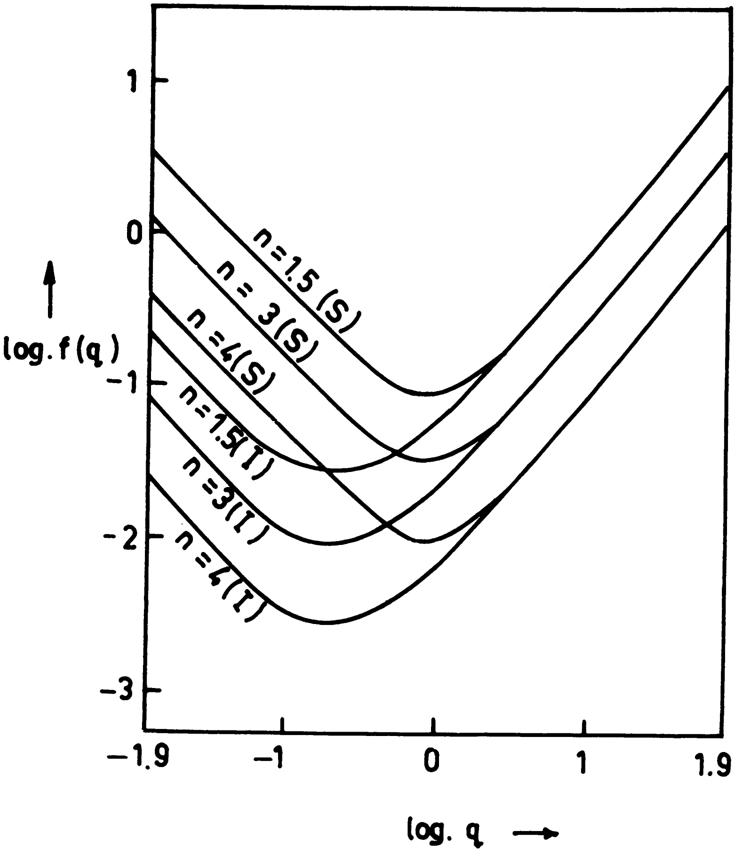

Fig. 5. Variation of log $f(q)$ with log q for semidetached systems. S and I denote upper and lower limits.

If q differs sufficiently from unity, and the concentration of the matter towards the star centres is moderate ($n \sim 3$), then the ratio J_{rot}/J_{orb} is not negligible, that is the angular momentum J_{rot} is not negligible.

4.4. SYSTEMS CONTAINING A SUPERNOVA REMNANT COMPONENT AND A CONTACT COMPONENT

This case can be considered as a limit case of a semidetached system. The radius of the supernova remnant component can be neglected, that is $r_{det} \simeq 0$. Then (27) is reduced to

$$f(q) = (1+q)(0.38 + 0.2 \log q)^2 k^2 \qquad (28)$$

In Table VI, the numerical values of the function $\log f(q)$ are given, for $\log q \epsilon [0, 1.9]$ and taking the values of k^2 corresponding to the polytropic indexes 1.5; 3 and 4. The results obtained are presented graphically in Figure 6. One observes that for $q \sim 10$ and $n \sim 3$ the angular momentum J_{rot} is not negligible.

For example, for the system HD 153919 (2U 1700–37), taking $q = 20$ (Wolff and

TABLE VI

Values of log $f(q)$ as function of log q for systems containing a supernova remnant component

log $f(q)$ \ log q	0	0.3	0.6	0.9	1.0	1.3	1.9
$n = 1.5$	−1.23	−0.92	−0.59	−0.24	−0.12	0.25	0.98
$n = 3$	−1.66	−1.36	−1.02	−0.67	−0.55	−0.19	0.55
$n = 4$	−2.17	−1.86	−1.53	−1.17	−1.06	−0.69	0.04

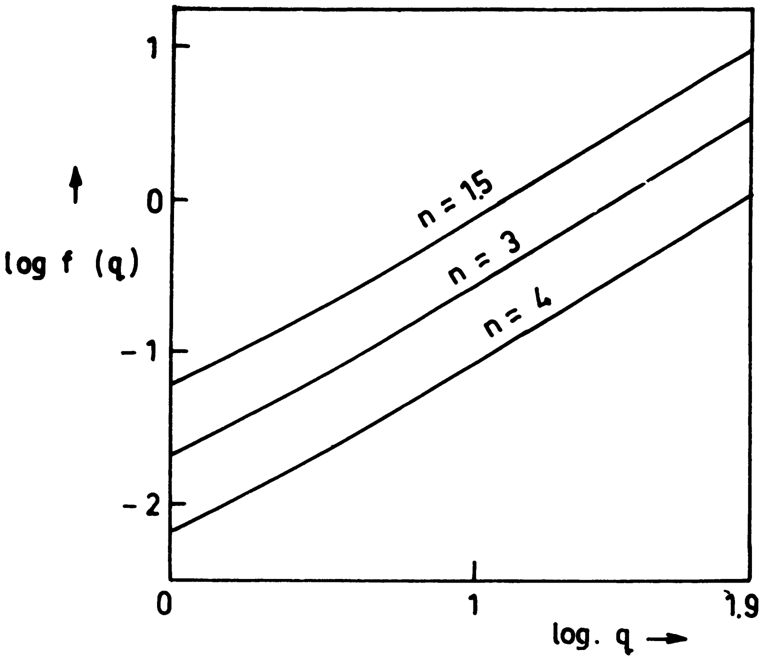

Fig. 6. Variation of log $f(q)$ with log q for systems containing a supernova remnant component and a contact component.

Morrison, 1974) and $3 \leqslant n \leqslant 4$, it results $0.65 \geqslant f(q) \geqslant 0.20$. This means that J_{rot} is comparable with J_{orb} for this system.

5. Conclusions

From the results obtained we see that the ratio J_{rot}/J_{orb} is sensitive to the variation of the mass ratio q. The rotational angular momentum J_{rot} is not negligible, if q differs much from unity and if the concentration of the stellar matter is moderate ($n \sim 3$). The non-synchronism between rotation and revolution (if $\tilde{\omega}_i > 1$) as well as the presence of rings or disks around the components of some close binary systems will contribute to the increase of the ratio J_{rot}/J_{orb}. During the evolution of a close binary system, this ratio

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varies on the one hand because of the variation of the internal structure (non-dimensional gyration radii) and of the dimensions of the components, and on the other hand because of the variation of the mass ratio. The last variation begins at the moment when the initially more massive component reaches the Roche limit. So, in some phases of the evolution, the rotational angular momentum of the system can increase until it becomes a considerable fraction of total angular momentum. In such phases the neglect of J_{rot} is not justified. In these phases a redistribution of the angular momentum takes place, the mass transfer between the components being accompanied by the angular momentum transfer. Evidently, the mass loss from the system will also affect the distribution of the angular momentum.

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