payable yearly, for $\left[(1+i)^{\frac{1}{n}}\right]^n = (1+i)$ equally for all values of n.

In my former communication, then, I was content to allow Mr. Sang the credit of having avoided this error; but, after the express testimony of his friend Mr. Thomson, we are forced upon the consideration that Mr. Sang's use of the facility of dividing the yearly logarithm, coupled with his own silence on the subject, are not circumstances wholly beyond reproach, when viewed in connection with the construction of numerical tables upon which others may have to depend.

London, May 1853. EDWIN JAS. FARREN.

ON THE SAME SUBJECT AS THE FOREGOING.

To the Editor of the Assurance Magazine.

SIR,—Observing in No. XI. of your *Magazine* a paper by Mr. E. J. Farren, containing some very neat and concise remarks "On the Period intervening between the date of Death and Payment of Sum assured," I feel called upon to say a few words on the subject, more particularly with reference to the mode in which it was treated by Mr. Sang, in the hope that you will give them a place in your pages.

I have since minutely examined the construction of Mr. Sang's Assurance Table, and am unable to discover what precise meaning he attached to the element introduced by him ($\sqrt{1\cdot03}$) to adjust the values for sums payable at the *instant* of death. If it was intended to express the amount of £1 with six months' interest at 3 per cent. per annum, he must have assumed the conversion of interest to be twice in the year; for a somewhat smaller rate of interest than 3 per cent. per annum, if improved half yearly, will be sufficient to bring up the quantity $\sqrt{1\cdot03}$ to the desired amount, 1·03, at the end of one year. In short, the half yearly rate of interest is $\sqrt{1\cdot03}-1$, or ·014889. But I notice that Mr. Sang, in the illustration of his published tables, in conformity with the practice and principles of previous computers, assumes the interest to be accumulated yearly, in which case the amount of £1 with six months' interest must be held to be 1·015, and not $\sqrt{1\cdot03}$.

This will be more evident on examination of his formula representing the value of an assurance of £1 payable at the *instant* of death, viz.—

$$\frac{v^{\frac{1}{2}}d_x + v^{1+\frac{1}{2}}d_{\overline{x+1}} + v^{2+\frac{1}{2}}d_{\overline{x+2}}}{l_x}, \text{ \&c.,}$$

which is equal to

$$\frac{1}{\sqrt{1+i}} \times \left(\frac{vd_x + v^2d_{\overline{x+1}} + v^3d_{\overline{x+2}}}{l_x}, \text{ \&c.}\right)$$

In this formula, the quantity enclosed within ( ), being the ordinary expression for the value of an assurance payable *six months after* death (see Jones, vol. i., page 155), most clearly assumes the interest to be capitalized only once a year. Hence the other element in that expression, or $\frac{1}{\sqrt{1\cdot03}}$, the present value of £1 payable six months hence, at ·014889 per cent.

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interest for each half year, will make the value of the assurance at the instant of death somewhat too great. It appears to me, therefore, that the symbol $\sqrt{1+i}$ is not a proper element in the consideration of this subject, in the note appended to my paper read some time ago before the Institute of Actuaries, where I took occasion to refer to this matter; but since reading Mr. Farren's paper, I am persuaded that it contains the correct solution of the problem.

I observe in the same number of the *Magazine* there is a very interesting paper "On the Valuation of Life Contingencies," by Dr. Charles Jas. Hargreave, who seems to have held views similar to those of Mr. Farren and Professor De Morgan, as to the meaning of the quantity $1-i\text{A}$. In page 214 it is stated, "When we represent by $\text{M}(1-\text{A}_n)$ the value of a sum M payable on the death of A_n, we mean that it is payable at the commencement of the year of A's death; and if it be not payable until the end of that year, we must take a year's discount from it"—that is, we must divide it by $1+i$: and if payable at the instant of A's death, we must of course take half a year's discount, or divide it by $1+\frac{i}{2}$. It will easily be perceived that $(1-\text{A}_n)$ here, and $(1-i\text{A})$ in Mr. Farren's paper, are identical expressions for the value of the reversion payable at the *beginning* of the year of A's death; and I have no doubt that your readers will have observed also the singular agreement of the writers on the subject now under consideration.

The object of the present inquiry seems to me to rest on the correct determination of the discount of £1 for half a year—a matter certainly of no great difficulty; for if ·03 be the interest for one year, one half, or ·015, must be the interest for half a year, and therefore the rate supposed to be paid on the sum assured from the *instant* of death to six months after death, when the claim is settled.

The expression for the value of an assurance of £1 payable at the *instant* of death, correctly stated, will therefore be

$$\frac{1}{1+\frac{i}{2}} \times \left(\frac{vd_x + v^2d_{x+1} + v^3d_{x+2}}{l_x}, \text{ \&c.}\right);$$

and the most convenient form for the construction of the table, according to the mode pursued by Mr. Sang, will be to write the colog. of 1·015 or 0·0064660 on a piece of card, and add it to every value of log. v^x, adding the sum to log d_x, and then proceeding as described in my former paper. The values thus brought out will necessarily be less than those given by Mr. Sang, though after all the difference is not great, the ratio being only as 1·014889 to 1·015, or as 1 to 1·000109.* Mr. Sang's tables, therefore, though slightly erroneous in certain parts, and perhaps only in the opinion of a few, may practically be considered as valuable as ever they were, the difference being of so small moment.

At all events, Mr. Todd, in his *Investigation Tables*, having eliminated the quantity $\sqrt{1\cdot03}$ in precisely the same way in which it was introduced, has not only removed the principal objection to the use of Mr. Sang's

* That is, Mr. Sang's tables give the premiums for insuring £1000. 2*s*. 2*d*. at the *instant* of death, in place of £1000, with the values of a corresponding policy, other sums being in proportion.

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tables, with reference to the value of policies, but has, in my opinion, freed them to the same extent from a slight existing inaccuracy—a consideration which perhaps should tend to make Mr. Todd's tables the more valuable.

I am, Sir,

Your very obedient Servant,

North British Insurance Office, DAVID CHISHOLM.
Edinburgh, 27th May, 1853.

ON THE SAME SUBJECT.

To the Editor of the Assurance Magazine.

SIR,—Having recently perused the introduction to the most valuable work of W. T. Thomson, Esq., I met, in page 25, with a foot note referring to a certain paper by Mr. Farren, as inserted in the last number of your *Magazine*, and was accordingly induced to read that paper itself.

To speak candidly, I read the paper alluded to over and over again, but to my mortification could not discover the results which Mr. Farren is anxious to deduce. This circumstance causes me to apply humbly to you for an explanation on the subject, either by Mr. Farren himself or any of your mathematical readers.

First Mr. Farren says, that Simpson and Dodson imagined that De Moivre assigned $1-i\mathrm{A}$ as the present value of £1 payable at the *end* of the year of death, while he (Mr. Farren) has reason to assert that no such error (?) could emanate from so celebrated an analyst as De Moivre—satisfying himself, that by $1-i\mathrm{A}$ is meant the present value of £1 due at the *beginning* of the year of death. In support of his argument, Mr. Farren quotes part of a paragraph from De Moivre's work, saying, "This conclusion may be deduced from the method he (De Moivre) has adopted in solving the following problem (XVI.), as it occurs in his *Treatise on Annuities*." Now, unfortunately, the works of Simpson and Dodson are not in my possession; but on perusing De Moivre's work itself, I must confess my inability to trace Mr. Farren's conclusions. The paragraph in question of De Moivre's, *in extenso*, runs thus:—

"*Problem* XVI.—A borrows a certain sum of money, and gives security that it shall be repaid at his decease, with the interests. To fix the sum which is then to be paid, let the sum borrowed be s; the life of the borrower, M years' purchase; d the interest of £1: then the sum to be paid at A's decease will be $\frac{s}{1-d\mathrm{M}}$; thus, supposing $s=800$, $\mathrm{M}=11\cdot83$, $d=0\cdot05$, then $\frac{s}{1-d\mathrm{M}}$ would be found $=$£1,958. In the same manner, if the sum to be paid at A's decease was to be an equivalent for his life, unpaid at the time of the purchase, that sum would be $\frac{\mathrm{M}}{1-d\mathrm{M}}=$ £2,895, supposing the annuity received to be £100, as also the life of A 11·83 years' purchase."

You will perceive that the two examples just named—rather essential in the present case—are omitted in the extract made by Mr. Farren, who,

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