## A ERV MODEL FOR STELLAR MAGERTISS

P.F. Browne
Department of Physics, UMIST, Manchester M60 1QD.

ABSTRACI Different drift velocities of electrons and ions in response to viscous forces exerted by neutral atoms generate current density $j$ and magnetic field B, where $B$ is proportional to vorticity $\omega$ of the fluid. Magnetic vortex tubes (MVTs) form arrays on a hierarchy of scales. MVTs are basic to the magnetism of all astrophysical systems, conferring a structure of aligned filaments. In the Sun a torsional oscillation generates oscillatory vorticity, and hence an oscillatory magnetic field. The same mechanism is proposed for the Ap stars, but with "pole-on" viewing. Resonance-line radiation pressure segregates elements within MVTs of Ap stars, where the anomalous concentrations are preserved. However, variation of the 30 MG magnetic fields of AM Her white dwarfs may be due to precession of an MVT. There is reason to attribute common magnetic flux to the Sun, Ap stars, white dwarfs and neutron stars

## 1. INTRODUCTION

In a differentially rotating partially ionized plasma the different drift velocities of electrons and ions in response to viscous forces exerted by non-gyrating neutral atoms leads to current density ( $0, \mathrm{j}_{\phi}, 0$ ) with magnetic field ( $0,0, \mathrm{~B}_{2}$ ), where components are cylindrical polar. It can be shown that
 1968; 1986)

$$
\begin{equation*}
\underline{B}=b \omega, \quad b=\sqrt{2} \pi e \theta / 3 c Q_{0} Q_{n} n_{n} \tag{1}
\end{equation*}
$$

where $Q_{c}$ is the cross section for electron-ion collisions, $Q_{n}$ that for collisions between neutral particles, $n_{n}$ is the number density of neutral particles, and $\theta$ is a dimensionless quantity whose value approximates unity provided that electron-neutral and ion-neutral collisions have crass sections $\approx Q_{n}$. For temperature $T$,

$$
\begin{equation*}
Q_{0}=\Lambda \pi p_{0}^{2}, \quad P_{0}=e^{2} / 3 \mathrm{kT}, \quad \Lambda=8 \ln \left(P_{m} / P_{c}\right) \tag{2}
\end{equation*}
$$

where $p_{m}^{2}=k T / 4 \pi n_{e} e^{2}$, the Debye length.
If $T \simeq 10^{4} \mathrm{~K}$ then $p_{0}=5.6 \times 10^{-8} \mathrm{~cm}$, and if $n_{e}=10^{8} \mathrm{~cm}^{-3}$ then $p=6.9 \times 10^{-2} \mathrm{~cm}$. It follows that $\Lambda=112$, and hence that $Q_{0}^{m} \simeq 10^{-12} \mathrm{~cm}^{2}$. Taking $Q_{n} \simeq 3 \times 10^{-16} \mathrm{~cm}^{2}, n_{e}=10^{8} \mathrm{~cm}^{-3}$, and $\theta \simeq 1$, (1) yields $b \simeq 0.8 \mathrm{G} \mathrm{s}$. Adopting $\omega=1400 \mathrm{~s}^{-1}$, which is vorticity measured on the sun (Brandt et al., 1988), the prediction from (1) is $B=1.1 \mathrm{~kg}$. Such elemental flux tubes are basic not only to the sun, but to all astrophysical systems. All fields are filamentary, being concentrations of elemental "magnetic vortex tubes" (MVTs).

The proportionality $B \mathcal{C} \underline{\propto}$ remains valid as the fluid velocity field $\underline{u}(r, \phi, z, t)$ distorts lines of $\underline{\omega}$ and $\underline{B}$, because

$$
\begin{align*}
& \delta \underline{\omega} / \delta t=\underline{\nabla} \times(\underline{u} \times \underline{\omega})-K_{\omega} \nabla^{2} \underline{\omega}  \tag{3a}\\
& \delta \underline{B} / \delta t=\underline{\nabla} \times(\underline{u} \times \underline{B})-K_{B} \nabla^{2} \underline{B} \tag{3b}
\end{align*}
$$

Where $K$ is diffusivity of field lines through the fluid. The time scale $t$ for significant change of $\underline{\omega}$ or $\underline{B}$ is obtained from the first and last terms of (3), writing $\delta / \delta t \simeq 1 / t$ and $\nabla^{2} \simeq$ $1 / L^{2}$, so that $t \simeq L^{2} / K$. Normally $K_{i v} \gg K_{B}$.

A crucial phenomenon, not at present recognized in astrophysics, is the role of hierarchical vorticity. The nonlinear term ( $\underline{u} . \underline{\underline{\eta}}$ ) $\underline{\text { in }}$ in the equations of motion transfers energy between vortices of different scale lengths $L$ without dissipation (Kraichnan and Montgomery, 1980). The energy oscillates back and forth among all scales. Dissipation rate varies as $L^{-2}$. In the steady state a dissipation on some scale $L_{d}$ balances input on a larger scale $L_{0}$. It seems that exchange of energy between vorticity and magnetic field can occur more rapidly than the magnetic variations.

Current density $\frac{j}{}$ and magnetic field $B$ in an MVT both are helical. Magnetic force is $j \times B=\left(j_{\phi} B_{z}-j_{z} B_{\phi}\right) \hat{r}$, assuming $j_{r}=B_{r}=0$, with $\hat{\underline{r}}$ a unit vector. Thus MVTs pinch or expand depending on whether $j_{z} B_{\phi}$ or $j_{\phi} B_{z}$ is dominant. This is also the condition for attraction or repulsion between parallel

MVTs of the same polarity. Thus attraction occurs when twisting of field lines is increasing, and repulsion when twisting is decreasing. An example is aggregation of magnetic knots during formation of sunspots, followed by dispersal during decay.

MVTs originate at an early epoch of stellar evolution specifically, when bipolar outflows and discs are associated with the star (Browne, 1992). A rotating gas cloud collapses into a disc with approximately Keplerian velocity field, $u_{\phi}(r) \propto r^{-1 / 2}$. Because $\omega^{2}$ is transferred rapidly between scales of hierarchical vorticity, and bacause viscous damping time varies as $L^{2}$, the velocity field quickly evolves to that for zero vorticity, namely $u_{\phi}(r) x^{\prime} r^{-1}$. How angular momentum is equally shared among particles of the same mass (say 1 per unit mass). The transfer of angular momentum inwards strongly increases centrifugal force $1^{2} / r^{3}$. Centrifugal force becomes dominant for $r$ < $r$ *, and gravitational force becomes dominant for $r>r^{*}$. Gas at radii near $r^{*}$ is squeezed, leading to increased pressure gradient $\delta \mathrm{p} / \delta \mathrm{z}$ in the axial directions. When $\delta \mathrm{p} / \delta \mathrm{z}>\mathrm{g}_{2}$, where $\mathrm{g}_{\mathrm{z}}$ is the axial component of gravity, a bipolar outflow is driven hydro-dynamically. The phenomenon is analogous to a terrestial tornado.

The outflow is maintained by an inflow of gas in the disc, and conservation of angular momentum of indrawn gas maintains the vorticity. The core of the MVT channels a high velocity outflow, because here there is least mass to entrain due to centrifuging. Angular momentum as well as mass is lost during the outflow. If the direction of $\underline{l}$ changes as outlying gas begins to be expelled, then the MVT will precess about the new direction of $\underline{l}$ because vortex lines comove with the fluid.

## 2. SOLAR MAGMETIC CYCLE

It came as a surprise that all of the magnetic flux through the surface of the Sun is concentrated into isolated tubes of diameter $\simeq 300 \mathrm{~km}$ within which the field strength is $\simeq 1500 \mathrm{G}$, corresponding to flux $\simeq 10^{18} \mathrm{Mx}$. The number density of these flux tubes determines the mean field, perhaps 5 G in quiet regions. Flux tubes of diameter $\simeq 300 \mathrm{~km}$ intersect the solar surface at "magnetic knots". Magnetic knots tend to accummulate around the periphery of supergranules of diameter $\simeq 30,000 \mathrm{~km}$, forming the "magnetic network". A large sunspot, including its penumbra, has diameter $\simeq 30,000 \mathrm{~km}$ and flux $\simeq 3 \times 10^{22} \mathrm{Mx}$.

Brandt et al. (1988) report evidence for a classical vortex in the surface velocity field of the Sun. Vorticity of $\omega \simeq 1400 \mathrm{~s}^{-1}$ is constant out to $\mathrm{R} \simeq 1500 \mathrm{~km}$, and for $\mathrm{r}>\mathrm{R}$ circulation is constant at $\simeq 4000 \mathrm{~km}^{2} \mathrm{~s}^{-1}$. Also peak rotational
velocity is $u_{q}(R)=0.46 \mathrm{~km} s^{-1}$. Outlying elements spiralled to the center with velocities $0.27-0.60 \mathrm{~km} \mathrm{~s}-1$.

Since $\underset{\sim}{B} \propto \underline{\omega}$, the 22 y oscillation of $B$ implies a 22 y oscillation of $\underline{\omega}$. The variation of $\omega$ can be attributed to a "torsional oscillation" (Howard and Labonte, 1980). Cylindrical layers of fast and slow rotation alternate from the axis outward, intersecting the surface in different latitude zones. Such oscillatory nonuniform rotation with angular velocity $\Delta \Omega_{2}(r, t)$ is superimposed on steady nonuniform rotation $\Delta \Omega_{1}(r)$, and on uniform rotation $\Omega_{0}$. The sense of vorticity due to $\Delta \Omega_{2}(r, t)$ reverses from one 11 y half-cycle to the next. The torsional oscillation may be coupled to weak pulsation due to angular momentum conservation.

The steady component of differential rotation $\Omega_{i}(r)$ was exploited by Babcock (1961) in an explanation of sunspot polarity laws. The different rotation speed of an equatorial outer layer stretches a kink in an axial MVT into an increasingly elongated loop (fig. 1). Because the outgoing and return arms of the loop are on opposite sides of the equator, the Sun has a sub-surface azimuthal magnetic field which reverses polarity from one hemisphere to the other. Secondary loops in the nearly azimuthal flux tube rise and break through the surface at bipolar magnetic regions (BMRs), with preceding (p) and following (f) members of opposite polarity (fig. 1).


Fig. 1. Common model for magnetism of Sun and Ap stars

## 3. KILOGAUSS FIELDS OF AP STARS

The mechanism of the solar cycle is proposed also for the magnetic variations of Ap stars, but with the difference that viewing is "pole-on". Then variation of magnetic field is real, not an aspect effect. The period is that of a torsional oscillation. Observed periods span $2-2500$ d.

The prediction of "pole-on" viewing is supported by the low rotational velocities for Ap stars inferred from their narrow lines. Slettebak (1955) finds a mean of $50 \mathrm{~km} \mathrm{~s}^{-1}$ for Ap stars as opposed to a mean of $176 \mathrm{~km} \mathrm{~s}^{-i}$ for normal A stars.

It can be shown that resonance-line radiation pressure in the core of a polar MVT is strong enough to force certain elements upward, until self-screening limits the pracess. In this way a cloud of anomalously abundant elements attains quasi-equilibrium at some height above the photosphere. Being trapped in the MVT there is no question of turbulent mixing. Centrifuging varies through the magnetic cycle, causing opacity to vary in the MVT core, which in turn causes resonance-line radiation pressure to vary. Consequently, elements segregated by radiation pressure move upward and downard. In fact two groups of such elements move up and down in antiphase, being confined to MVTs of opposite polarity, which explains why two spectra which vary in antiphase in respect of intensity, line profile, and radial velocity. Such a mechanism was suggested many years ago, but in a different model (Browne, 1968b).

## 4. MEGAGAUSS FIELDS OF WHITE DWARF STARS

White dwarfs have radil about $1 / 100$ th those of Ap stars. If the radius of a polar MVT contracts by this factor, magnetic field of 3 kG is amplified magnetohydrodynamically (flux conservation) to 30 MG.

Magnetic fields in the range $10-100$ MG have been detected on several white dwarfs by Zeeman effect or by cyclotron harmonics. AM Her stars, a subgroup with 16 members, show strong ( $5-20 \%$ ) circular polarization which varies cyclically with a period in the range $80-200 \mathrm{~min}$, occasionally with reversal of sense. Magnetic fields are typically of order 30 MG (Schmidt et al., 1986).

The conventional interpretation of the reversal of sense of circular polarization is alternate viewing of opposite magnetic poles of a dipolar field whose axis is misaligned with the rotation axis. Instead of a rotating dipolar field, we now propose a precesing MVT field (fig. 2). In order that opposite polar caps should come into view alternately, the axis of precesion must be roughly transverse to the line of sight.


Fig. 2. Precessing MVT model for AM Her stars

Convincing evidence for MVT precesssion in these stars comes from their strong emission lines of H and He , which have a narrow peak superimposed on a broad base. The source gas for the peak must differ from that for the base, because the peak and base show different Doppler curves. The Doppler curves severely constrain the model, rather as happens in SS433, A fit is obtained if source gas is trapped in a precessing MVT, that on one side of the star having only precessional motion and that on the other side having outflow motion superimposed on precessional motion.

AM Her stars usually are Xray sources. Assuming that X-rays come from a degenerate sub-layer, both $X$-ray source $X$ and cyclotron source $C$ in fig. 2 should be eclipsed at the same phases, as observed (Szkody et al., 1980).
Two eclipses occur each precession, one when the stellar limb obscures $C$ or $X$ and the other when trapped gas obscures $C$ or X . The latter type of eclipse is recognized by frequencydependence; the absorption cross sections vary as $\omega^{-355}$ for bound-free and as $\omega^{-2}$ for free-free transitions, so that the eclipse can disappear for hard X-rays. On occasion very soft X-rays, with a square wave light curve, are seen (Heise et. al., 1985). Emission phase coincides with eclipse phase for normal flux, suggesting reprocessing of the latter.

## 5. TERRAGAUSS MAGNETIC EIELDS

Cyclotron lines in the range 11 - 35 kev have been observed in absorption in 4 X-ray pulsars, and at $27-70 \mathrm{kev}$ in some $19 \gamma$-ray bursts (Mazets et al., 1981). From $\mathrm{H} \omega_{\mathrm{c}}=35$ kev and $\omega_{\mathrm{L}}=e \mathrm{~B} / \mathrm{mc}$, one infers $\mathrm{B}=3 \mathrm{TG}$ (1 TG $=10^{12} \mathrm{G}$ ).

During evolution of a white dwarf to a neutron star the field lines of the MVT are compressed by a factor $\simeq 300$, so that the field increases from 30 MG to 3 TG ( $1 \mathrm{TG} \equiv 10^{12} \mathrm{G}$ ).

The magnetic flux may be the same for all magnetic stars, and equal to that of a large sunspot (fig, 1). Putting $\pi R^{2} B=$ $3 \times 10^{22} \mathrm{Kx}$, values of B and R are typically ( $3 \mathrm{kG} ; 30,000 \mathrm{~km}$ )
for Ap stars, ( $30 \mathrm{MG} ; 300 \mathrm{~km}$ ) for white dwarfs, and (3 TG; 1 km ) for neutron stars.

A blackbody source at temperature 1 kev with radius 300 km radiates $3 \times 10^{38} \mathrm{erg} \mathrm{s}^{-1}$, parameters typical of an X -ray burst; a blackbody at temperature 100 kev with radius 1 km radiates $3 \times 10^{41}$ erg $s^{-1}$, parameters typical of $\gamma$-ray bursts. In the burst model previously proposed (Browne, 1990) the Xray and $y$-ray fluxes come from the degenerate interiors of white dwarfs and neutron stars respectively, escaping through "windows" provided by polar MVTs after quasi-periodic "blow outs". The basic cause is imbalance between internal power generation and surface luminosity in a degenerate system.

## REFERENCES

Babcock, H. W., 1961, Ap. J. 133, 572.

Brandt, P.N. et al., 1988, Nature 335, 238.

Browne, P.F., 1968a, Ap. Lett. 2, 217.
Browne, P.F., 1968b, Nature 220, 1296.

Browne, P.F., 1986, in Interstellar Magnetic Fields, eds. R, Beck and R. Grave, Springer-Verlag, Heidelberg, p. 220.

Browne, P.F., 1990, in Galactic and Intergalactic Magnetic Fields, eds. R. Beck et al., IAU Colloquium No. 140, P. 136.

Browne, P.F., 1992, in "Stellar Jets and Bipolar Outflows", eds. A. Vittone, D.L. Errico, Kluwer Publ., Dordrecht.

Heise, J. et al., 1985, Astron. Ap. 148, 114.

Howard, R. and LaBonte, B.J., 1980, Ap. J. 239, L33.
Kraichnan, R., Montgomery, D., 1980, Rep. Prog. Phys. 43, 547.
Mazets, E.P. et al., 1981, Nature 290, 378.
Schmidt, G.D., Stockman, H. and Grandi, S., 1986, Ap. J. 300, 804.

Slettebak, A., 1955, Ap. J. 121, 653.

Szkody, P. et al., 1980, Ap. J. 241, 1070.

